Проявление в природе законов случайных блужданий Колмогорова 1934 года

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In 1933 A. N. Kolmogorov has published first paper on the stochastic motions of a particle.

Next two years he was developing a basics for the theory of random motions

combining the trajectories and velocities for particles under action of random forces.

It is culminated in a two-page paper "Zufallige Bewegungen" Ann. Math. **35**, 116-117, 1934.

He considered there stochastic process with homogeneous probability $p(\underline{\mathbf{r}}, \mathbf{u}, \tau)$

invariant to galilean transformations of the coordinate system.

The Fokker – Planck equation was derived:

$$\frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x_i} = D \frac{\partial^2 p}{\partial u_i \partial u_j} \tag{1}$$

The solution of this equation at the initial condition $p(\underline{\mathbf{r}}, \underline{\mathbf{u}}, 0) = p(0, 0, 0)$

$$p(r,u,\tau) = \left(\frac{\sqrt{3}}{2\pi D \tau^2} \right)^3 \exp\left[-\left(\frac{3r^2}{D\tau^3} + \frac{3ru}{D\tau^2} + \frac{u^2}{D\tau} \right) \right] \tag{2}$$

with $2D = \langle u_i, \dot{r}_i \rangle$ the diffusion coefficient in the velocity space

Later A. M. Obukhov (1959. Adv. Geophys. **6**, 113-115) that for turbulent flows in the inertial interval $D = \varepsilon/2$, the diffusion in the velocity space is equal to a half of the rate of the generation/dissipation of the turbulent kinetic energy. He also noted that there are scales for the mean position and velocity (or energy) and diffusion coefficient in the coordinate space

$$\overline{r^2} \approx \varepsilon\tau^3 \tag{3}$$

$$\overline{u^2} \approx \varepsilon\tau \tag{4}$$

$$K = \overline{ru} \approx \varepsilon\tau^2 \tag{5}$$

Express time from (3) and put it to (4) and (4):

$$u^2 = \varepsilon\left(\frac{r^2}{\varepsilon}\right)^{1/3} = (\varepsilon r)^{2/3},$$

$$K = \varepsilon\left(\frac{r^2}{\varepsilon}\right)^{2/3} = \varepsilon^{1/3} r^{4/3}$$

i.e. we obtain expressions for the velocity structure function

and for the Richardson-Obukhov turbulent diffusion laws.

The expression (3) $\overline{u^2} \approx \varepsilon\tau$ was first published in 1944 by L. D. Landau

and is well known for theoreticians

as the time gain of energy under stochastic forcing.

The expressions (3) – (5) are derived for an ensemble of particles

as asymptotes for $\tau \to \infty$. But these both conditions we have never in reality.

Therefore we (Gledzer and Golitsyn, 2010) have undertaken

prolonged numerical experiments by solving system of equations

$$\dot{u}_i = a_i, \ \dot{x}_i = u_i \text{ at } \tau = 0,\ a_i = 0,\ u_i = 0, \qquad (6)$$

$i = 1, 2, \ldots, N$ – the general number of couples of such equations.

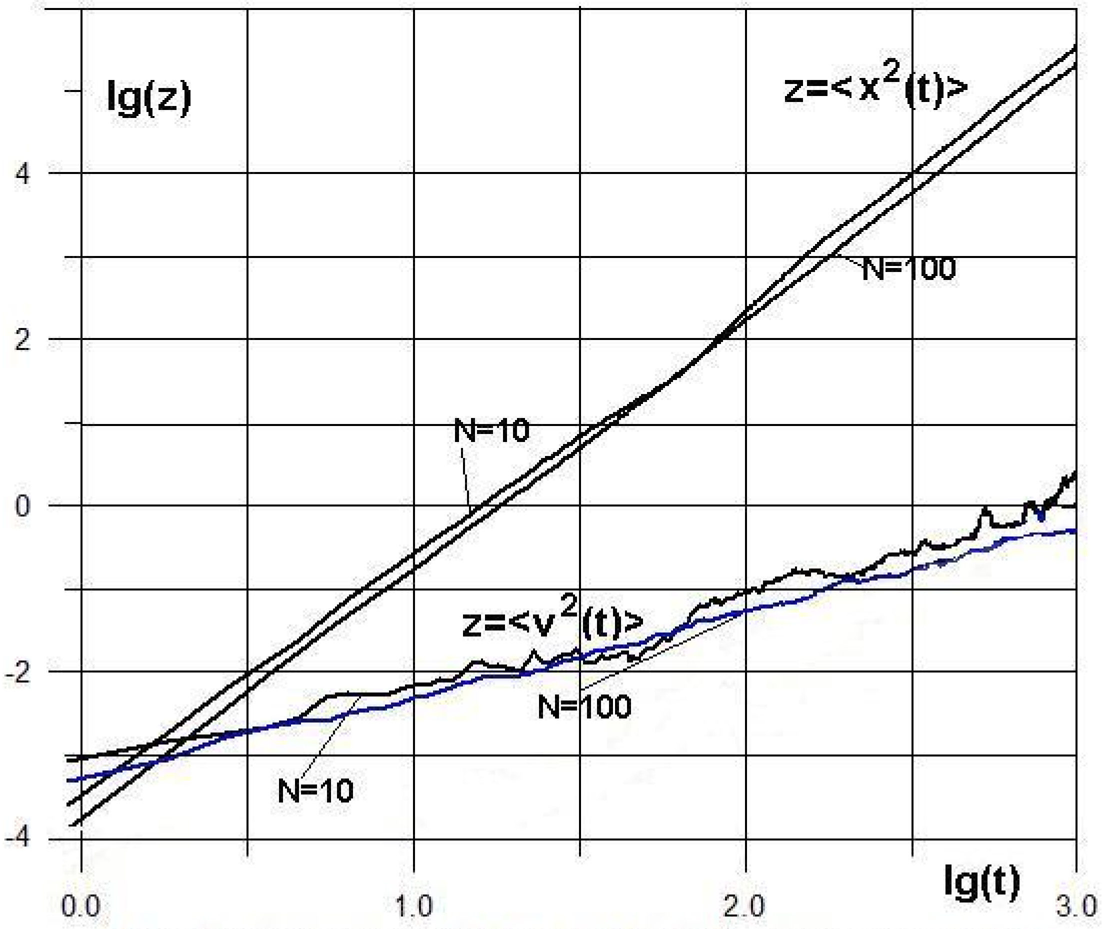
The accelerations a_i for every particle and at each time step

had independent probability distribution functions.

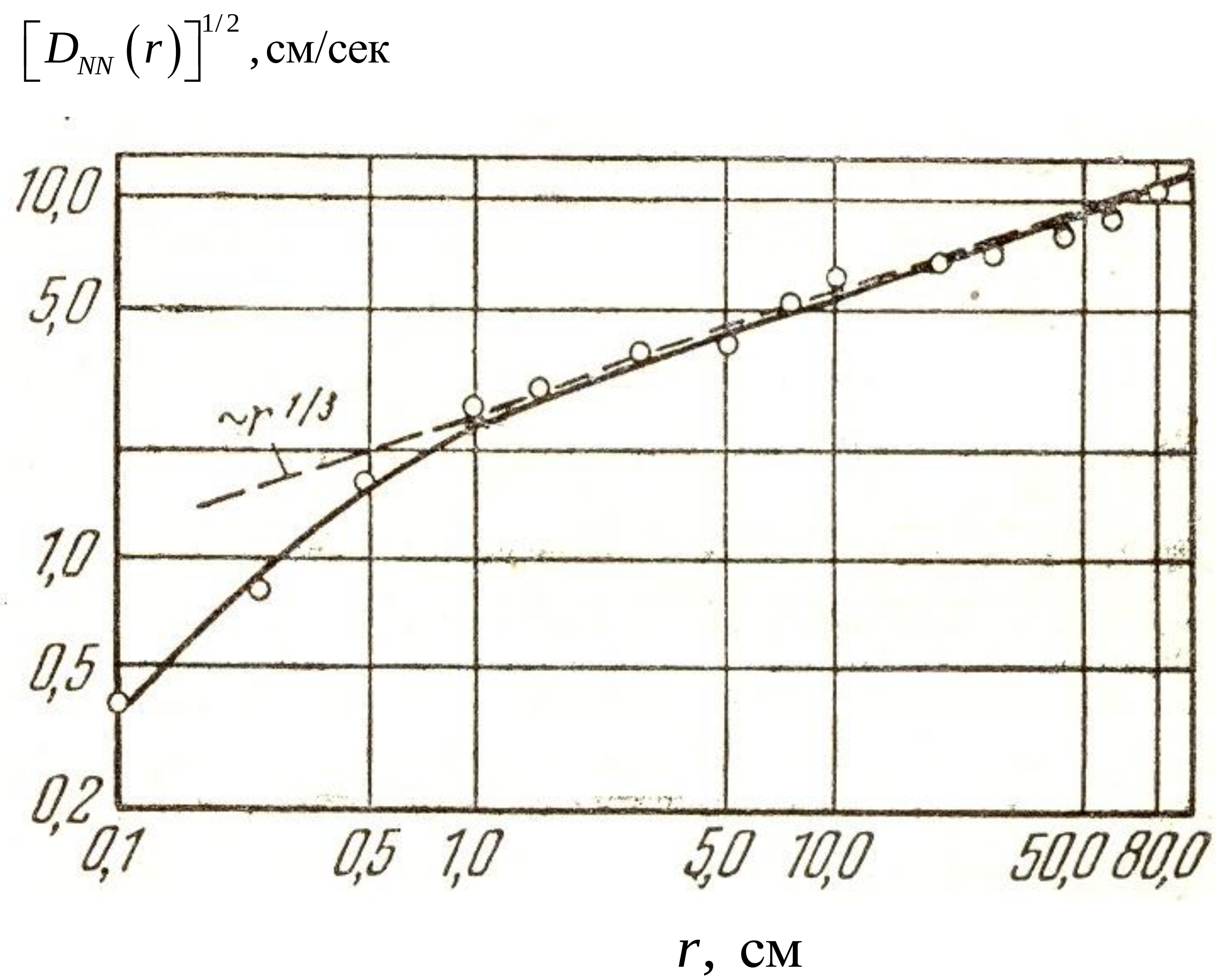
PDF were tried as normal, β and γ distributions with varying numbers of parameters.

The second moments for coordinate and mean velocities

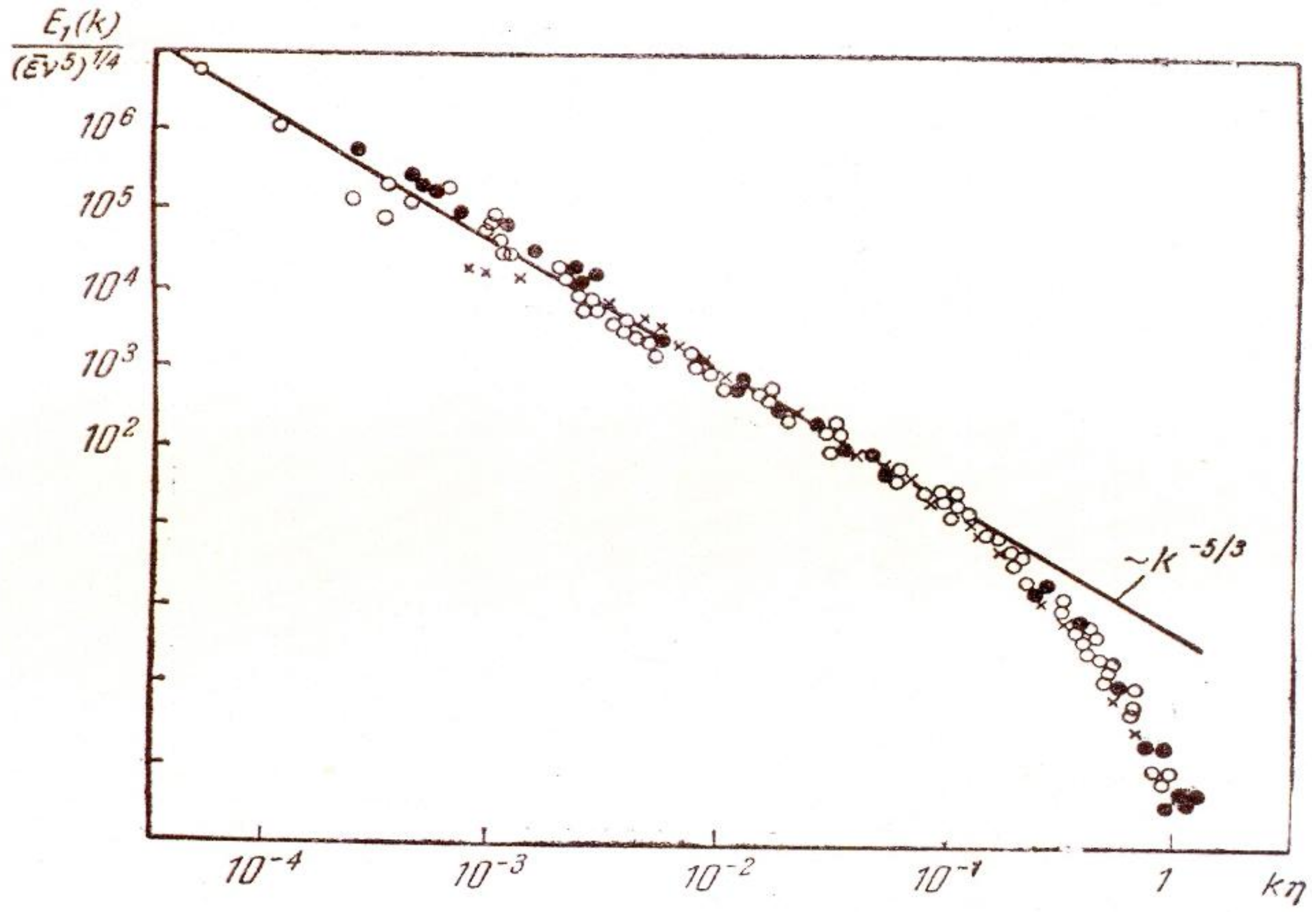
were calculated with time τ.



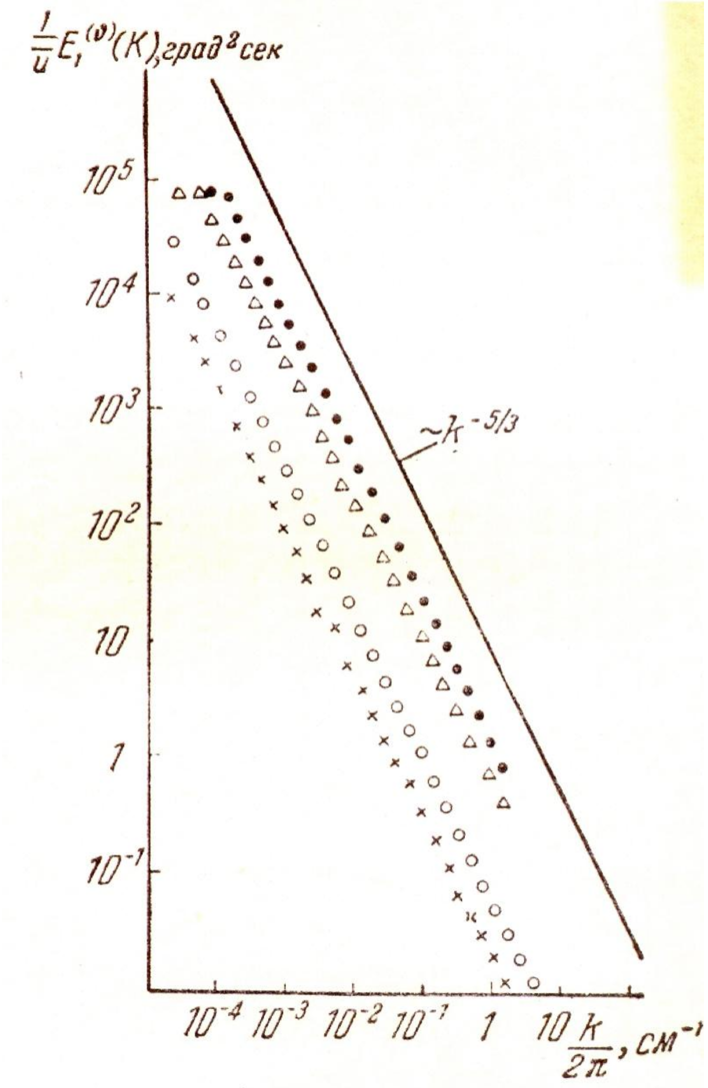
The second moments for velocities and displacements in time for $N=10$ and 100 for β-distributions of random forcings



The structure function for the transversal velocity component obtained by A.M. Obukhov (1949a) after measurement data by Gödecke (1935)

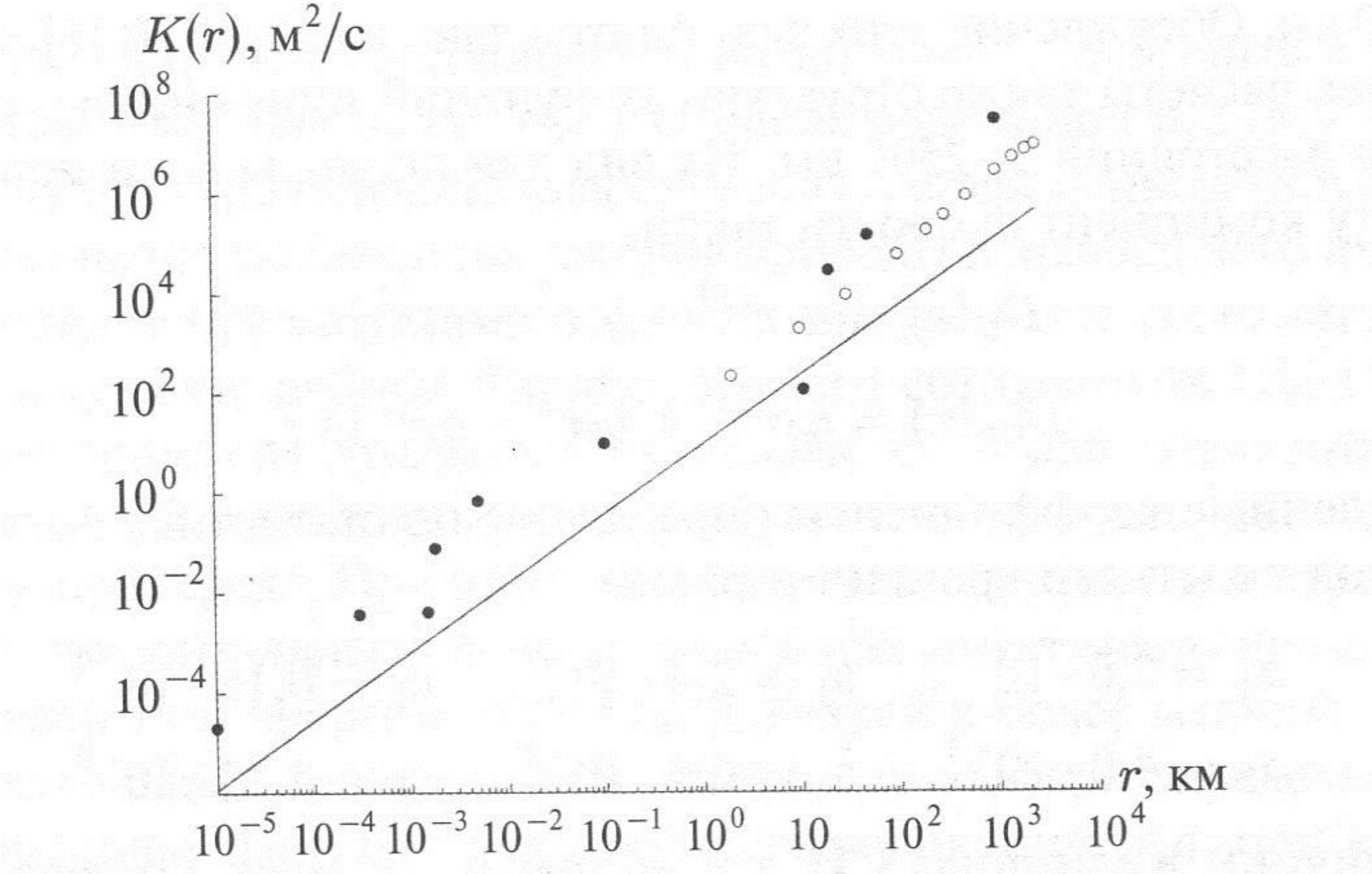


Normalized velocity spectrum obtained (1) in aerodynamic tube, (2) in near water air layer, (3) in ocean tidal current G. I. Taylor hypothesis $\omega = ku$



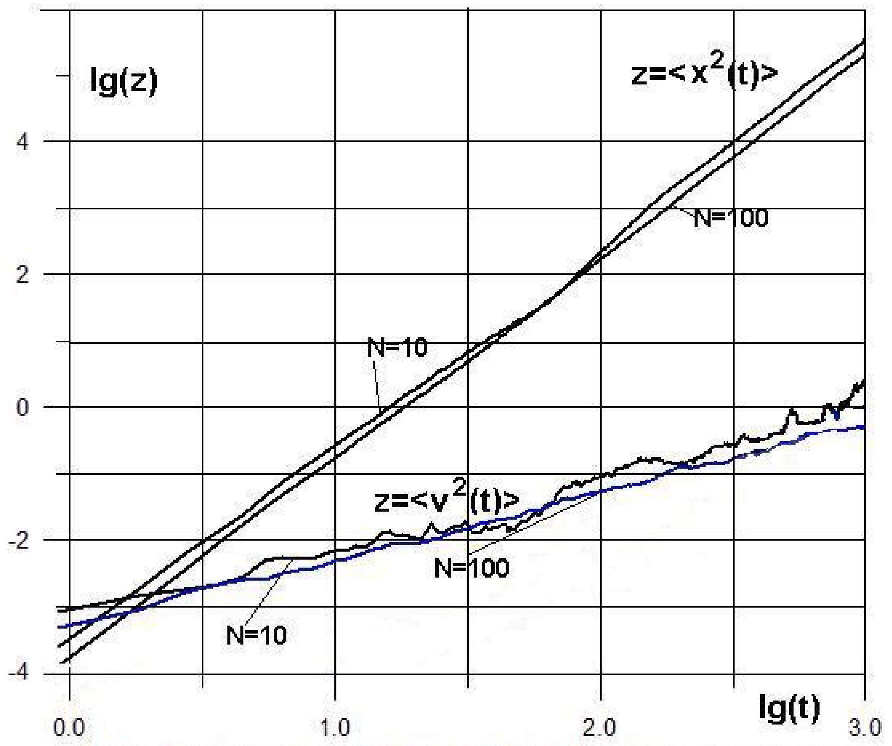
Temperature spectra in atmospheric surface layer

(A. S. Gurvich, T. K. Kravchenko, 1961)



Dependence of coefficients of relative turbulent diffusion on the tracer spot size,

●Richardson (1926, 1929), ○ Golitsyn (2004) after data by E. Lindborg (1999).



All is very good with our computations (Gledzer&Golitsyn, 2010) illustrating Kolmogorov (1934), but… strictly speaking comparisons with reality is not quite correct:

in calculations with numbers of particles the third moment $\overline{U^3} > 0$,

but in real turbulence the asymmetry $\overline{U^3} / \left(\overline{U^2}\right)^{3/2} < 0$!

In our calculations <u>NO</u> dissipation!

However, there are models of 2D turbulence with $S > 0$

Kraichnan (1966): Isotropic turbulence and inertial range structures

(Phys. Fluids, **9**, 1937-1943).

In the model there is the wave number k_0 of vorticity generation.

The vorticity input $M = \frac{d}{dt}\overline{\omega^2}$ and at $k > k_0$ the velocity energy spectrum is $E_n(k) = Mk^{-3}$.

For $k < k_0$ there is the inverse energy cascade with ε and

$E_n(k) = \varepsilon^{2/3} k^{-5/3}$, $k < k_0$

The basic scales, at least in theory, do not depend

on the sign of the energy cascade, whether it goes

from larger scales to smaller once, direct, or vice versa – inverse cascade!

Anyway, our calculations, Gledzer&Golitsyn, 2010,

are quite illuminating the results of Kolmogorov (1935)

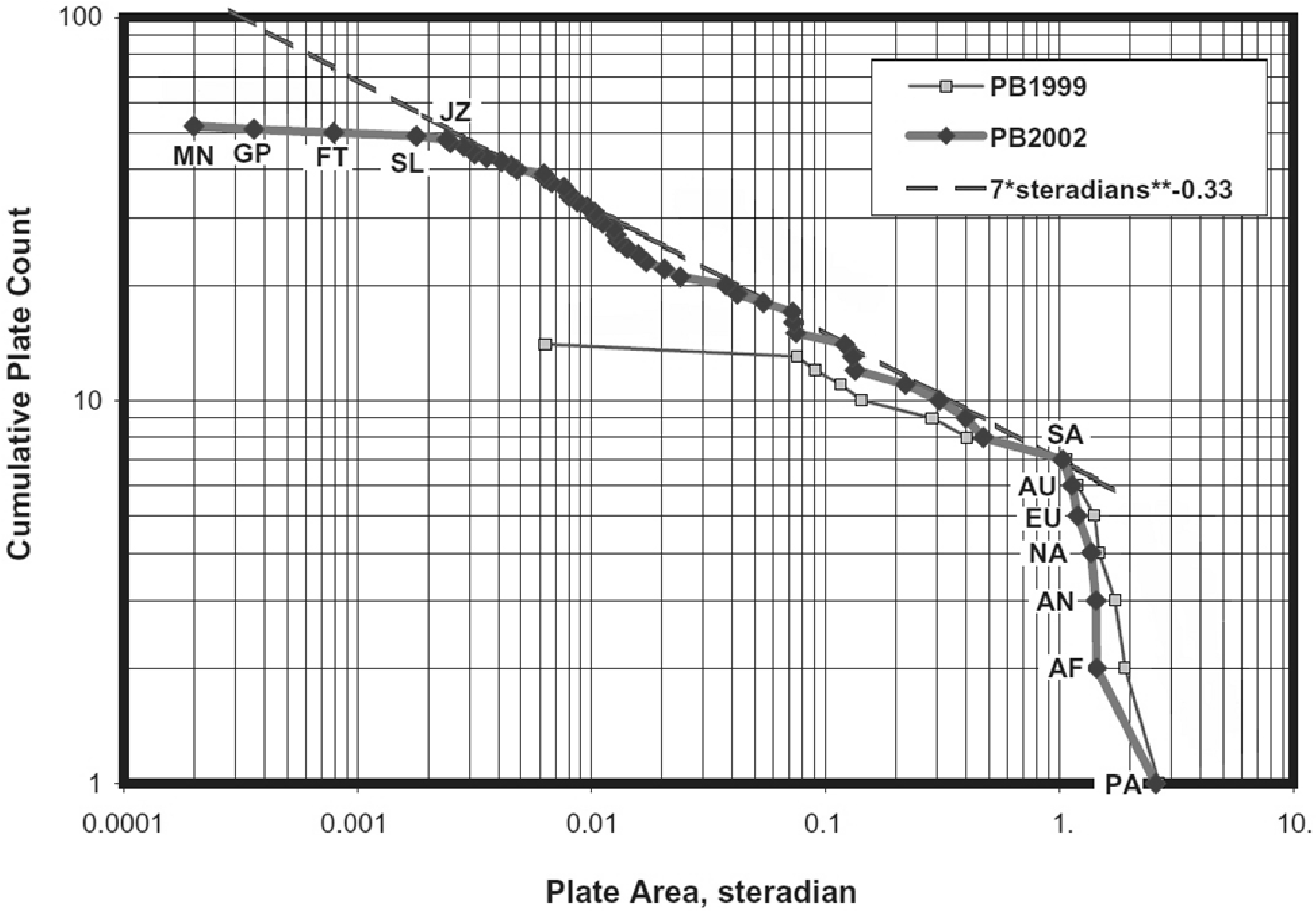
showing how close he has approached the results of 1941

which was first demonstrated by Obukhov in 1958 at Oxford, International Symposium on air pollution (Adv. Geophys., **6**, 1959)

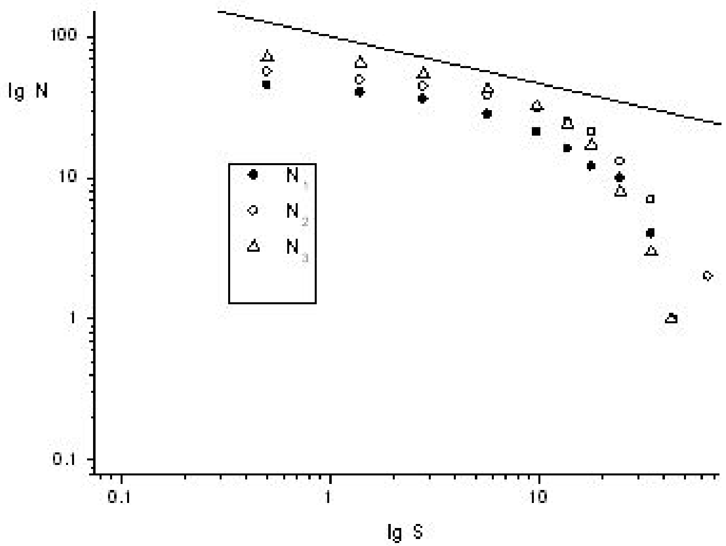
The most striking example of regularities in chaos is demonstrating the cumulative size distribution of lithospheric plates found by Bird (2003):

$$N(\geq S) = 7S^{-n},\ n = 0.33. \qquad (7)$$

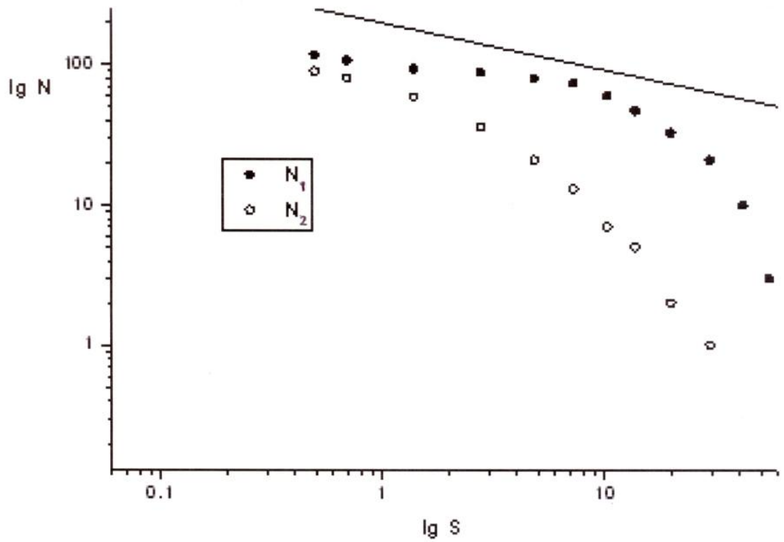
Bird has determined 52 plates with 6 large ones: Pacific (2.53), African (1.44), Antarctican (1.43), North American (1.37), Eurasian (1.20) and South American (1.20). These are of 1 By age or more.
The rest have been formed during the breack of the last supercontinent about 100 My ago. The largest is Somali 0.47. Numbers are in steradians, the whole Earth's surface is 4π steradians, or 1 str = $40.6 \cdot 10^6$ km^2. RF is 0.42 str.



Cumulative distribution on areas of lithospheric plates by Bird (2003).



lg N
lg S
100
10
1
0.1
0.1
1
10
N1
N2
N3



lg N
lg S
100
10
1
0.1
1
10
N1
N2

Cumulative distribution of plates by their areas has the dimension of frequency, or inverse time obtained from histograms

$$\left[N(\geq S)\right]=\frac{1}{T}, \text{ histograms } \left[N(S)\right]=\frac{1}{TS};\ N(\geq S)=\int_{S_{\min}}^{S_{\max}} N(S)\,dS.$$

We have in Kolmogorov (1935) solution the scale

$$\overline{r^2}=\varepsilon\tau^3, \qquad (3)$$

the mean square of coordinate. Equate it to the plate area and obtain

$$N(\geq S)=a\left(\frac{\varepsilon}{S}\right)^{1/3}. \qquad (8)$$

An estimate of the energy generation rate ε for geotectonic and seismicity produces $\varepsilon \approx 10^{-11}$ m^2/s^3 = 406 str$\cdot$$s^{-3}$, Golitsyn (2007).

Compare (8) with estimate by Bird (7) and obtain the numerical coefficient *a* in (8) as *a* = 0.95 $\approx$ 1. Einstein (1911) was saying that the right dimensional (here an exact) formula should have the numerical coefficients neither very small or large, better O(1).

- Kazansky A. B., Archiv of Glaciology, **19,** 239, (1987). $D_u(r) \sim r^n$, $n \approx 2/3$.
- Taguchi, Y-h. Physica D, **80,** 61 (1995). $E_u(k) \sim k^{-5/3}$ box of small balls.

  Energy spectrum with no mean flow.

- Galaxies, Golitsyn (2017), *L* luminocity, *M* mass, *R* size, *U* velocity, σ – random velocities.

  $\frac{L}{M} = \varepsilon \sim 10^{-5}$ m^2s^{-3} ≈ const for most of objects

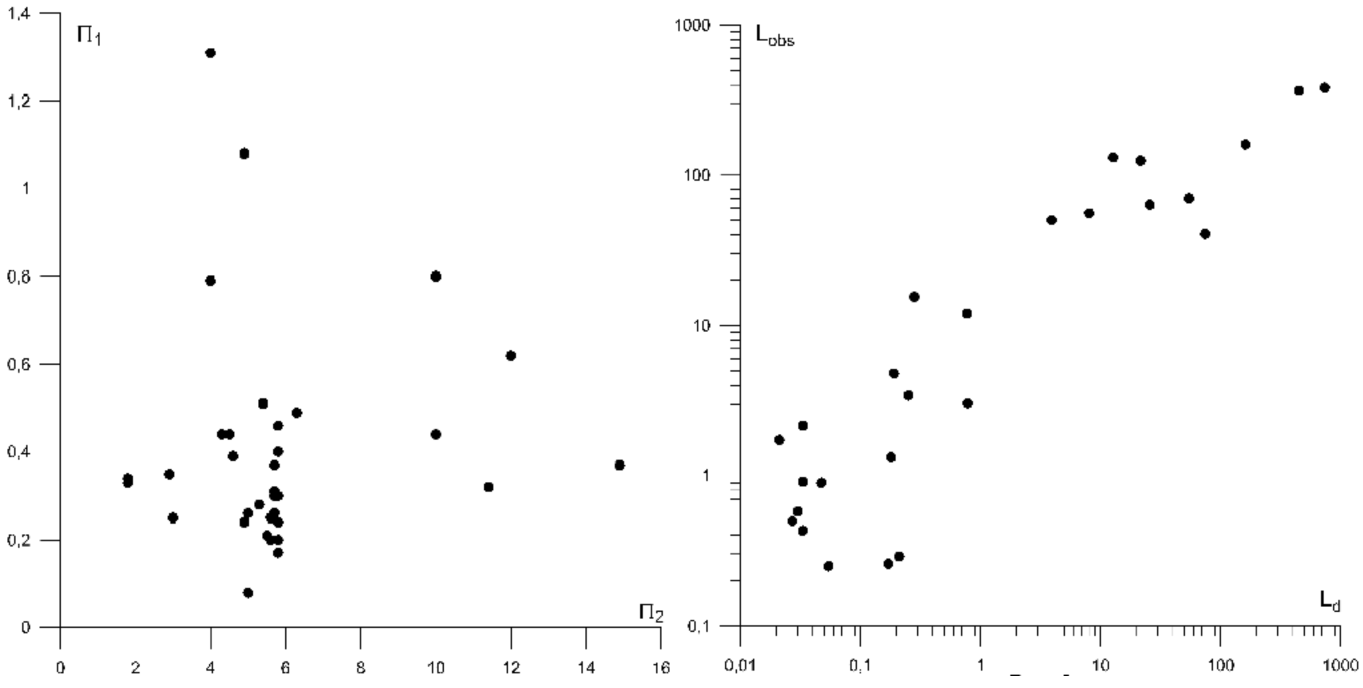
  $L \sim U^3 / R$ Tully – Fisher (1977) or $\frac{RL}{M} \approx \varepsilon R \sim U^3$

  $\sigma \sim (\varepsilon R)^{1/3}$ Golitsyn (2017) Kolmogorov (1941)

There is a partial solution of (1) numbered (4):

$$\langle u^2 \rangle = \varepsilon\tau$$

we obtain that energy per unit mass in stochastic forcing increases with time linearly, result known for long time. The numerical coefficient at rhs may depend on similarity parameter(s) if any. In such a way a Gutenberg – Richter law for frequency size distribution of earthquakes was obtained (Golitsyn, 1996, 2001) for very strong quakes. For lesser quakes the coefficient depends on the similarity parameter: the ratio of the fault length to the width of the lithosphere. Analogously the energy spectrum of cosmic ray particles was explained (Golitsyn, 1998, 2005).


Π_1
Π_2
1,4
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1
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0 2 4 6 8 10 12 14 16
L_{obs}
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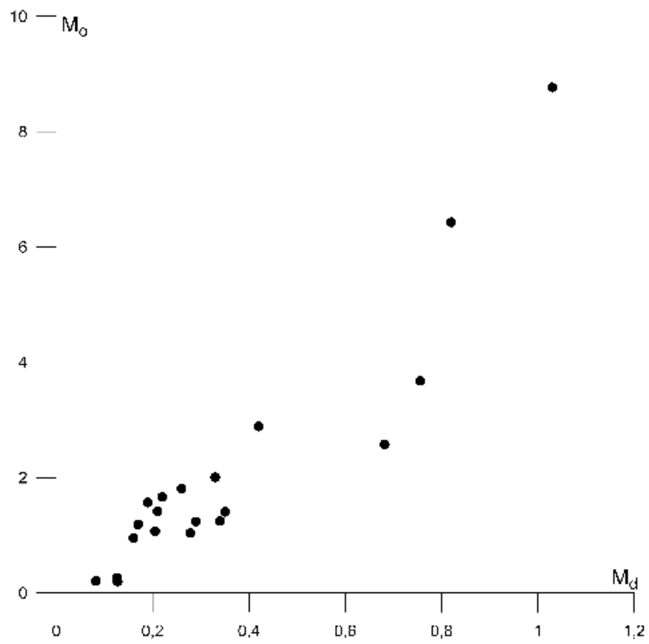
10
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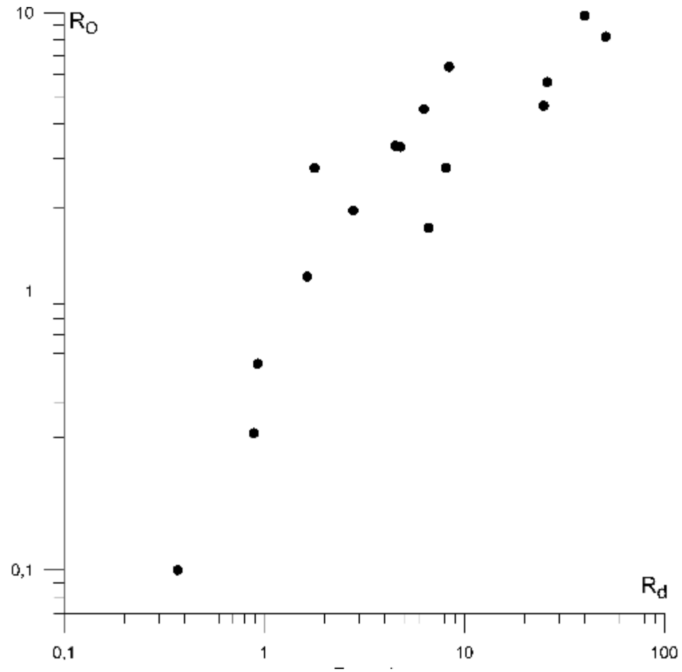
1
1,2
M_o
M_d


10
1
0,1
0,1
1
10
100
R_O
R_d

Burning gas mixture flame propagation (Gostintsev and Fortov, 2008)

$$u = (\varepsilon t)^{1/2}$$

$$R = (\varepsilon t^3)^{1/2}$$

Here is nothing random, the same dependencies are dynamical and produced just from dimensional analysis.

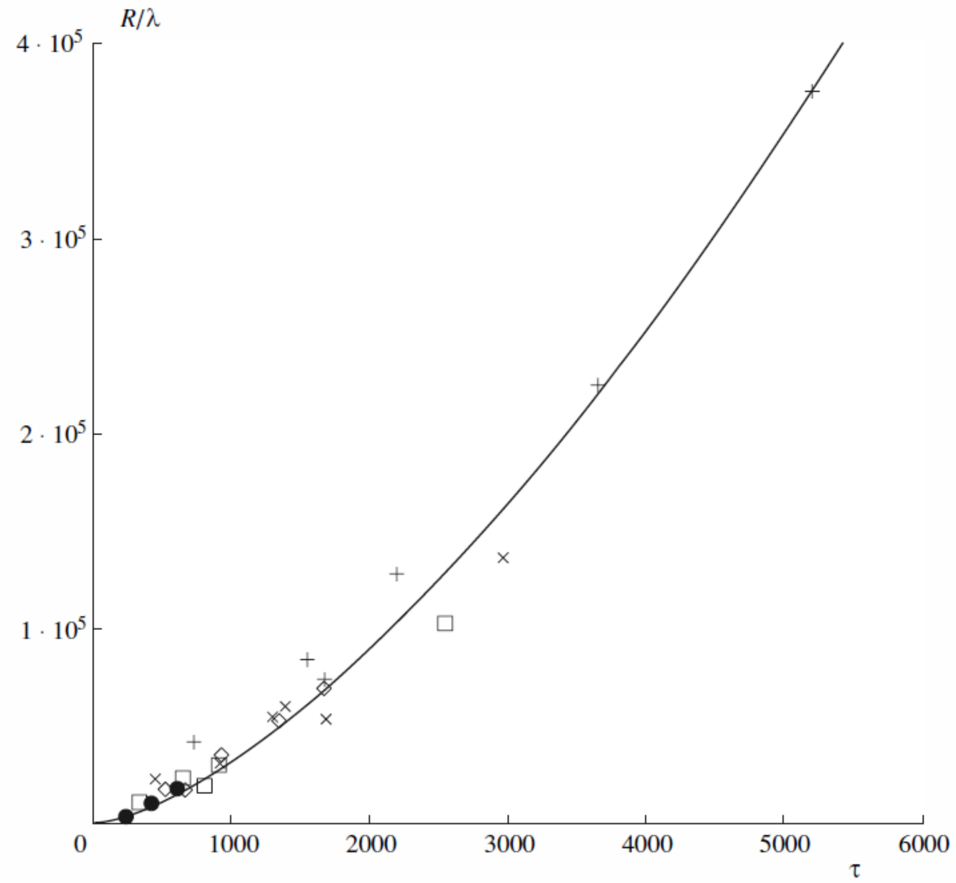


Рис. 1. Зависимости R/λ от τ для свободных турбулентных сферических пламен (экспериментальные точки, обозначенные различными символами, соответствуют горючим смесям из таблицы); теоретическая кривая построена в соответствии с (6).

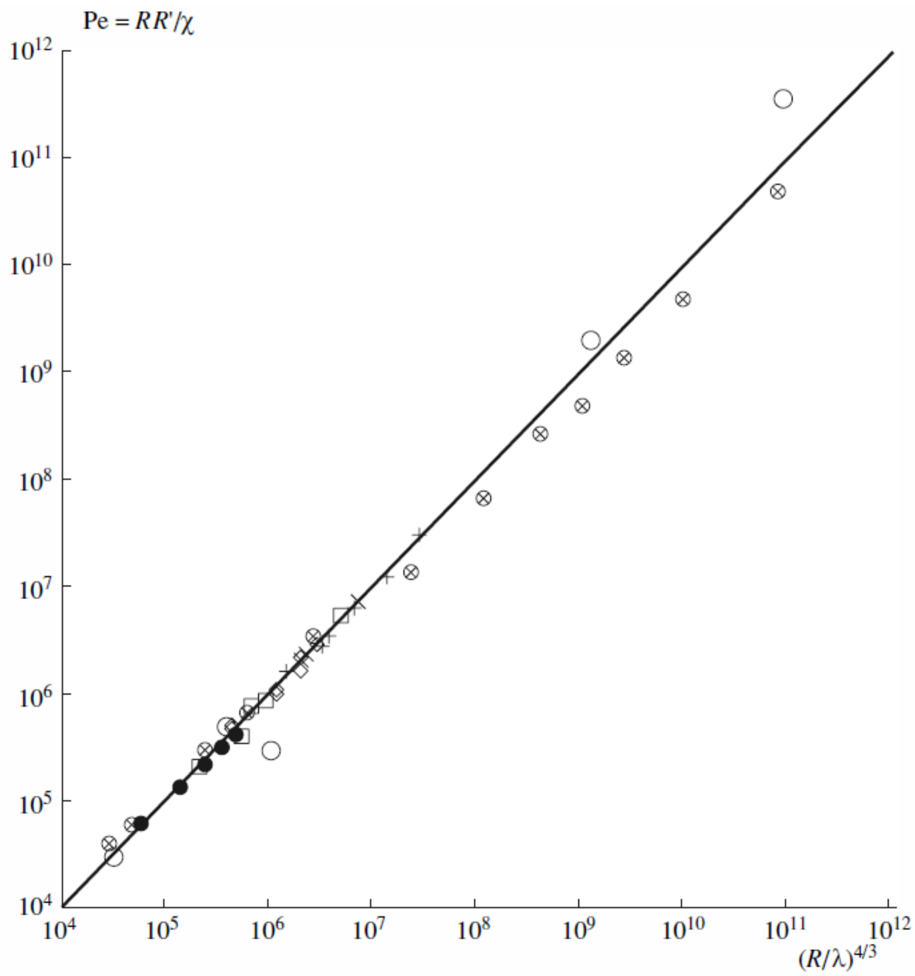
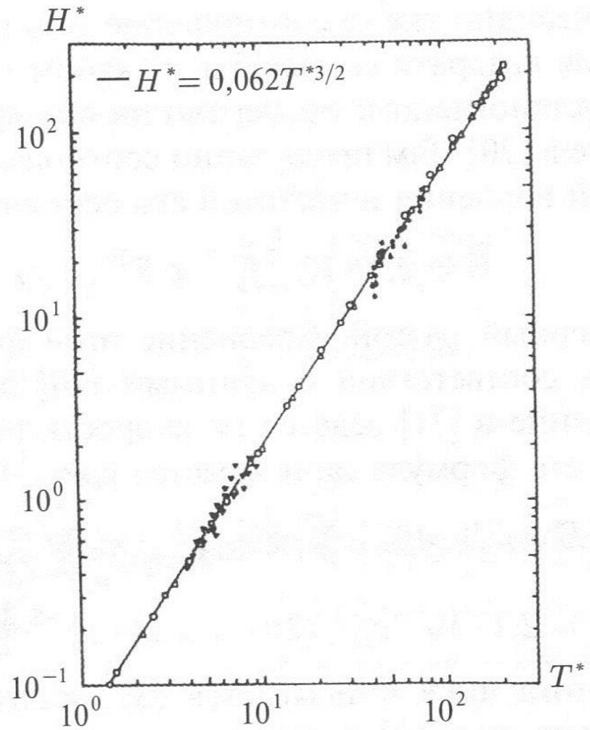


Рис. 2. Логарифмическая зависимость числа Пекле Pe от $(R/\lambda)^{4/3}$. Прямая – теоретическая зависимость, построенная в соответствии с (6). Точки – то же, что и на рис. 1.



H^*
— $H^* = 0{,}062T^{*3/2}$
10^2
10^1
10^0
10^{-1}
T^*
10^0
10^1
10^2

Wind sea waves: complicated phenomenon. Detailed studies since 1960 for strategy purposes. The most striking discovery in 1978 Toba Y. The mean square of the peak wave height - the proportionality to the cube of its period

$$h^3 \propto T^3$$

needs a simple fundamental explanation. Basic notions

$$F = \frac{gx}{U^2}\text{, fetch,}$$

x distance from the leeward shore, U wind, g = 9.8 m/s^2

Fetch laws:

$$\frac{Uf_p}{g} = AF^{-\alpha}, \; f_p = T^{-1}, \text{ wave period}$$

$$\frac{g^2\varepsilon}{U^4} = BF^{\beta}, \; \varepsilon = \frac{h_s^2}{16} \text{ - substantial wave height}$$

After dozens of field experiments $3\alpha \approx \beta$.

After eliminating the fetch *F* from two experimental laws

$$h_s^2 = \varepsilon_t T^3, \; \varepsilon_t = 16A^3BgU, \; t$$

stands for Toba

$$s = \frac{1}{2}kh_s \propto \Omega^{-n}, \; n = \frac{\alpha}{\alpha - 1}, \quad \alpha = \frac{1}{3} \quad n = \frac{1}{2}, \qquad \alpha = \frac{1}{4} \quad n = \frac{1}{3}$$

The last relationship between growing with fetch the wave height
and its period is the sequence of the random forcing by the wind spectral component
of the water surface, reflecting slow growth of the second moment of the wave
height with increasing time of the wave period.
There are basic internal relationships in the wind waves established first empirically.
The growth increase with time the first moments of velocity, i.e. of energy
proportional to time and of the position as t^3 is the basic feature of the processes
with permanent supply of energy. Another words were used by A. M. Obukhov:
the diffusion at the velocity (or momentum) space, or random walks in such a space.

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with $2D=\langle u_i,\dot{r}_i\rangle$ the diffusion coefficient in the velocity space, equal to ε

Thank you for attention