Instability of a weak collapse and the generation of non-condensate particles in the collapse of Bose-Einstein condensates

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OUTLINE

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Introduction: history and experimental data

- Study of collapse of the gaseous Bose-Einstein condensates (BECs) became possible due to using the Fano-Feschbach resonance technique when the scattering lengths a_s can be effectively changed from positive to negative values. For $a_s > 0$ interaction of atoms is repulsive and the BECs in traps are stable. In the opposite case, $a_s < 0$, BECs become unstable due to attraction between atoms. This instability results in collapse for condensate atoms, when small "singular" regions with high atomic density are formed. Development of collapse, as was shown in many experiments, is accompanied by escape of almost all atoms from the magnetic traps (more than 50 %). Therefore it was necessary to explain these experimental facts.

Introduction: history and experimental data

- The first explanation was given by Yu.M. Kagan with co-authors in 1997. They suggested the mechanism based on recombination of three atoms with the formation of dimer (like H_2) and one atom carrying out the momentum access.
- When the mean distance between atoms $\sim n^{-1/3}$ much larger the scattering length a_s then, in the leading order relative to small gas parameter $na_s^3 \ll 1$, for temperatures $T \to 0$, the condensate dynamics can be described within the Gross-Pitaevskii (GP) approximation,

$$\hat{H} = \int \mathbf{dr} \left[-\frac{1}{2}\hat{\psi}^\dagger(\mathbf{r})\nabla^2\hat{\psi}(\mathbf{r}) + \frac{U_0}{2}\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r}) \right],$$

where $U_0 = 2\pi a_s$ and $\hbar = m = 1$.

Introduction: history and experimental data

- Hence by decomposition of $\hat{\psi}$,
  $\hat{\psi} = \phi + \hat{\chi}$,
  we give the condensate wave function (c-number)
  $\phi = \left\langle \hat{\psi} \right\rangle$ and the operator $\hat{\chi}$, responsible for the non-condensate atoms, has zero expectation value,
  $\langle \hat{\chi} \rangle = 0$.
- For small density of the non-condensate atoms ϕ satisfies the GPE (which coincides with the NLSE),

$$i\phi_t + \frac{1}{2}\Delta\phi + |\phi|^2\phi = 0,$$

where we use dimensionless units when $U_0 = 1$.

Introduction: history and experimental data

- It follows immediately that the three-body recombination, as inelastic process, appears in the next order relative to na_s^3. As the result, the GPE gets additional term $iK_3|\phi|^4\phi$ with constant $K_3 > 0$:

$$i\phi_t + \frac{1}{2}\Delta\phi + |\phi|^2\phi + iK_3|\phi|^4\phi = 0.$$

Main goals

- In this paper we suggest a new mechanism within the Gross-Pitaevskii approximation (GPA) which does not contain additional small gas parameter. It is connected with generation of non-condensate particles due to the coherence destruction of collapsing condensate. We show that the generation of non-condensate particles in the framework of the GPA (this is a quantum problem) for small density of non-condensate particles reduces to the linear problem for the normal and anomalous correlators:

$$\begin{aligned} n(\mathbf{x},\mathbf{x}',t) &= \left\langle \hat{\chi}^{\dagger}(\mathbf{x},t)\hat{\chi}(\mathbf{x}',t)\right\rangle, \\ \sigma(\mathbf{x},\mathbf{x}',t) &= \left\langle \hat{\chi}(\mathbf{x},t)\hat{\chi}(\mathbf{x}',t)\right\rangle. \end{aligned}$$

Main goals

- These correlators due to inhomogeneous background depend on both time and two coordinates $\mathbf{r_1}$ and $\mathbf{r_2}$, but not on their difference as in the case of homogeneous background. The normal correlator at $\mathbf{r_1} = \mathbf{r_2}$ represents density of non-condensate particles n and the anomalous correlator σ (at $\mathbf{r_1} = \mathbf{r_2}$) is responsible for particle exchange between condensate and non-condensate reservoir:

$$\partial_t \int n(\mathbf{r}) d\mathbf{r} = -2 \int \mathrm{Im}[\phi^2 \sigma^*] d\mathbf{r} = -\partial_t \int |\phi|^2 d\mathbf{r}.$$

Just the anomalous correlator describes the coherence "transfer" from condensate to non-condensate particles. Note that between normal and anomalous correlators there exists the following inequality: $n \geq |\sigma|$.

Main goals

- In the case of small density of non-condensate particles the corresponding linear equations for correlators admit separation of variables that leads to the linearized GP equation on the background of the collapsing solution.
- Hence, the question about the non-condenstate particles generation - a quantum problem - reduces to the linear stability analysis for collapsing regime, describing by the Gross-Pitaevskii equation. As it was shown by Zakharov and Kuznetsov in 1986, in the 3D NLSE, a few regimes of collapses are possible, starting from quasi-classical strong collapse when a finite amount of particles is captured into singularity, quasi-classical weak collapse when formally zeroth amount of particles is captured into singularity and ending by a boundary weak collapse with the self-similar behavior which follows directly from NLS.

Main goals

- Amplitude in the latter regime grows like $(t_0 - t)^{-1/2}$, and respectively the condensate density behaves $\sim (t_0 - t)^{-1}$, where t_0 is the collapse time.
- Quasi-classical approximation for the GPE corresponds to the Thomas-Fermi approach. In Zakharov-Kuznetsov paper it was shown that all quasi-classical regime of collapses are unstable except probably the only one, i.e., the weak collapse for which the density at the collapse center grows inverse proportionally to square of the collapse size. Asymptotics of this solution as $r \to \infty$ was found analytically and the solution in the whole region was obtained numerically (Zakharov, Kuznetsov 1986).

Weak collapse

- Thus, in the Thomas-Fermi approximation, we immediately arrive at the generation of non-condensate particles. As for the stability of a weak collapse and, accordingly, the formation of non-condensate particles in this regime, this problem remained open until recently.
- It is worth noting that the recent experimental observations (Phys. Rev. **X 6**, 041058 (2016)) show that collapses in gaseous BECs with low densities should be related to the weak collapse regimes. Also note that the first numerical simulations (Zakharov, Kuznetsov, Musher, 1985) demonstrated that collapse in the 3D NLSE has the behavior corresponding to a weak collapse.

Weak collapse

- Solution of the GPE corresponding to a weak collapse has a self-similar form

$$\phi = \frac{1}{(t_0-t)^{1/2+i\alpha}}\chi\left[\frac{\mathbf{r}}{(t_0-t)^{1/2}}\right].$$

- In the spherically symmetric case ($\xi = r/(t_0-t)^{1/2}$) $\chi(\xi)$is defined from the following ODE,

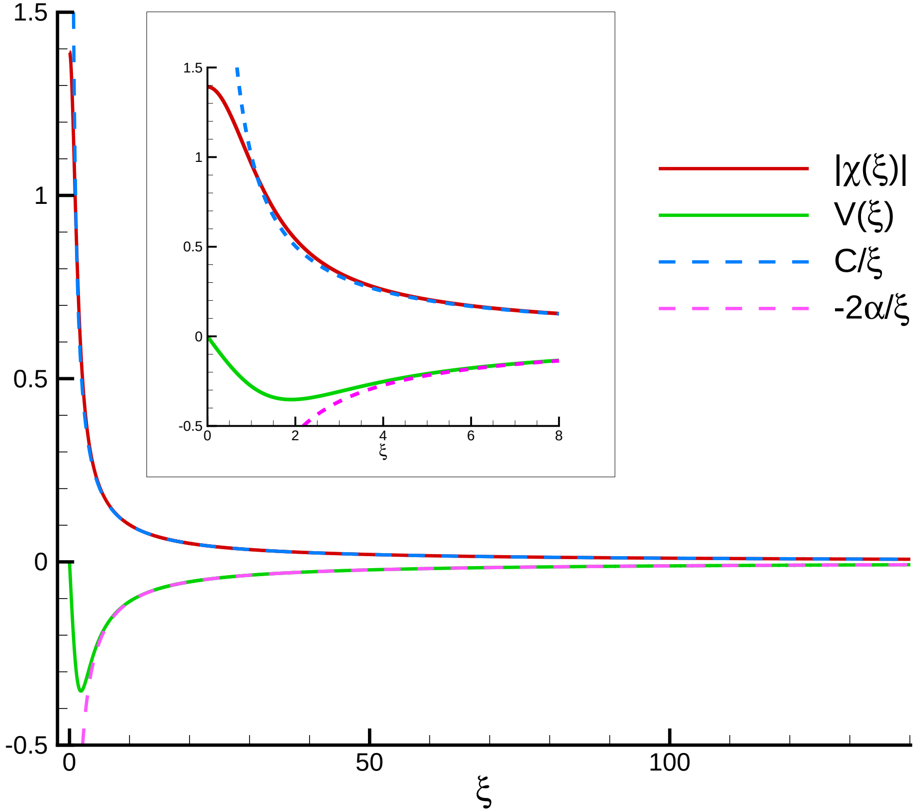
$$i\left[\left(\frac{1}{2}+i\alpha\right)\chi+\frac{1}{2}\xi\chi_\xi\right]+\frac{1}{2}\chi_{\xi\xi}+\frac{1}{\xi}\chi_\xi+|\chi|^2\chi=0,$$

with $\chi \to 0$ as $\xi \to \infty$ and bounded χ at $\xi = 0$. This equation represents the nonlinear spectral problem where α plays the role of eigen-value. This solution was analyzed by Zakharov and Kuznetsov 1986 (its plot is demonstrated below).

Weak collapse

The solution has the asymptote $\chi \to C/\xi^{1+2i\alpha}$ with $C = 1.0077$. Remarkably, this asymptote for ϕ does not depend on time: $\phi \to C/r^{1+2i\alpha}$ - frozen tail.

Dependences of $|\chi(\xi)|$, $V(\xi) = (\arg\chi)'$



Weak collapse

- For this solution the number of particles ΔN, captured into singularity, at $t \to t_0$ formally tends to zero like $(t_0 - t)^{1/2}$. Numerical simulation of collapse in the NLSE (Malkin, Shapiro, 1990) shows that while approaching singularity a new regime is formed which was called as the ”black hole” regime. It was predicted first time by Talanov with co-authors and later Zakharov showed that there are some logarithmic corrections to their answer. The black hole regime can be considered as an indication to instability of a weak collapse.

Instability of weak collapse

- Consider now the linear stability problem for the weak collapse solution putting
$$\phi = (t_0 - t)^{-(1/2+i\alpha)} \left[\chi(\xi) + g(\xi, \Omega, t)\right],$$
where g is a small perturbation depending on radius ξ, spherical angle Ω and time t. Next, expand g over spherical harmonics with the given l: $g = u_l(\xi, t) Y_{lm}(\Omega)$. As the result, for $u_l(\xi, t)$ we arrive at the following linear equations

$$i\frac{\partial u}{\partial \tau} + i(1/2 + i\alpha)u + \frac{i}{2}\xi u' + \frac{1}{2}\Delta u + 2|\chi|^2 u + \chi^2 u^* = 0,$$

plus c.c., where

$$\Delta = \frac{1}{\xi^2}\frac{\partial}{\partial \xi}\xi^2\frac{\partial}{\partial \xi} - \frac{l(l+1)}{\xi^2}.$$

Instability of weak collapse

- Here we introduce a new time $\tau = -\log[(t_0 - t)/t_0]$ which varies from $-\infty$ as $t \to -\infty$ up to $+\infty$ as $t \to t_0$. The 'potentials' in these equations are time independent and therefore $u, u^* \sim \exp(\gamma\tau)$ that results in the eigen-value problem for the growth rate γ. This is a very complicated eigen value problem, even for numerics. Two coupled linear equations for complex amplitudes u, u^* contain not-self-adjoint differential operators of the second order.
- Instead of solving such eigen value problem, we decided to find numerically solution of the Cauchy problem by taking initially ($\tau = 0$) u, u^* in the Gaussian form

$$u_{0l} = C \exp[-\xi^2/2\xi_0^2],$$

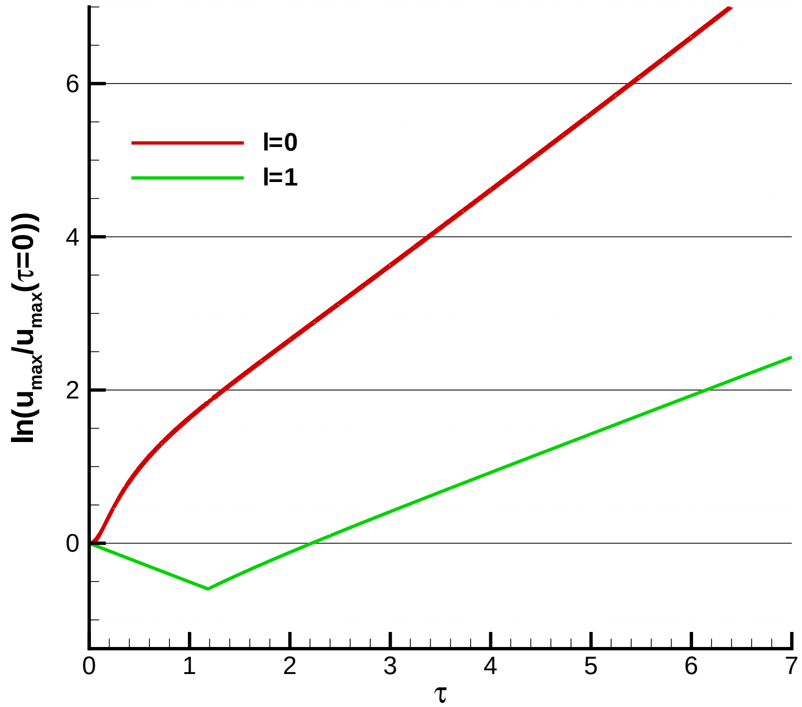
with $\xi_0 = 4$ much smaller the maximal $\xi_{max}(= 100)$ that models the «infinite» range of ξ.

Instability of weak collapse

- The linear equations for u, u^* were solved by the finite-difference method for $l = 0$, $l = 1$ and several $l \geq 2$. Information about γ was extracted from asymptotics of solution at $\tau \to \infty$. Indeed, the exponential growth (or decrease) of the maximal value of $|u|$ was observed already at low $\tau \sim 3 - 5$. Hence we got the growth rate. With increase of τ we observed also the formation of eigen modes for $l = 0$ and $l = 1$ (see figs below).

Instability of weak collapse

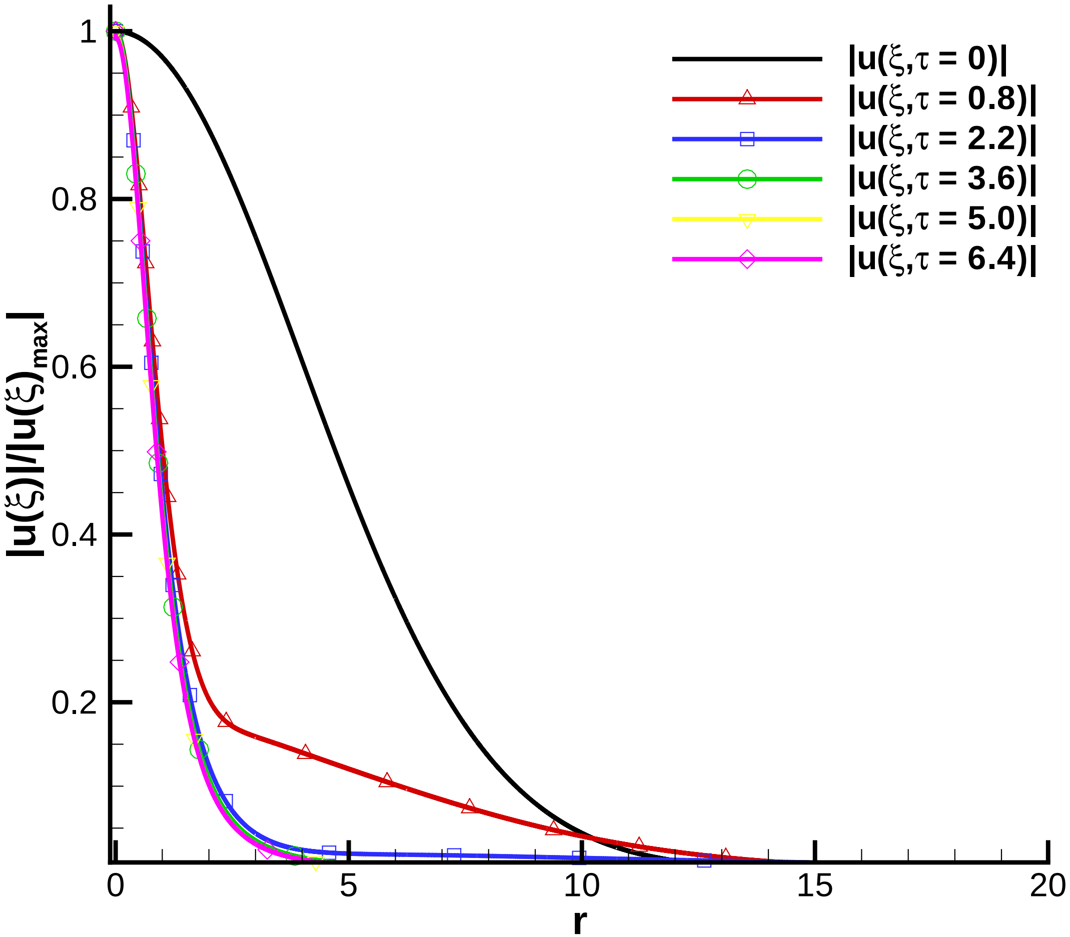
The temporal dependences of logarithms of the maximal amplitudes for $l = 0$ and $l = 1$.



It gives $\gamma_{l=0} = 0.984984$ and $\gamma_{l=1} = 0.512$, namely, we have instability.

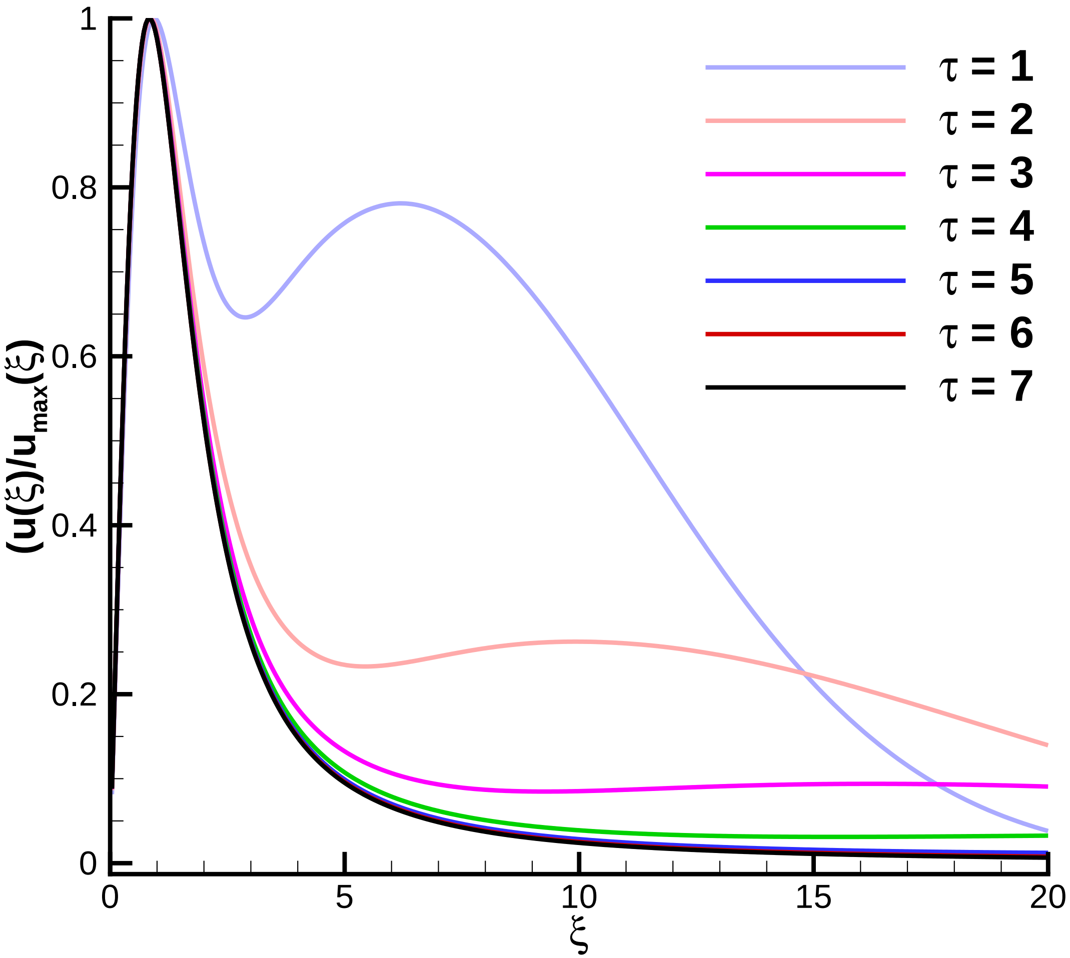
Instability of weak collapse

The formation of the eigen mode for $l = 0$.



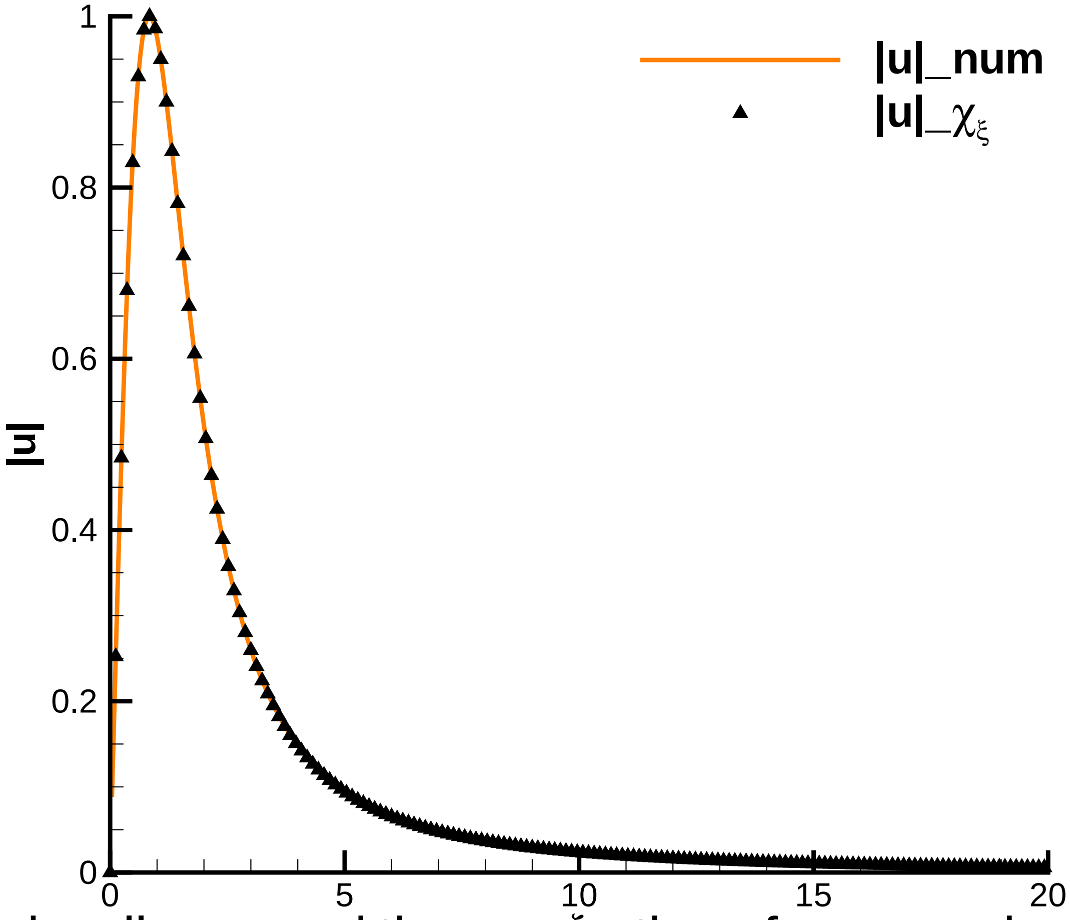
Instability of weak collapse

The formation of the eigen mode for $l = 1$: dependences of $|u_1|$ normalized by its maximum amplitude at different times.



Instability of weak collapse

Comparison of the found numerical mode for $l = 1$ with the shift mode obtained by differentiating the solution χ relative to ξ (under the normalization).



Concluding remarks

- We have thus solved two important problems: the instability of a weak collapse is proved to be one of the long-standing problems in the theory of wave collapse, and it is also shown that the generation of non-condensate particles is possible due to a new mechanism based on the destruction of the coherence of the collapsing condensate.
- Physically, this picture is quite transparent: in the collapse, a condensate particle acquires large kinetic energies and thus, from the quantum point of view, loses their coherent similarity with other collapsing particles. Those become individually independent. This is precisely the transformation of coherent particles into incoherent particles whose phases can be considered independent.

Concluding remarks

- If this is so, then, in accordance with the Fermi rule, the inverse characteristic time of their interaction, which is proportional to the square of the matrix element, will be substantially smaller, i.e. is proportional to the square of the density n^2, while for a collapse the characteristic inverse time is proportional to the first power of the density n. When approaching the point of collapse, the particles, gaining a large kinetic energy, practically become free. This is a qualitative explanation of why in the experiments as a result of the collapse there is an almost complete devastation of the traps. However, this mechanism should work at low densities, when the Gross-Pitaevsky approximation is still valid. At higher BEC densities, the mechanism proposed by Kagan and co-authors is significant, which, apparently, is realized in some experiments.

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