On another concept of Hasselmann equation source terms

An exploration of tuning-free models

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Klaus Hasselmann (1962)

$$\frac{\partial \varepsilon}{\partial t}+\frac{\partial \omega_k}{\partial \vec{k}}\frac{\partial \varepsilon}{\partial \vec{r}}=S_{nl}+S_{in}+S_{diss}$$

$$0+0=S_{nl}+0+0 \Rightarrow \varepsilon=C_K\frac{P^{1/3}}{\omega^4}$$

S_{nl} –derived from free surface Euler equations

S_{in} –multiple versions, differences up to 500%

S_{diss} –multiple LF and HF versions

Detailed discussion in Pushkarev, Zakharov 2016

Motivation :

Build S_{in} *consistent with mathematical properties of Hasselmann equation and requiring minimal tuning of the model*

Outline :

- *Background*
- *Theoretical approachs*
- *Experimental approach*
- *Numerical approach*
  - *“implicit” HF dissipation*
  - *Duration limited case*
  - *Limited fetch case*
- *Conclusions*

Background

Field Experiments	Theory	Numerics
$\varepsilon \sim \omega^{-4}$ $\varepsilon \sim \chi^{p}$, $\langle \omega \rangle \sim \chi^{-q}$ $0.74 < p < 1$ $0.2 < q < 0.3$ *Badulin, Babanin, Resio, Zakharov 2008*	$S_{nl} = 0 \Rightarrow \varepsilon = C_K \frac{P^{1/3}}{\omega^4}$ *Zakharov, Filonenko 1968* $\varepsilon \sim \chi^{p}$, $\langle \omega \rangle \sim \chi^{-q}$ $p = 1$, $q = 0.3$ *Zakharov 2005* *Zakharov, Resio, Pushkarev 2012*	$\varepsilon \sim \omega^{-4}$ $\varepsilon \sim \chi^{p}$, $\langle \omega \rangle \sim \chi^{-q}$ $p \approx 1$, $q \approx 0.3$ *Pushkarev, Resio, Zakharov 2003* *Badulin, Pushkarev, Resio, Zakharov 2005*

Experiment	p	q
Black Sea (Babanin & Soloviev 1998b)	0.89	0.275
Walsh et al. (1989) US coast	1.0	0.29
Kahma & Calkoen (1992) unstable	0.94	0.28
Kahma & Calkoen (1992) stable	0.76	0.24
Kahma & Pettersson (1994)	0.93	0.28
JONSWAP by Davidan (1980)	1.0	0.28
JONSWAP by Phillips (1977)	1.0	0.25
Kahma & Calkoen (1992) composite	0.9	0.27
Kahma (1981, 1986) rapid growth	1.0	0.33
Kahma (1986) average growth	1.0	0.33
Donelan et al. (1992) St Claire	1.0	0.33
Ross (1978), Atlantic, stable	1.1	0.27
Liu & Ross (1980), Michigan, unstable	1.1	0.27
JONSWAP (Hasselmann et al. 1973)	1.0	0.33
Mitsuyasu et al. (1971)	1.008	0.33
ZRP numerics	1.0	0.3

Exponents of wind-wave growth in fetch-limited experiments. Adapted from Badulin, Babanin, Zakharov, Resio 2007

Theoretical approach

Limited Fetch case: $$\frac{\partial \omega_k}{\partial \vec{k}} \frac{\partial \varepsilon}{\partial \vec{r}} = S_{nl} + S_{in}$$

Duration limited case: $$\frac{\partial \varepsilon}{\partial t} = S_{nl} + S_{in}$$

$$S_{in} \sim \varepsilon\, \omega^{s+1}$$

Existence of self-similar solutions is no guarantee of their realization!

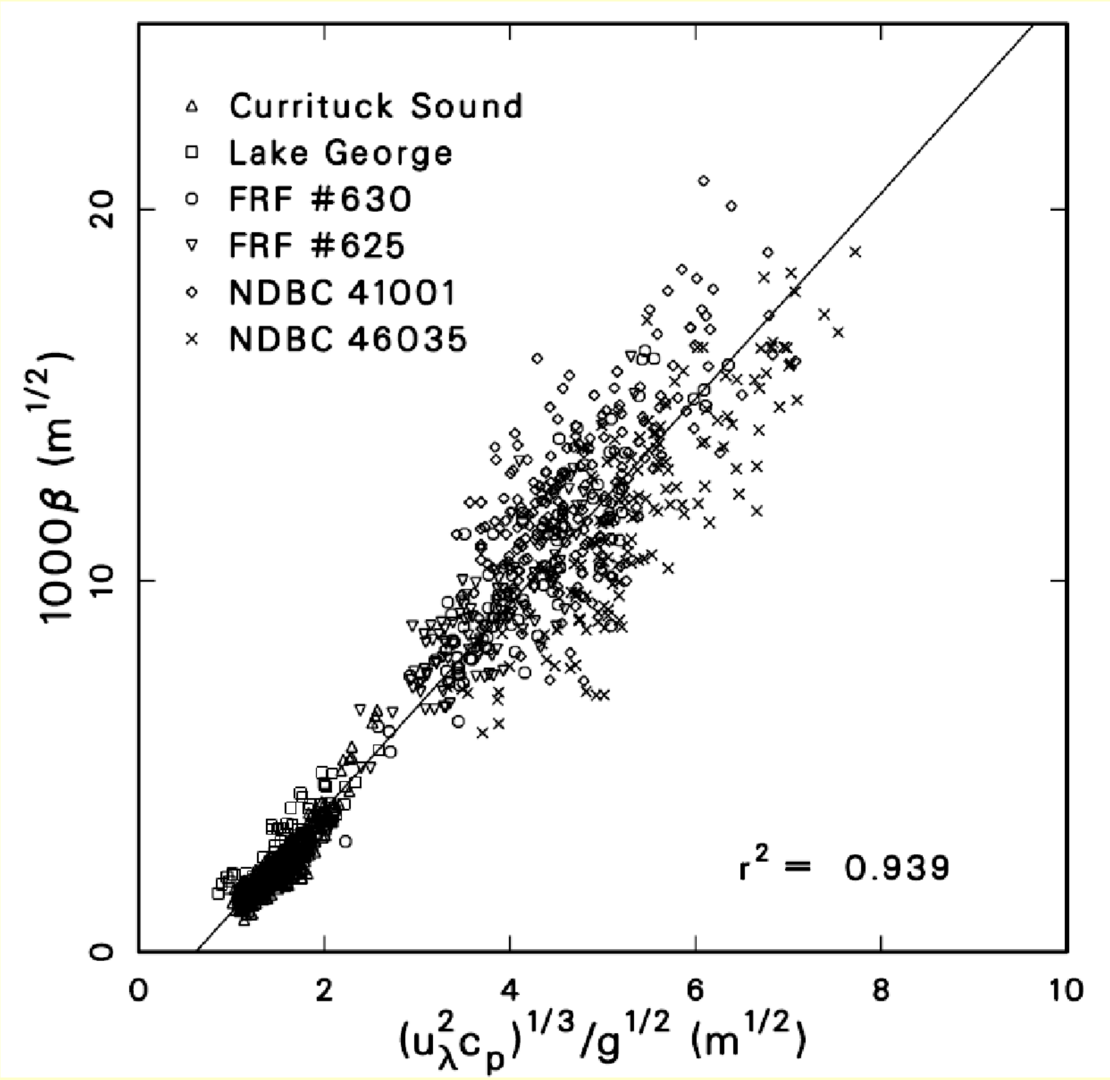
Example – wave collapse in NLS

Duration Limited Case	Fetch Limited Case
$\varepsilon = t^{p+q} F(\omega t^q)$	$\varepsilon = \chi^{p+q} G(\omega \chi^q)$
$\varepsilon = \varepsilon_0 t^p \quad \langle\omega\rangle = \omega_0 t^{-q}$	$\varepsilon = \varepsilon_0 \chi^p \quad \langle\omega\rangle = \omega_0 \chi^{-q}$
$9q - 2p = 1$	$10q - 2p = 1$
$s = s(p,q)$	$s = s(p,q)$

Experimental approach

Resio-Long 2004-2007 regression line

$$F(k)=\beta k^{-5/2} \quad \Leftrightarrow \quad \varepsilon = C_K \frac{P^{1/3}}{\omega^4}$$



Duration Limited Case	Fetch Limited Case
$p=10/7 \quad q=10/7$	$p=1 \quad q=3/10$
$s=4/3$	$s=4/3$

ZRP wind input term:

$$S_{in}(\omega,\theta) = A \cdot \frac{\rho_{air}}{\rho_{water}} \omega \left(\frac{\omega}{\omega_0} \right)^{4/3} f(\theta)\ \varepsilon(\omega,\theta)$$

$$f(\theta) = \begin{cases} \cos^2(\theta),\ for\ -\pi/2 < \theta < \pi/2 \\ 0,\ otherwise \end{cases}$$

$$\omega_0 = \frac{g}{U_{10}}$$

Numerical approach

The model still misses 2 features:

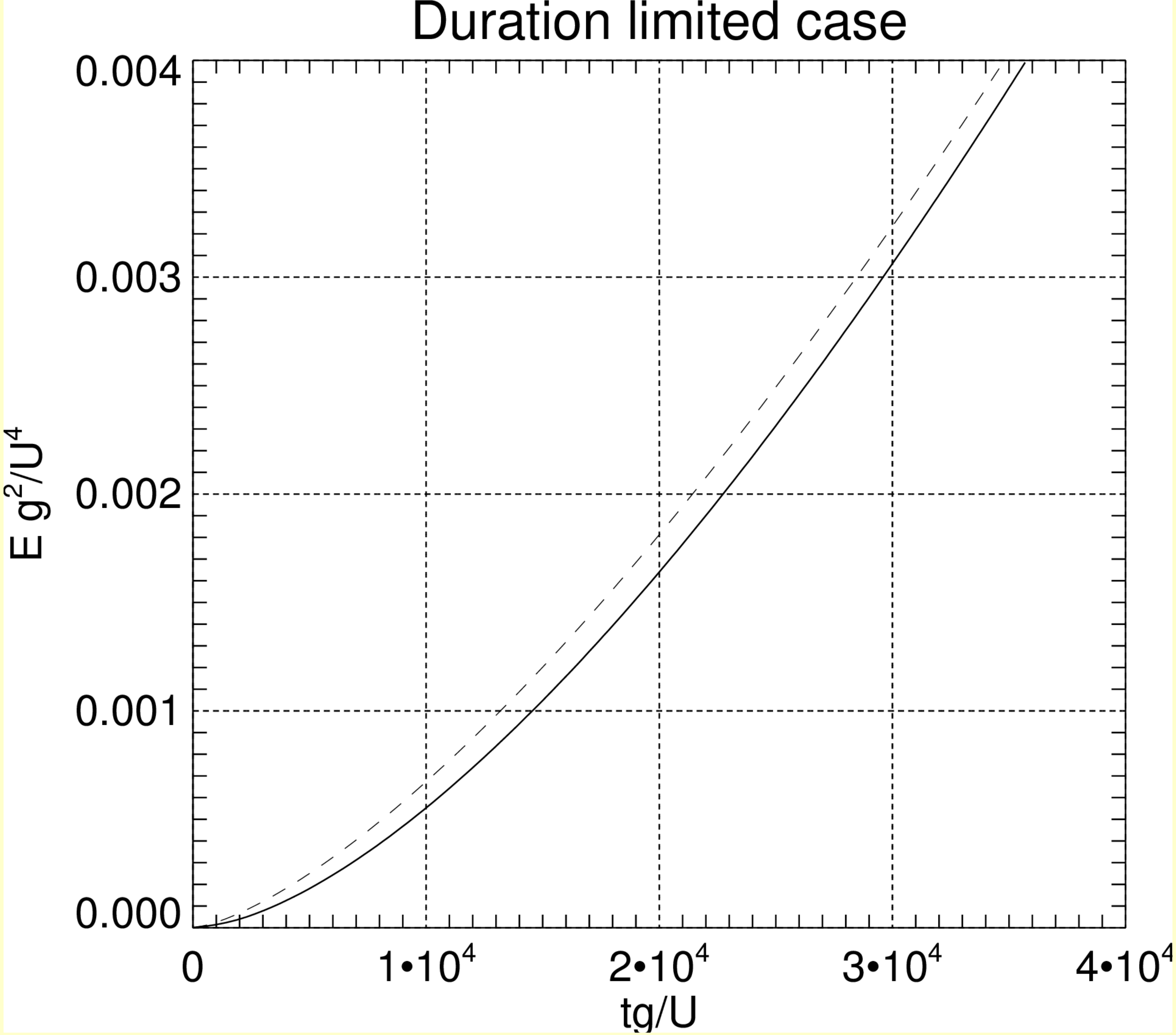
- *the coefficient in front of ZRP* S_{in}
- *dissipation function* S_{diss}

The coefficient 0.05 in front of ZRP term was chosen from field observations.

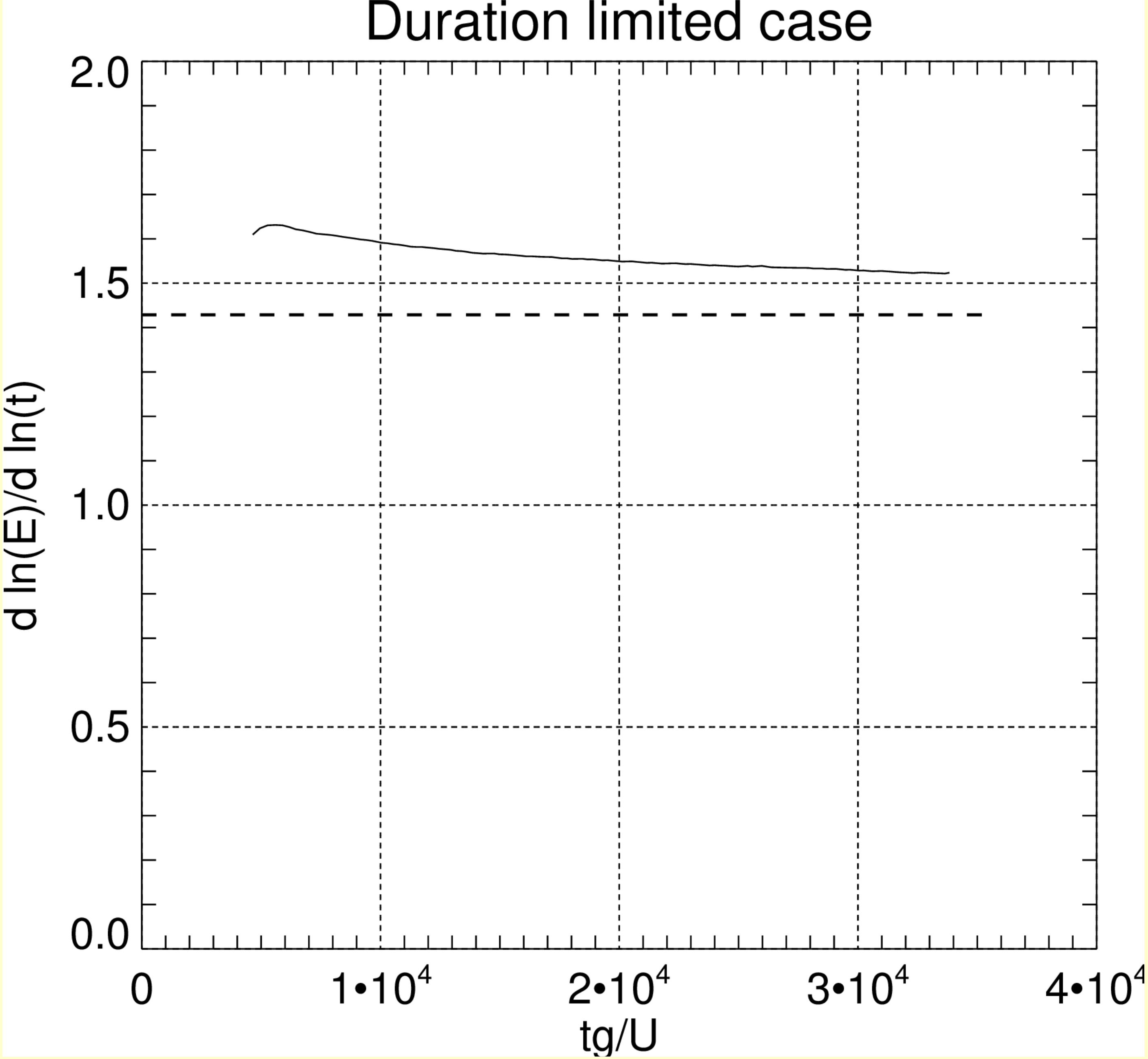
We used $\sim f^{-5}$ *HF "implicit dissipation" tail for* $f \geqslant 1.1\, Hz$*, working as the sigar cutter in Fourier space:*

Duration limited case

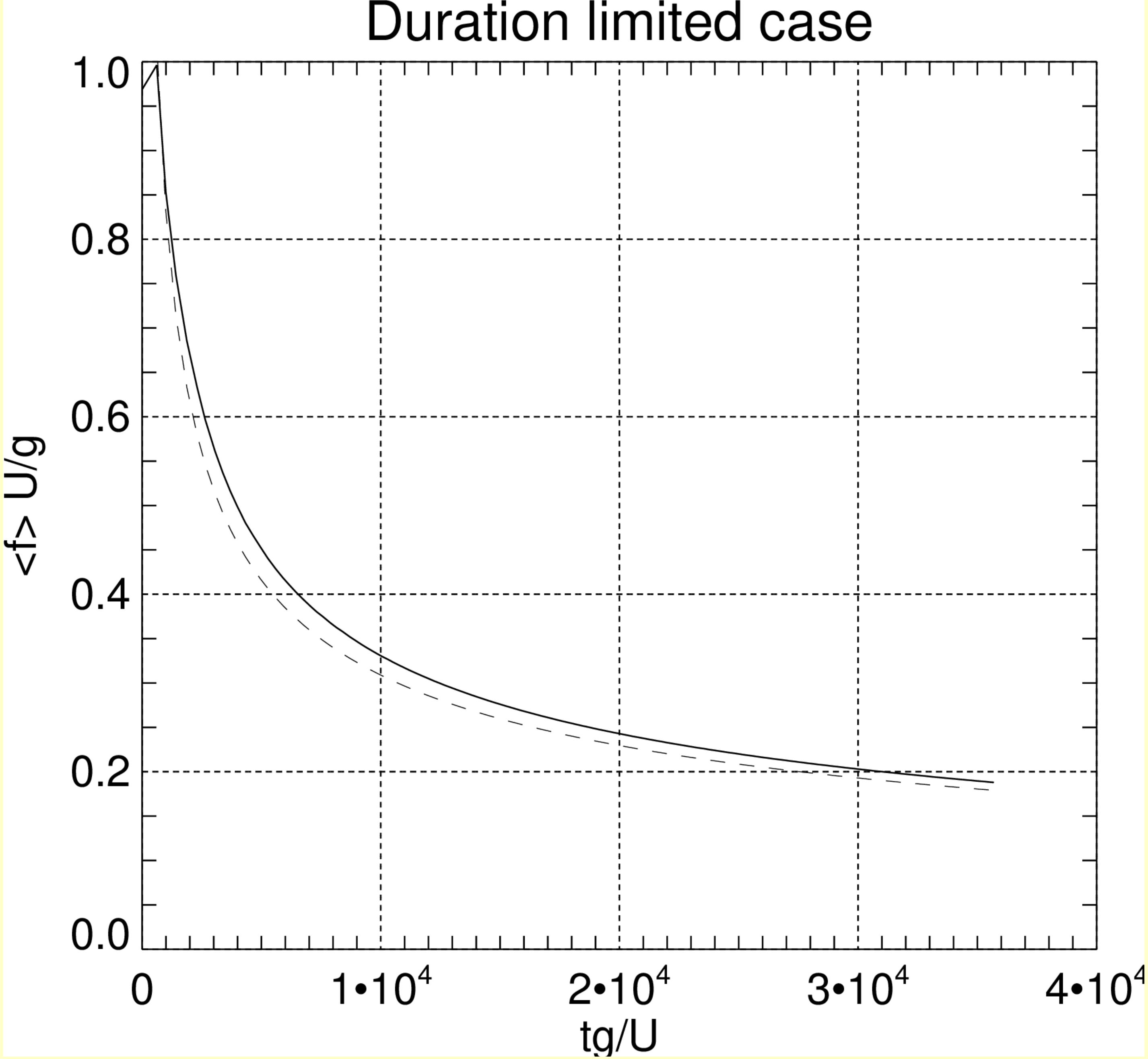
Wind speed 10 m/sec



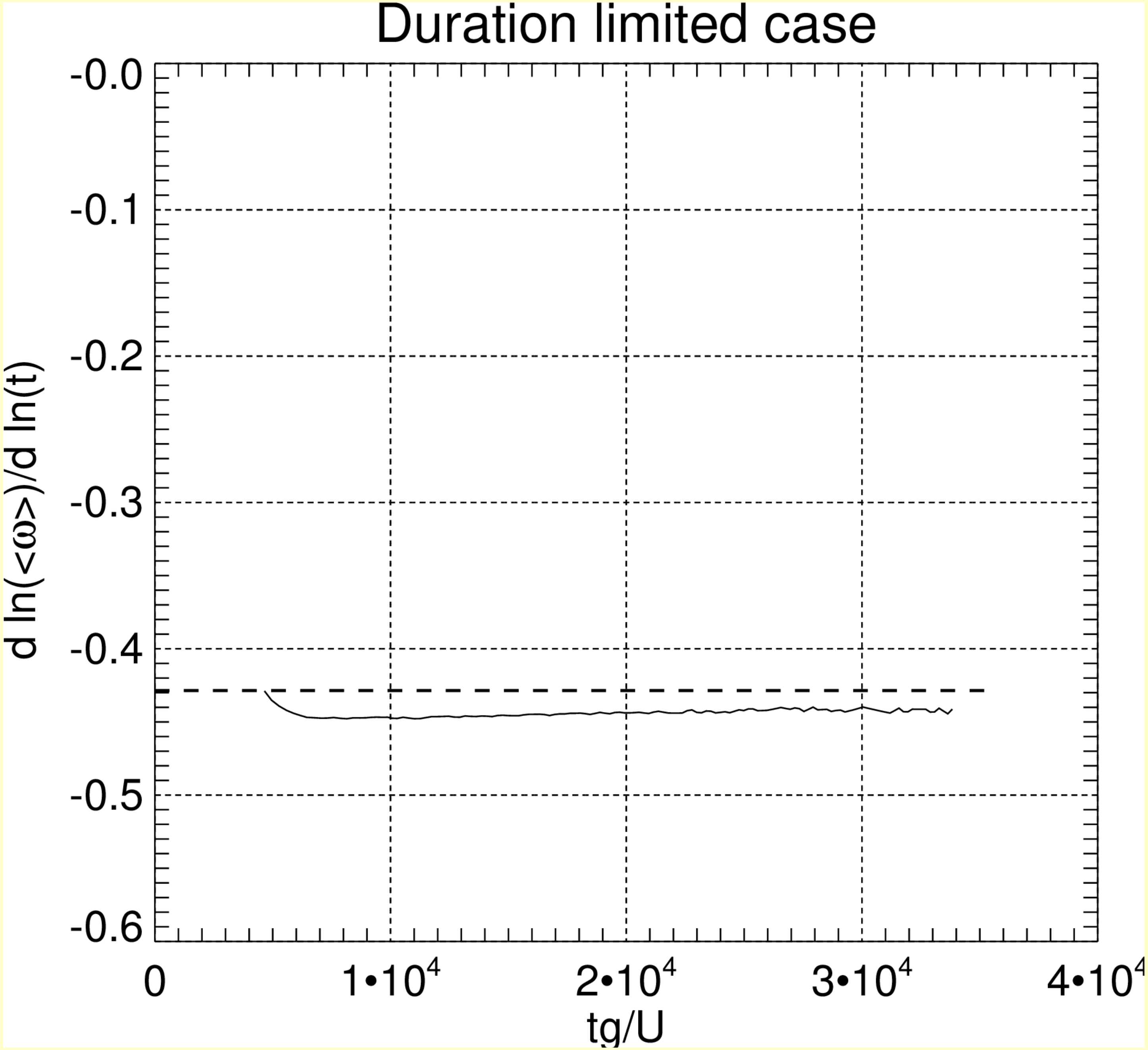
Dimensionless energy versus dimensionless solid line.
Self-similar solution - dashed line.



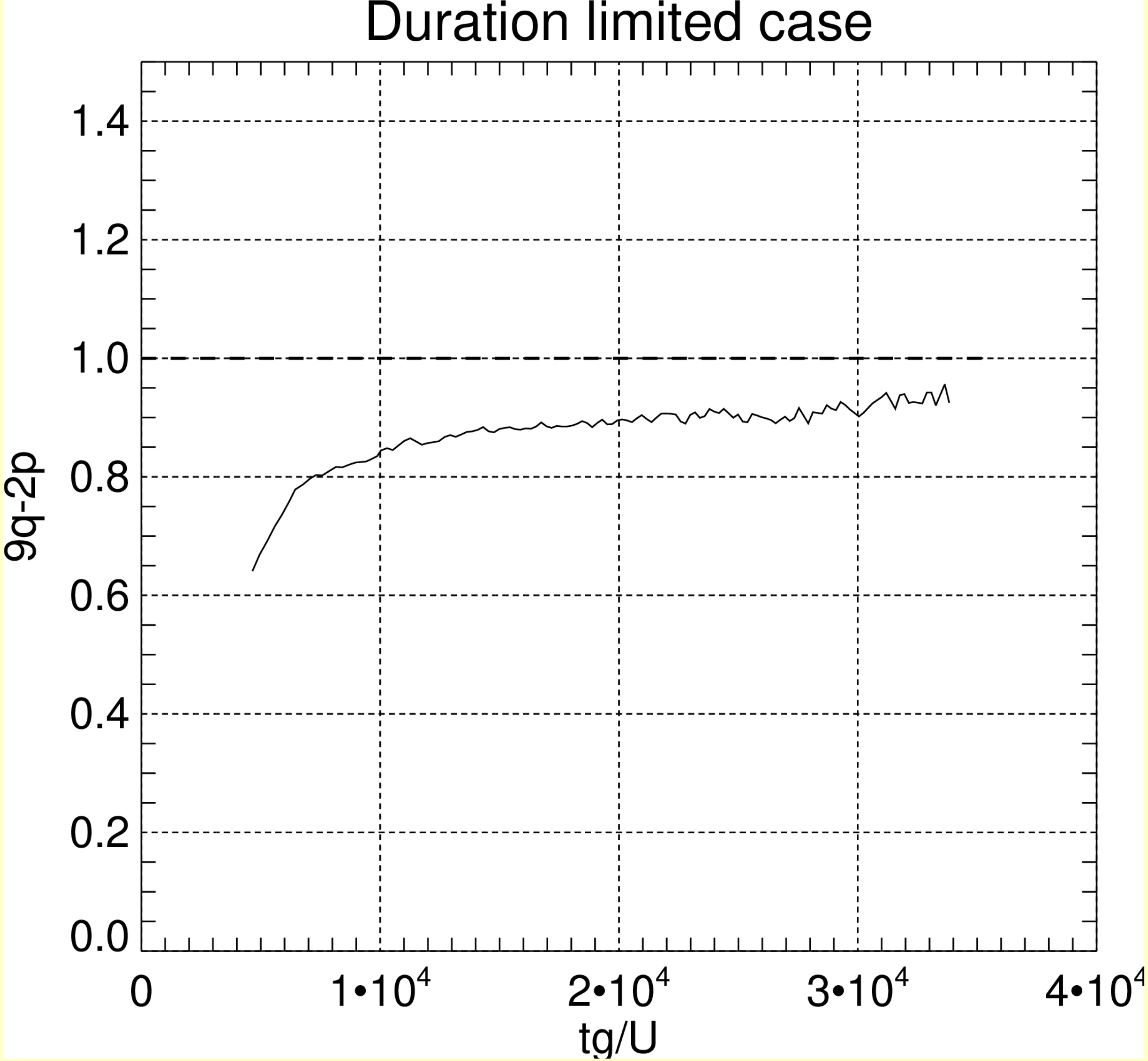
Total energy index as the function of dimensionless time - solid line. Self-similar index p = 10/7 - dashed line.



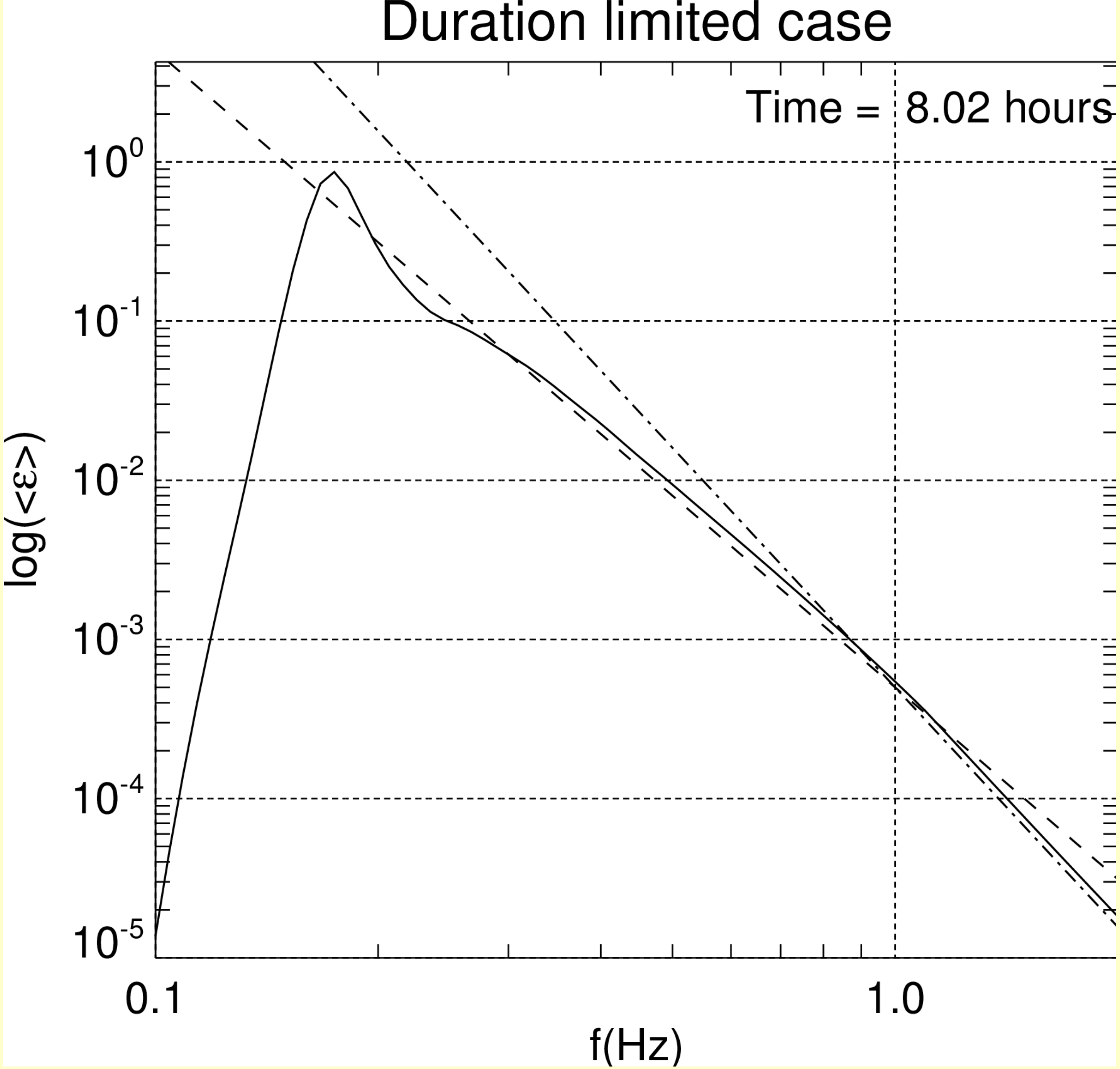
Dimensionless frequency versus dimensionless - solid line, self-similar solution - dashed line.



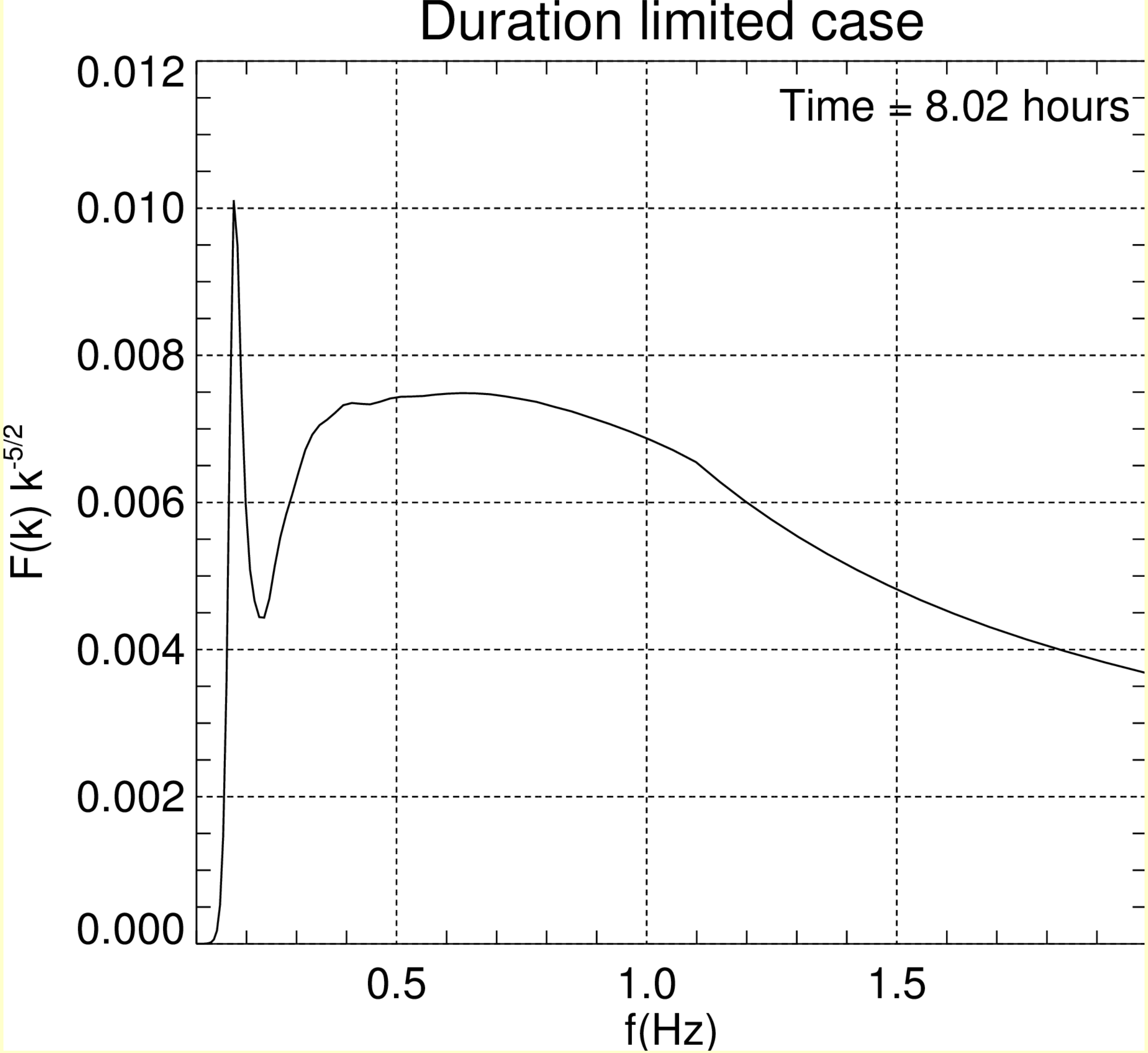
Mean frequency index versus dimensionless time - solid line.
Self-similar index q = −3/7 - dashed line



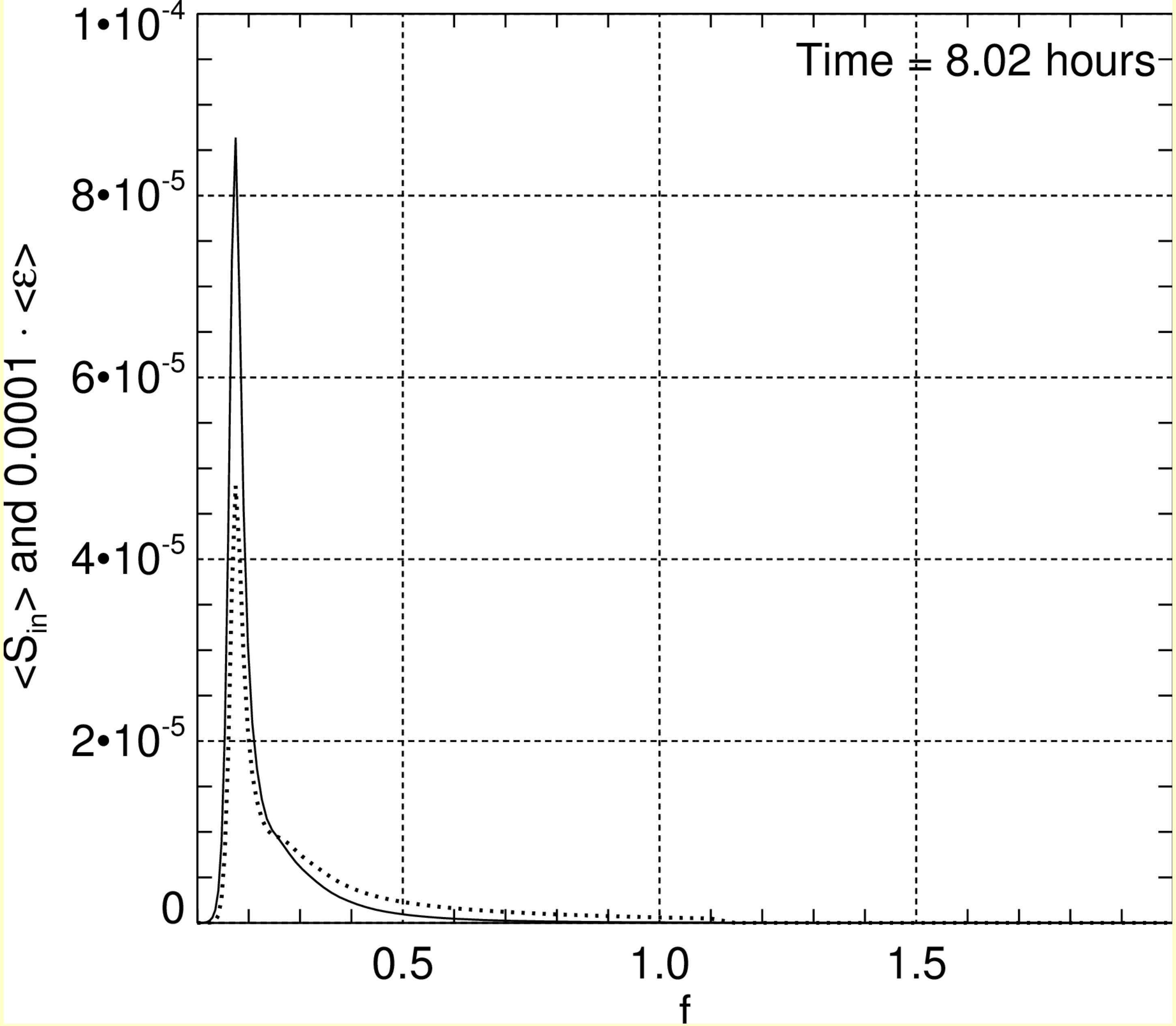
"Magic number" 9q – 2p versus dimensionless time – solid line.
Self-similar target - dashed line.



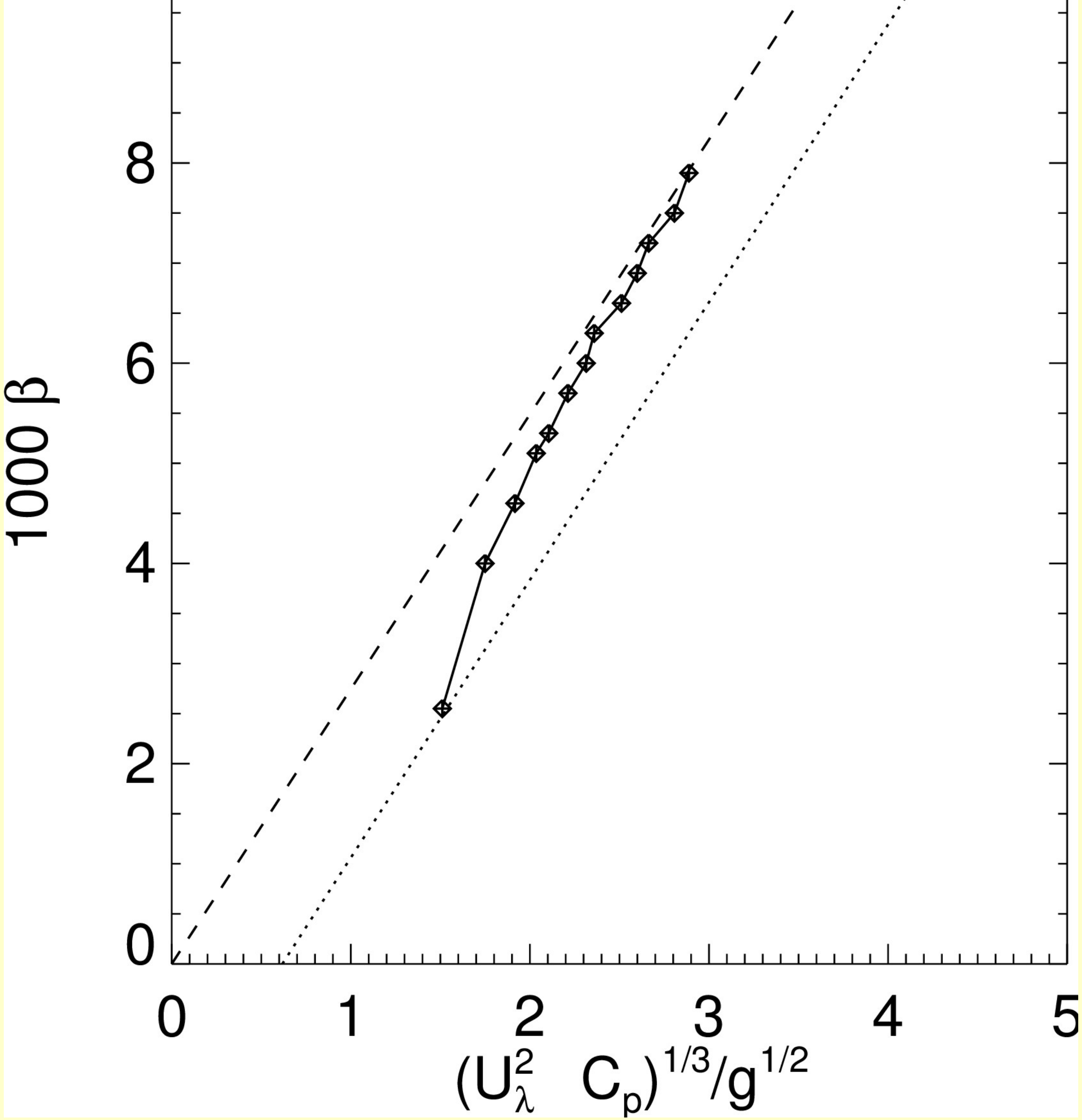
Decimal logarithm of the angle averaged spectrum versus decimal logarithm of the frequency - solid line. Spectrum f^{-4} - dashed line, spectrum f^{-5} - dash-dotted line.



Compensated spectrum versus frequency f.

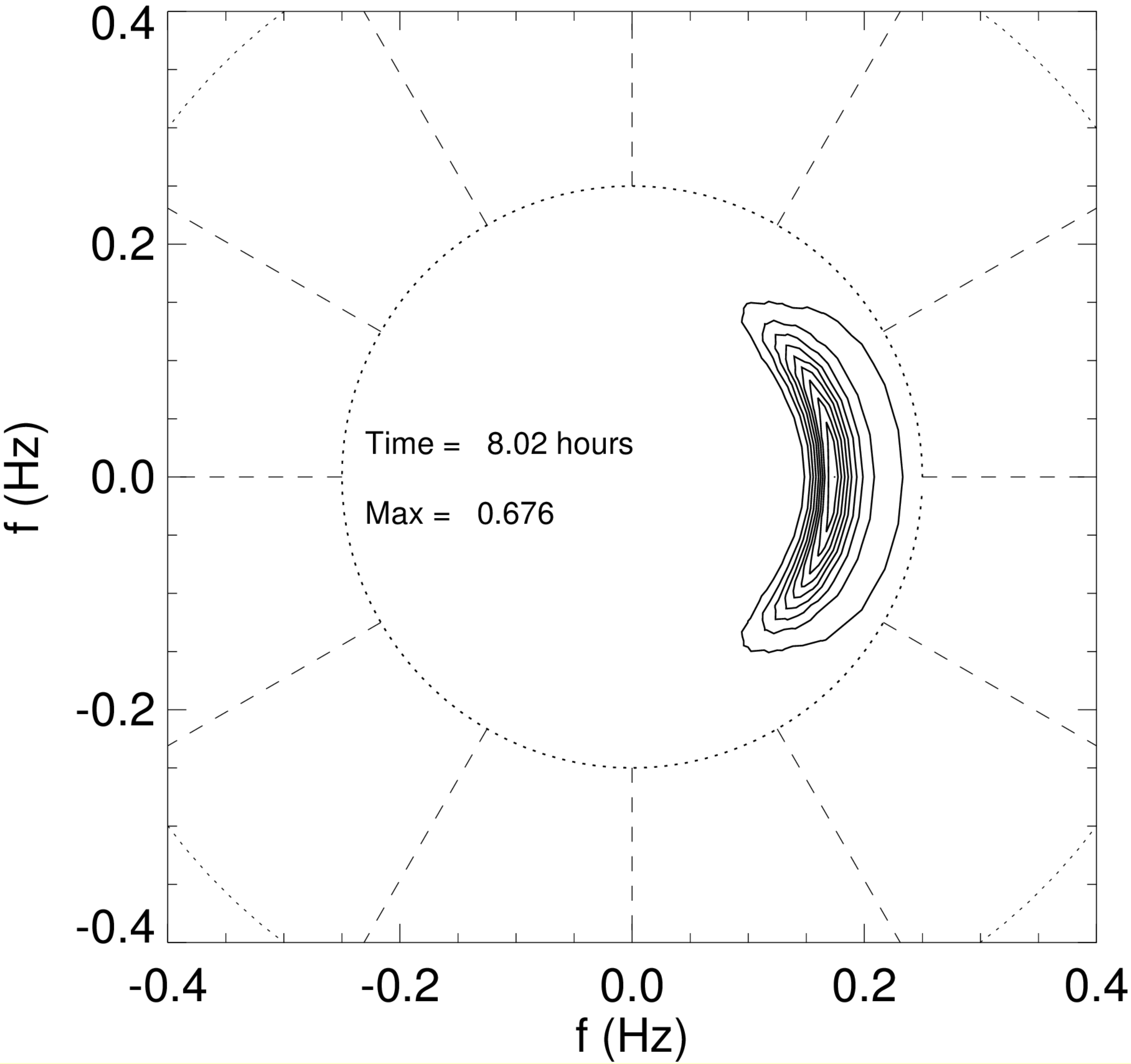


Angle averaged wind input function (dotted line) and angle averaged spectrum (solid line) versus frequency f.



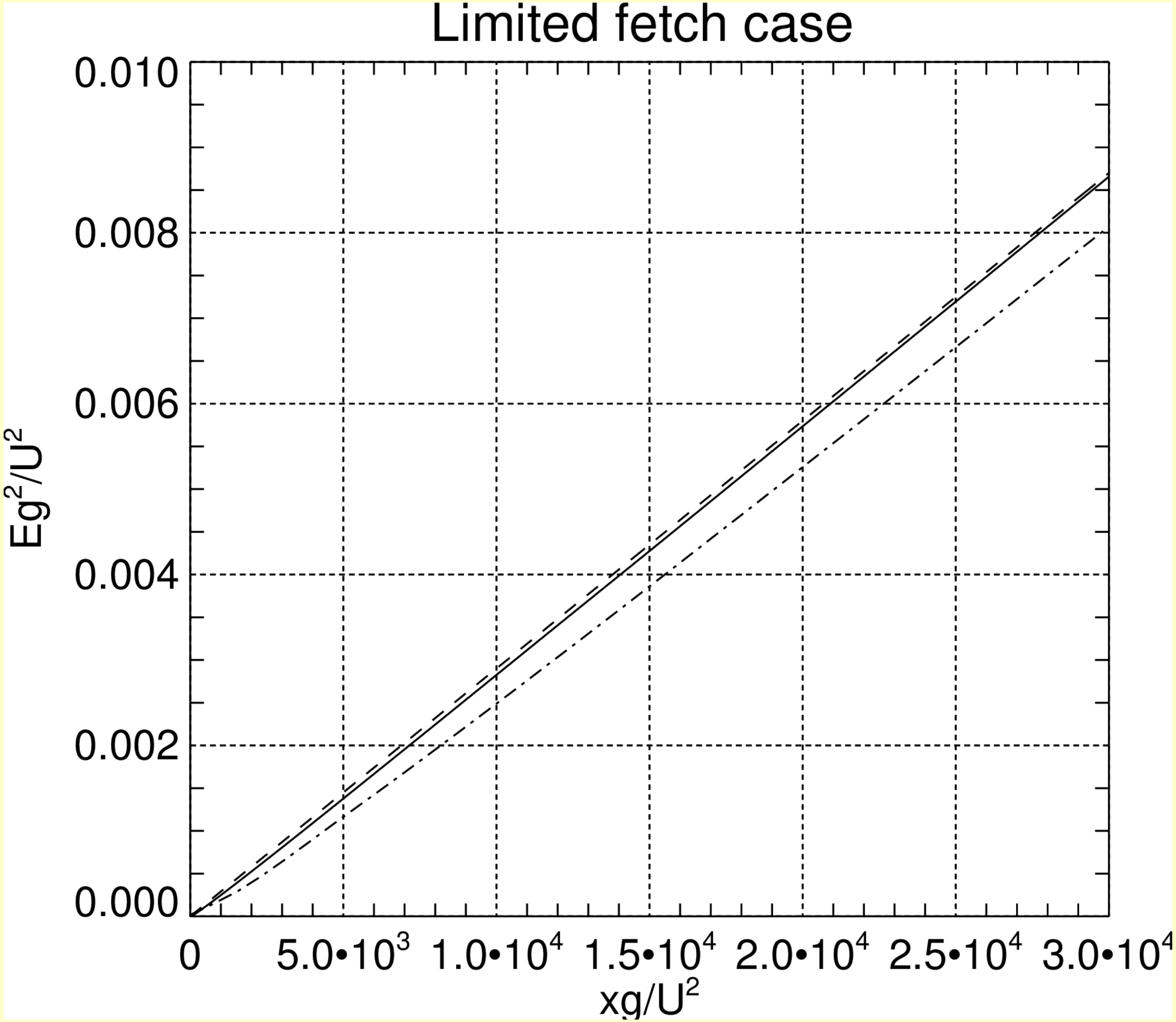
Experimental (dotted line), theoretical (dashed line) and numerical (diamonds) 1000β versus specific velocity for wind speed 10 m/sec.

Duration limited case

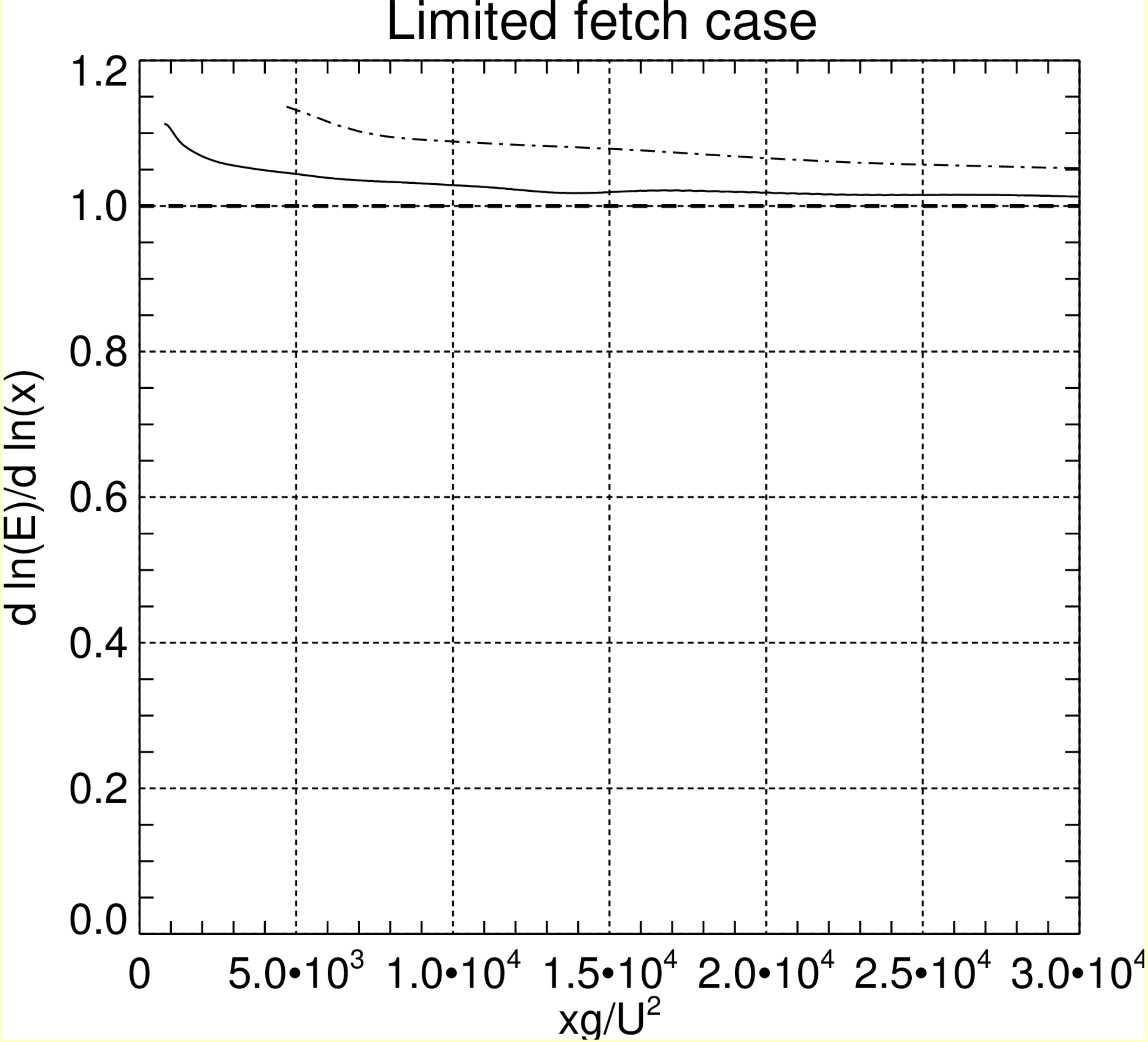


Limited fetch case

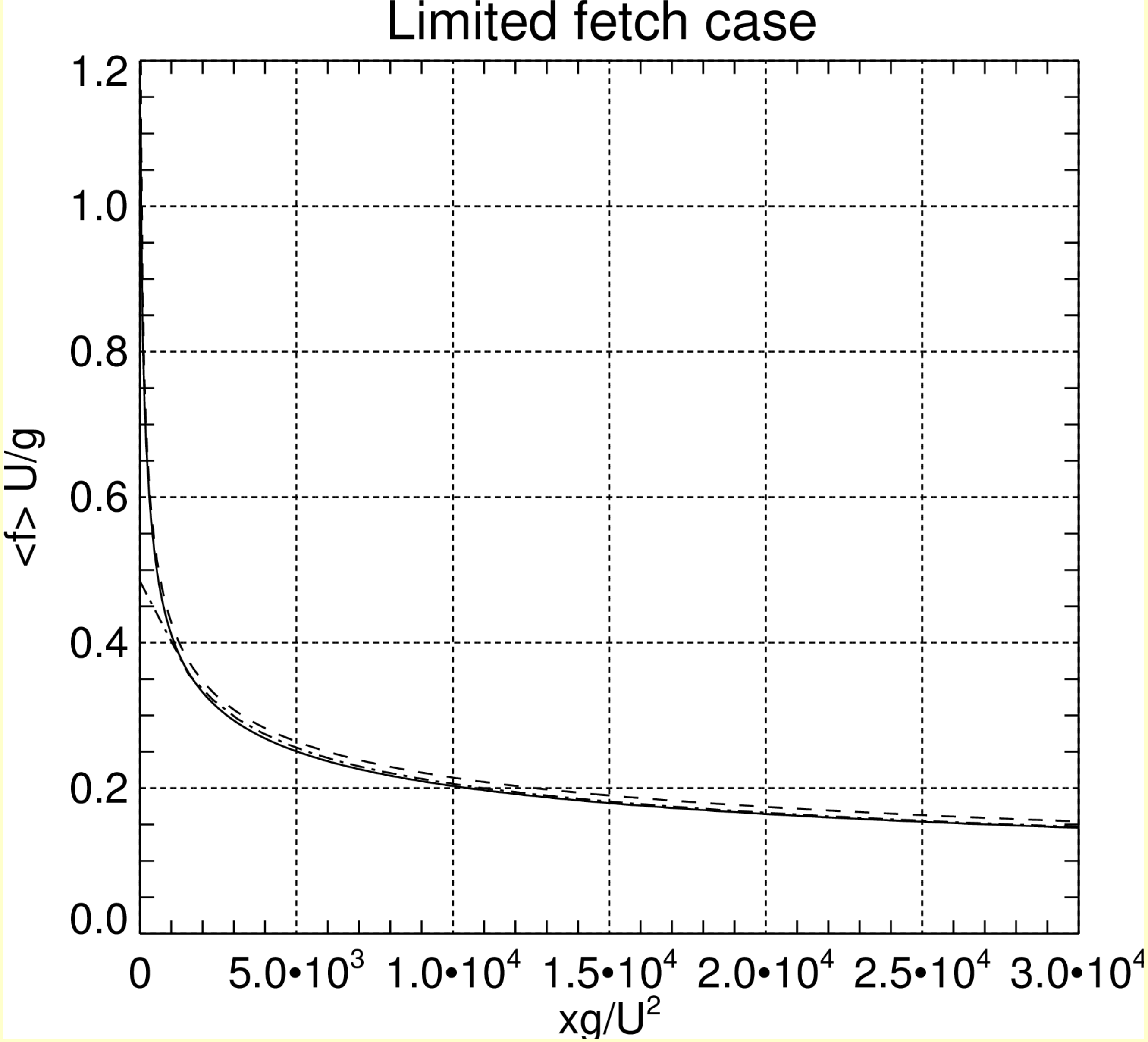
Wind speed 5 and 10 m/sec



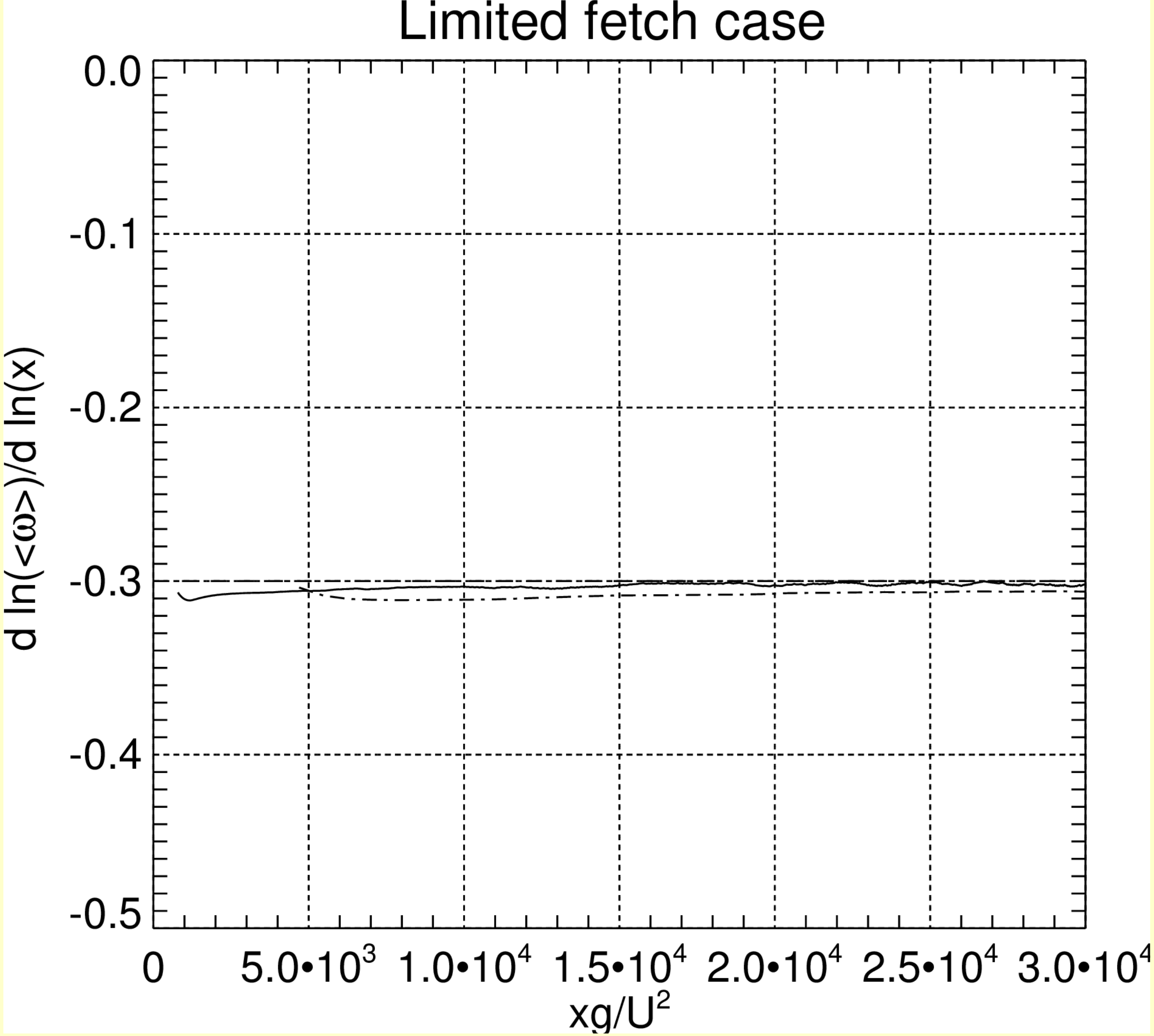
Total energy versus fetch: wind speed 10 m/sec - solid line, 5 m/sec - dash-dotted line. Self-similar solution - dashed line



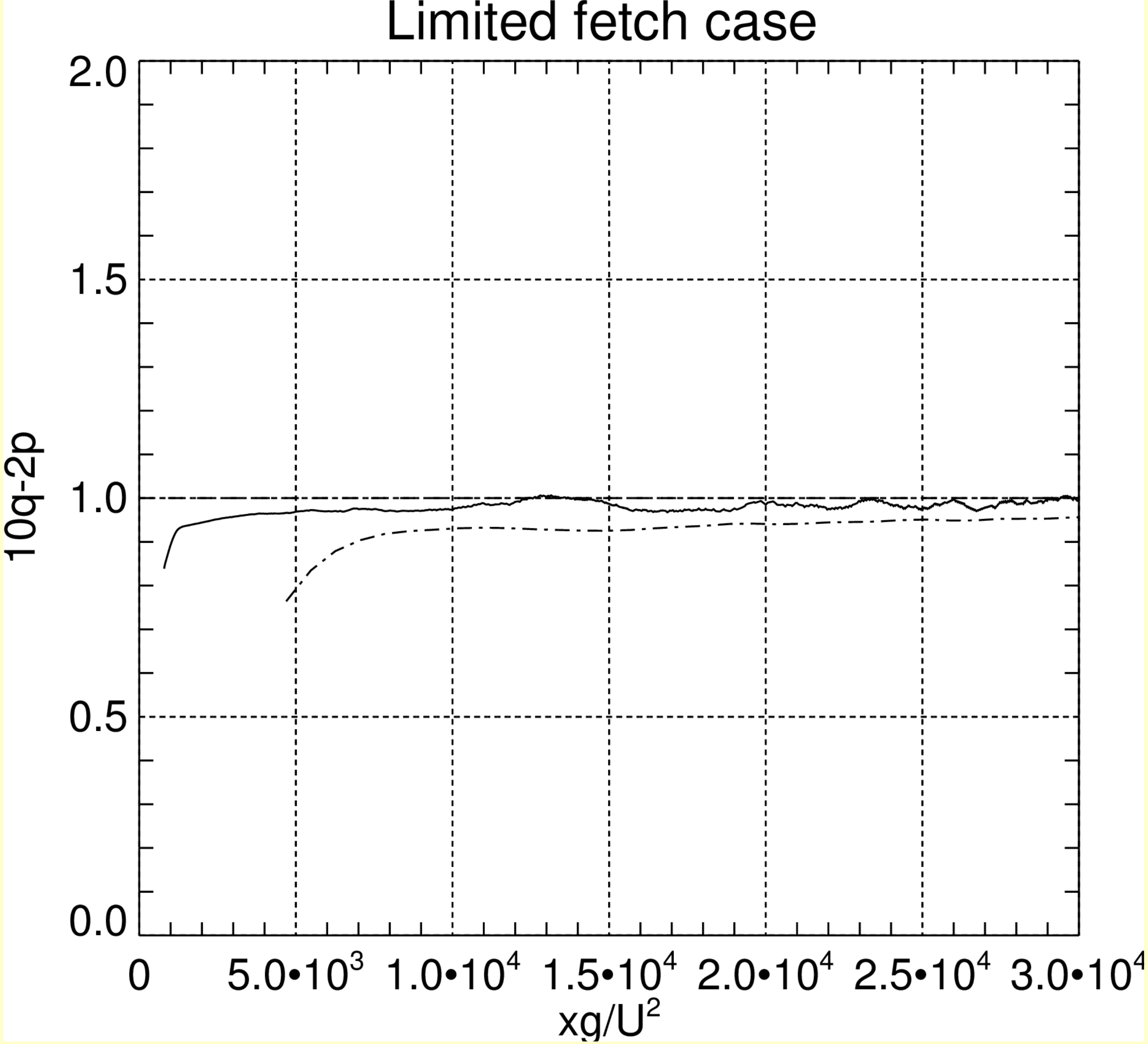
Local energy index versus fetch.



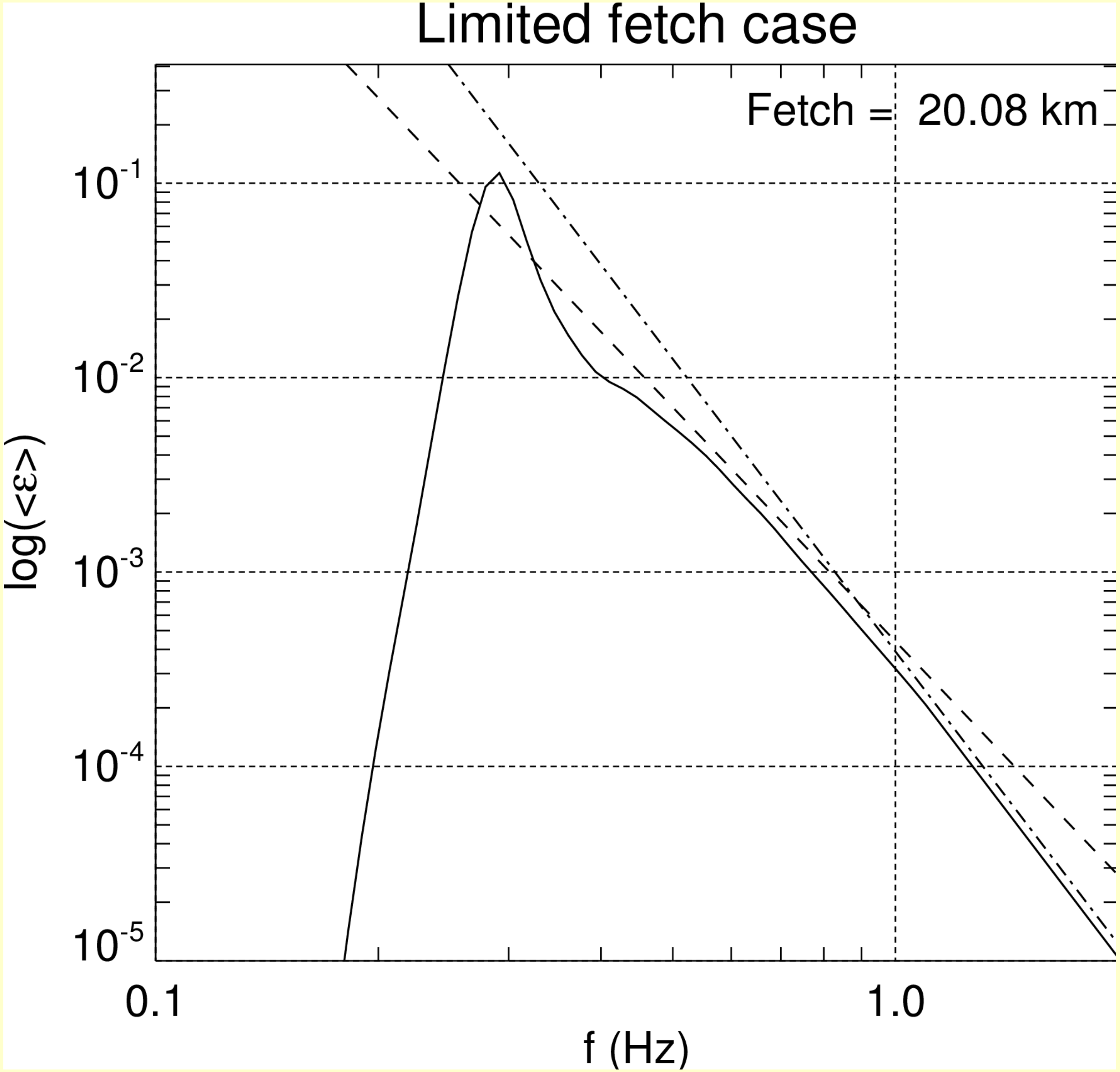
Mean frequency versus the fetch for wind speed 10 m/sec (solid line) and 5 m/sec (dashed line). Self-similar dependence - dash-dotted line.



Local mean frequency exponent −q = dln<ω> dlnx as the function of dimensionless fetch xg/U2 for fetch limited case. Wind speed 10 m/sec - solid line, wind speed 5 m/sec - dashed line. Horizontal dashed line - target value of the self-similar exponent -q = -0.3.



Magic number" 10q – 2p versus the fetch. Wind speed 10 m/sec - solid line, wind speed 5 m/sec - dash-dotted line. Self-similar target 1 – dashed line.



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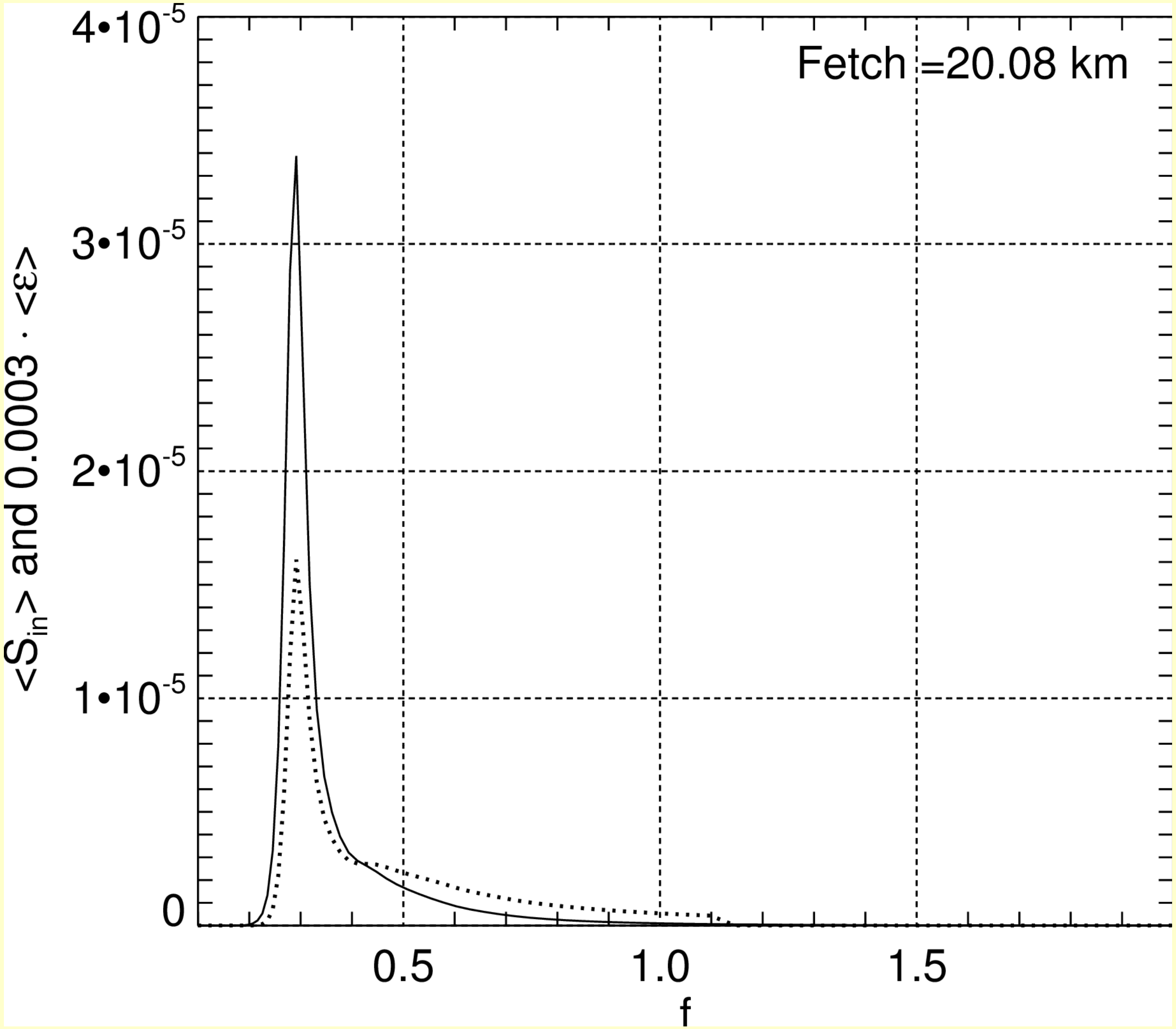

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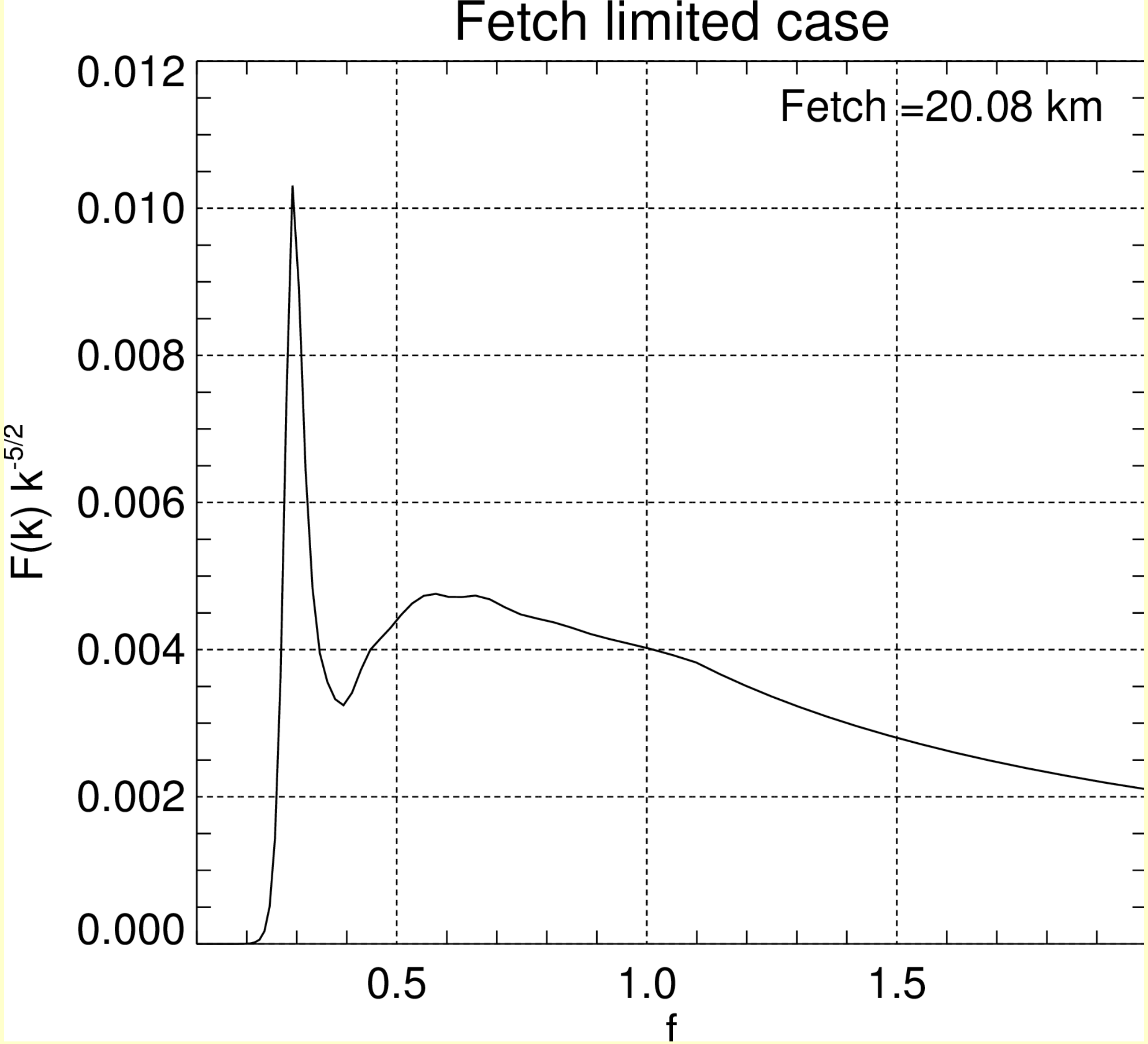

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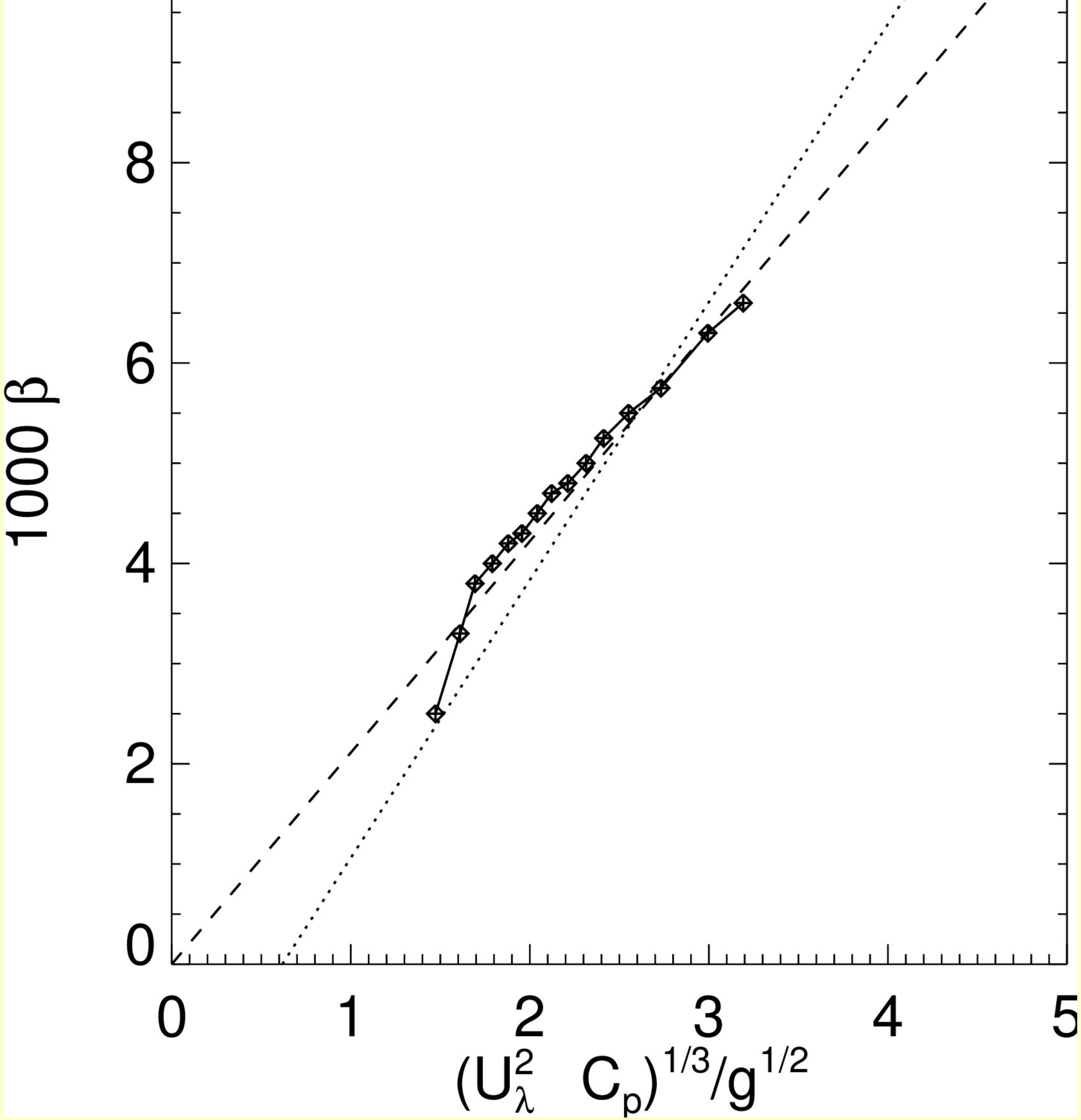
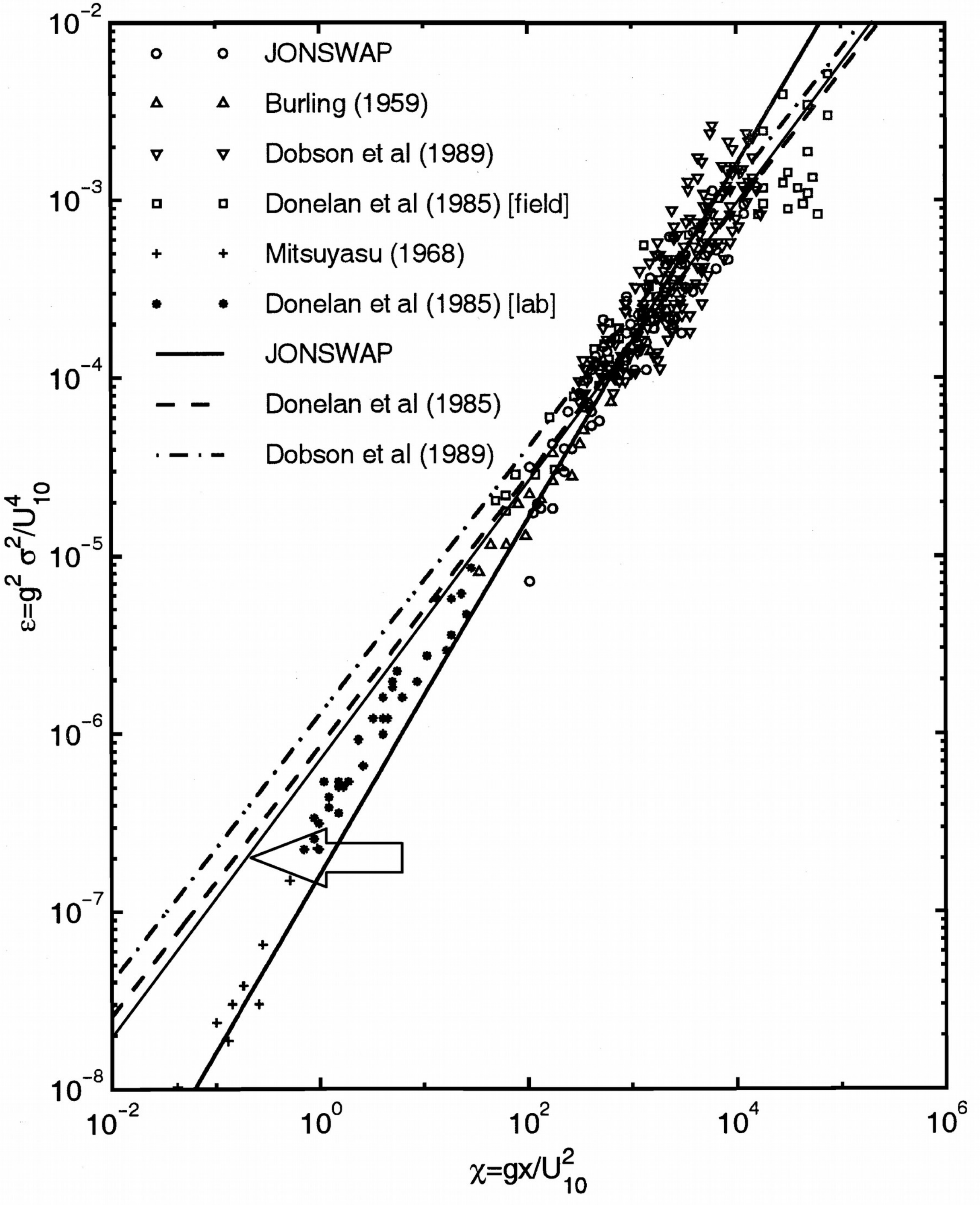
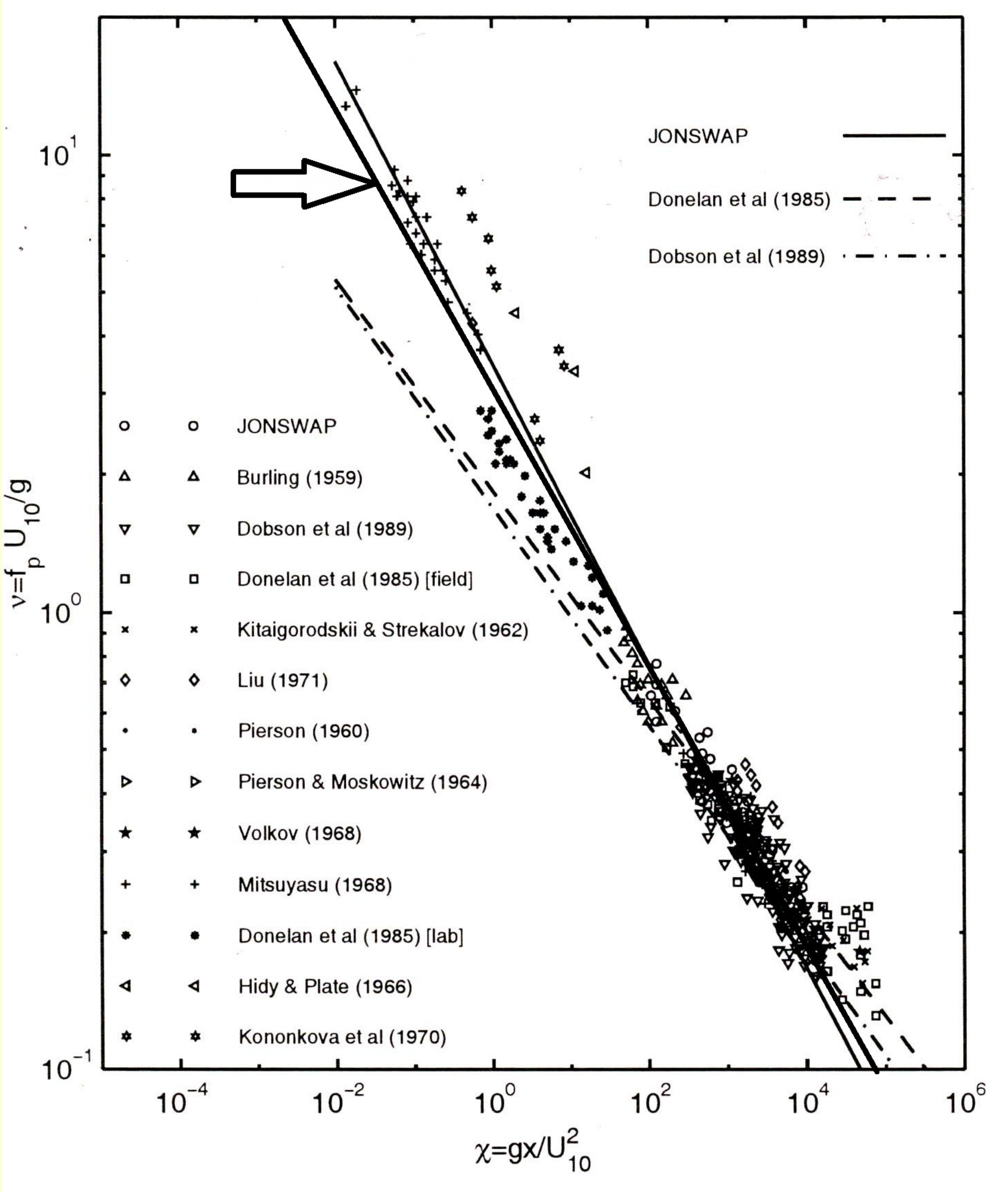


Experimental (dotted line), theoretical (dashed line) and numerical (diamonds) 1000β versus specific velocity for wind speed 10 m/sec.

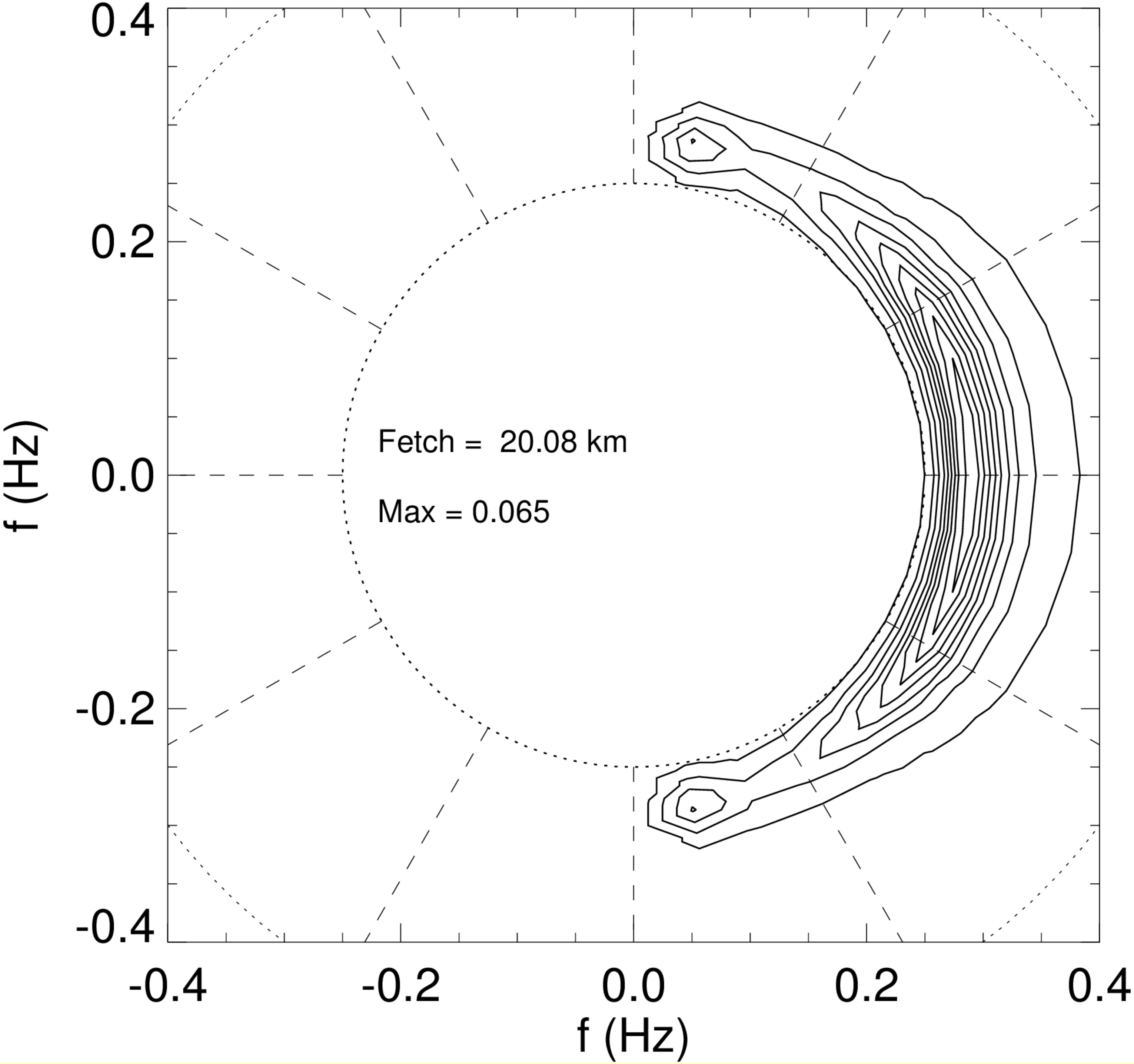


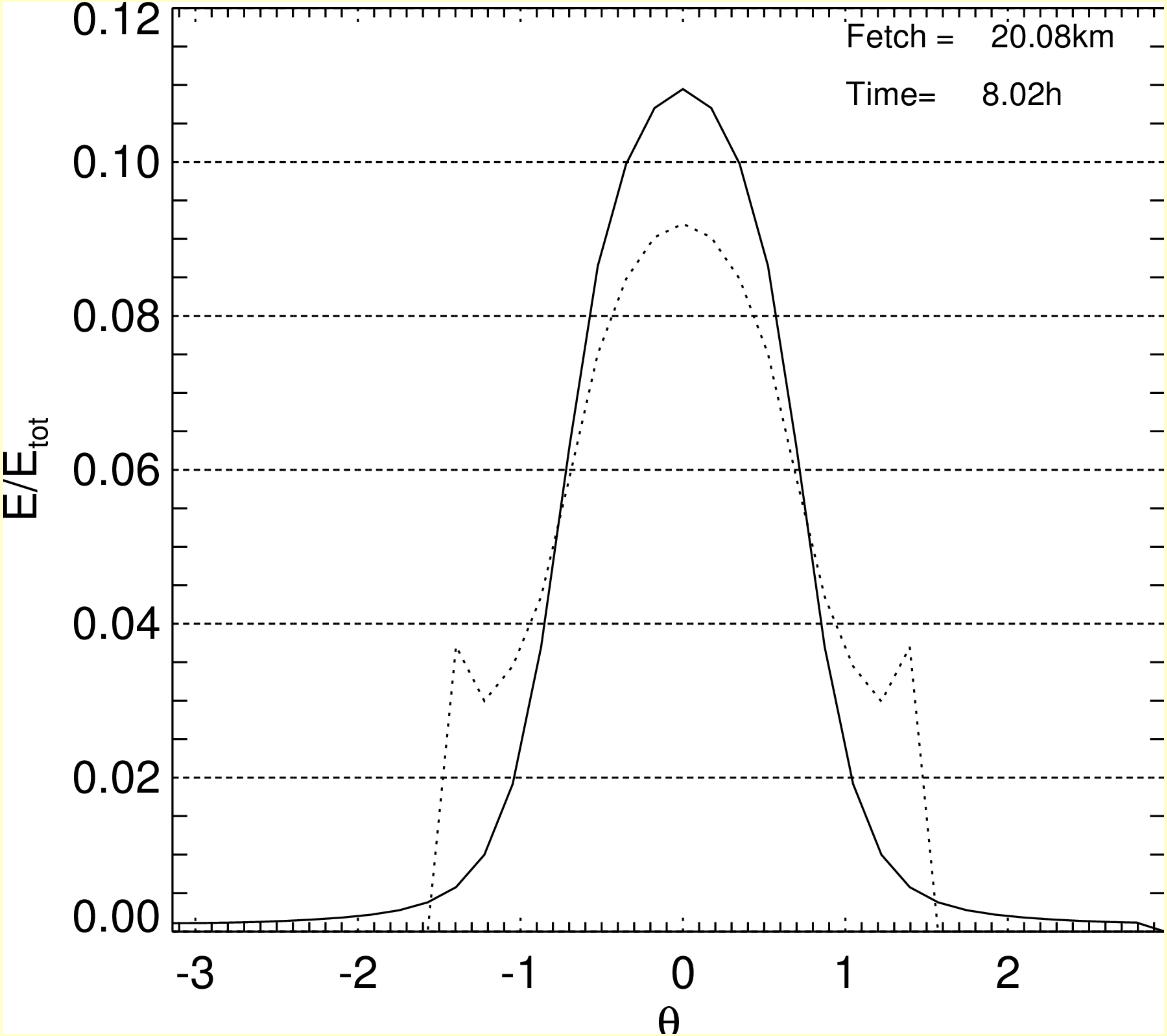
Energy versus fetch, adapted from Young 1999



Frequency versus fetch, adapted from Young 1999

Limited fetch case





Fetch = 20.08km
Time= 8.02h
0.12
0.10
0.08
0.06
0.04
0.02
0.00
E/E_tot
-3
-2
-1
0
1
2
θ

CONCLUSIONS:

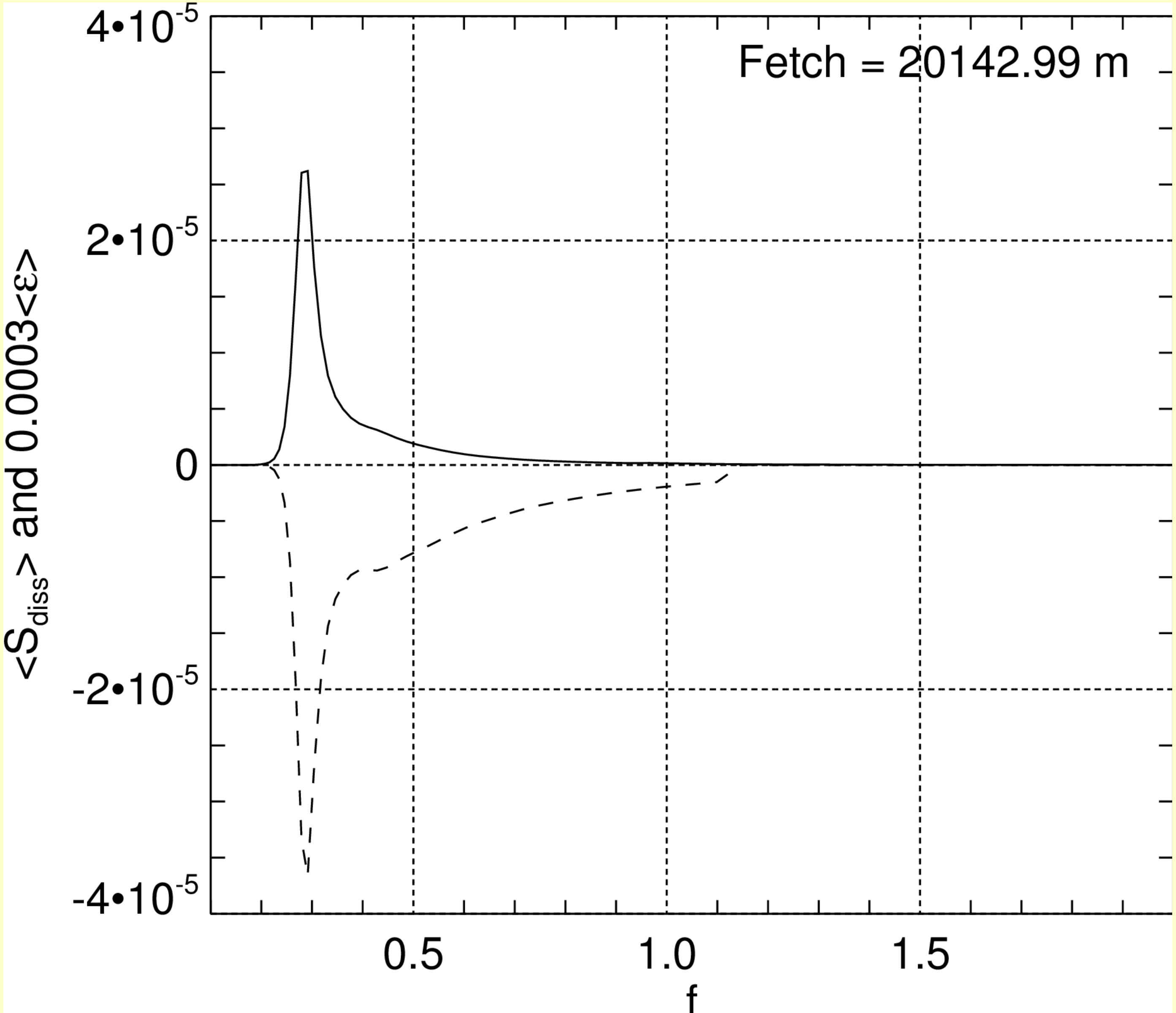
1. ***New set of Hasselmann equation source terms has been introduced, based on XNL, self-similarity analysis and experimental observations***

2. ***ZRP*** S_{in} ***is the same for limited fetch and time domain statements***

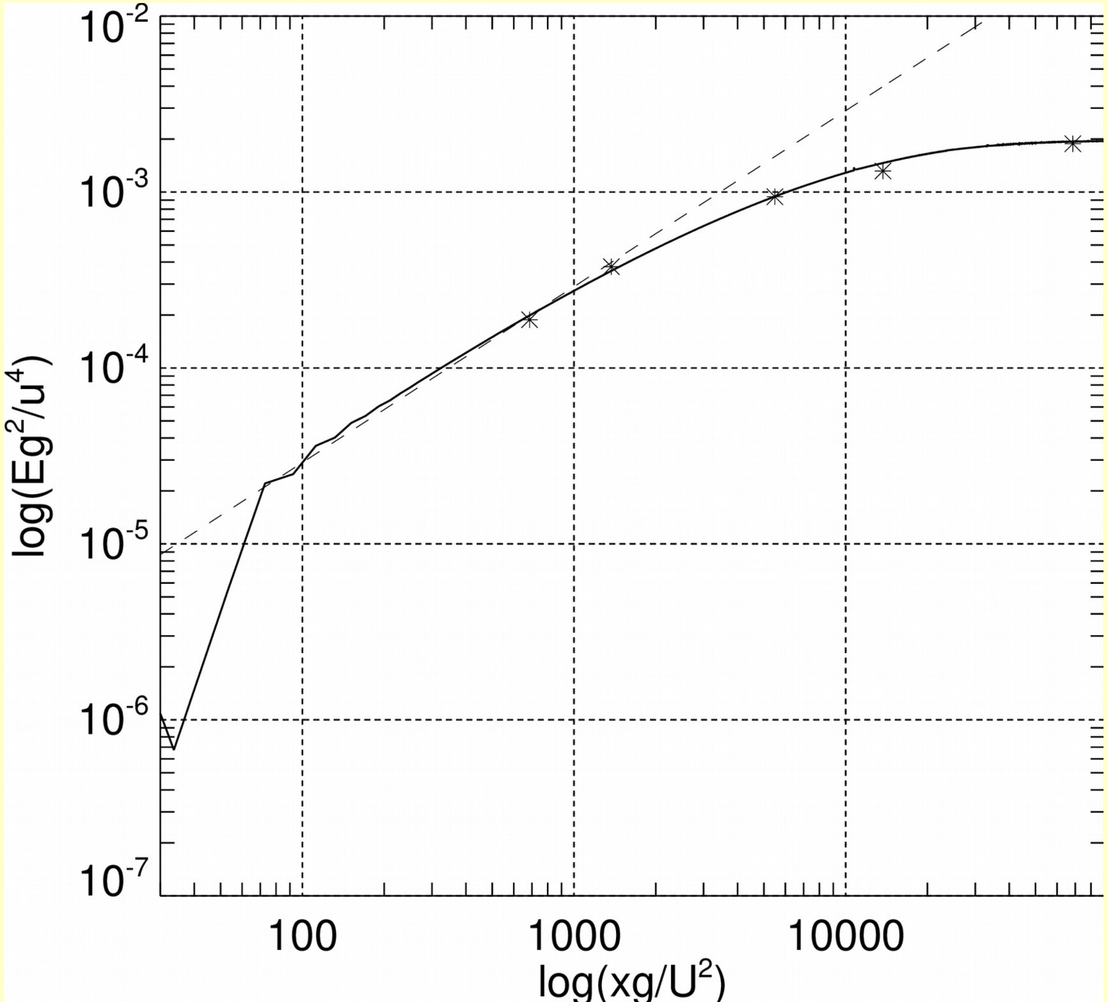
3. ***The new set of source terms reproduces self-similar properties of Hasselmann equation.***

4. ***ZRP*** S_{in} ***and "implicit" dissipation reproduce the results of a dozen of the field experimental predictions***

What we call long-wave, or spectral peak dissipation?



Is our simulation trustworthy?



Komen, S. Hasselmann, K. Hasselmann JPO (1984)