Численное моделирование процессов обмена между каплями и воздухом в приводном атмосферном пограничном слое

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Sea surface at strong wind-forcing conditions
Formation of spume drops at the water surface at wind speed about 25 m/s captured by high-speed video recording
Typical high-speed video image showing spray droplets shed by a breaking wave, captured at 200 Hz. 

Droplet volume concentration for wind speed 16m/s at different heights above the water surface (in cm)
Data on droplets generation by the wind (Andreas et al. 2010, JGR)

Droplet number density at different wind speeds at heights from 1 to 2m.

Droplet terminal velocity normalized by friction velocity and Karman constant (k=0.4)

\[ V_s = g \frac{d^2 \rho_w}{18 \nu \rho_a} \]
Figure 1. Processes in the droplet evaporation layer. Air and sea are always exchanging sensible \(H_{s,\text{int}}\) and latent \(H_{L,\text{int}}\) heat right at the interface. Both fluxes can go in either direction, depending on the local air–sea temperature and humidity differences. The labelled circles depict an individual spray droplet. This droplet cools rapidly, thereby giving up sensible heat. Its evaporation yields water vapour but extracts latent heat from the air. \(Q_L\) and \(Q_S\) are the latent and sensible heat fluxes associated with this single droplet. The interfacial and spray fluxes combine to give the total sensible \(H_{s,T}\) and latent \(H_{L,T}\) heat fluxes coming out of the top of the droplet evaporation layer.
Filed experiment during typhoons “Skip” and “Tess” (1988): cooling of the air boundary flow

FIGURE 2  The marine boundary layer cooling associated with strong winds in two tropical cyclones, shown as a scatter diagram of observed air-sea temperature difference plotted against wind speed (Pudov, personal communication, 1991).
Phenomenological models
(Fairall et al. 1994, Kudryavtsev & Makin 2011): consider Reynolds-averaged (RANS) equations where the impact of drops on the air momentum, temperature and humidity is modeled by source terms obtained by phenomenological closure assumptions.

Lagrangian stochastic models

Direct numerical simulation
Druzhinin et al. (2017): DNS of turbulent particle-laden Couette flow laden with monodisperse drops over waved water surface. Only momentum exchange between air and drops taken into account, but not the heat exchange and drops evaporation effects.
OBJECTIVE

To perform DNS of a turbulent, droplet-laden air flow over waved water surface taking into account momentum, heat and moisture exchange between air and drops.

3D Navier-Stokes equations for the air including the impact of discrete drops and the equations of motion of individual drops are solved simultaneously in curvilinear coordinates in a frame of reference moving the phase velocity of the wave.

Two-dimensional water wave with wave slope up to $ka = 0.2$ is considered. The shape of the water wave is prescribed and does not evolve under the action of the wind and/or drops.

Droplet mass concentration $0.03$ is attained (up to $12 \times 10^6$ drops with diameter from $O(10 \ \mu m)$ to $O(100\mu m)$ are considered) to get a significant impact of drops on the air flow.
**Schematic of numerical experiment**

- **T-temperature**
- **H-humidity**
- $U_0$- air bulk velocity

Domain sizes: 

\[ L_x = 6\lambda \quad L_y = 4\lambda \quad L_z = \lambda \]

\[ \text{Re} = \frac{U_0\lambda}{\nu} = 15000 \quad \text{- bulk Reynolds number} \]

**Tropical cyclon conditions:** $T_a=27^\circ C$, $T_w=28^\circ C$, $H_w=98\%$, $H_a=80\%$

**Polar low conditions:** $T_a=-20^\circ C$, $T_w=0^\circ C$, $H_w=98\%$, $H_a=70\%$

$\frac{c}{U_0}=0.05$ - wave celerity

$ka = 0.2$ - maximum wave slope
GOVERNING EQUATIONS: AIR FLOW
Eulerian framework

**Momentum**
\[
\frac{\partial U_i}{\partial t} + \frac{\partial(U_i U_j)}{\partial x_j} = -\frac{1}{\rho_a} \frac{\partial P}{\partial x_j} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} + \delta_{iz} g \frac{T}{T_a} + \sum_{n=1}^{N_d} f_{Ui}^n
\]

**Continuity**
\[
\frac{\partial U_j}{\partial x_j} = 0
\]

**Temperature**
\[
\frac{\partial T}{\partial t} + \frac{\partial(T U_j)}{\partial x_j} + \frac{\partial(T_{ref} U_3)}{\partial x_3} = \kappa \frac{\partial^2 T}{\partial x_j \partial x_j} + \sum_{n=1}^{N_d} f_T^n
\]

**Humidity**
\[
\frac{\partial H}{\partial t} + \frac{\partial(H U_j)}{\partial x_j} + \frac{\partial(H_{ref} U_3)}{\partial x_3} = D \frac{\partial^2 H}{\partial x_j \partial x_j} + \sum_{n=1}^{N_d} f_H^n
\]

\[
T_{ref}(z) = T_w + (T_a - T_w) \frac{z}{\lambda}
\]

\[
H_{ref}(z) = H_w + (H_a - H_w) \frac{z}{\lambda}
\]

- reference profiles

**Drops feedback contributions**

\[N_d = \text{const} - \text{total number of drops in DNS}\]
GOVERNING EQUATIONS: DROPS
Lagrangian framework

coordinate
\[
\frac{dr_i^n}{dt} = V_i^n
\]

velocity
\[
\frac{dV_i^n}{dt} = \frac{1}{\tau_n}\left(U_i(r^n) - V_i^n\right)\left(1 + 0.15\text{Re}_n^{0.687}\right) - \delta_{iz} g
\]

temperature
\[
m_n c_w \frac{dT_n}{dt} = 2\pi \kappa' d_n \left(T_a(r^n) - T_n\right)\left(1 + 0.25\text{Re}_n^{0.5}\right) + L_v \frac{dm_n}{dt}
\]

mass
\[
\frac{dm_n}{dt} = 2\pi D'd_n \rho_{sat}^v \left(H(r^n) - H^n_s\right)\left(1 + 0.25\text{Re}_n^{0.5}\right)
\]

\[
\text{Re}_n = \frac{|U(r^n) - V^n| d_n}{v} - \text{drop Reynolds number}
\]

\[
\tau_n = \frac{d_n^2 \rho_n}{18 v \rho_a} - \text{response time}
\]

\[
D' - \text{diffusivity of water vapor}
\]

\[
\kappa' - \text{thermal conductivity coefficient of air}
\]

\[
d_n - \text{diameter}
\]

\[
m_n = \rho_n \pi d_n^3 / 6 - \text{mass}
\]

\[
\rho_{sat}^v - \text{saturated vapor density}
\]

\[
H^n_s - \text{Humidity at the drop surface}
\]

\[
L_v - \text{latent heat of evaporation}
\]

\[
c_w - \text{specific heat of water}
\]

Air velocity, humidity and temperature at each drop location are obtained by Hermitian polynomial interpolation.
Feed-back contributions due to drops

**Momentum**

\[ f_{Ui}^n = \frac{\pi d_n^3}{6} \frac{\rho_d^n}{\rho_a} \frac{1}{\tau_n} \left( V_i^n - U_i (r^n) \right) \left( 1 + 0.15 \text{Re}_n^{0.687} \right) \frac{w(r^n, r)}{\Omega_g} \]

**Temperature**

\[ f_T^n = 2\pi \kappa' d_n \left( T_n - T_a (r^n) \right) \left( 1 + 0.25 \text{Re}_n^{0.25} \right) \frac{1}{\rho_a c_a} \frac{w(r^n, r)}{\Omega_g} \]

**Humidity**

\[ f_H^n = -\frac{1}{\rho_{sat}^v} \frac{m_n}{dt} \frac{w(r^n, r)}{\Omega_g} \]

\[ \text{Re}_n = \frac{|U(r^n) - V^n| d_n}{\nu} \quad \text{- drop Reynolds number} \]

\[ r^n - \text{drop coordinate} \]

\[ r - \text{grid node coordinate} \]

\[ w(r^m, r) - \text{geometrical weighting factor} \]

\[ \Omega_g(r) - \text{grid cell volume} \]

**POINT-FORCE APPROXIMATION is used:**

Each drop is considered as a point, and its feedback contributions to the momentum, heat and moisture of the air flow are distributed to the surrounding grid nodes.
CURVILINEAR COORDINATES

\[ x = \xi - a \exp(-k\eta) \sin k\xi \]
\[ z = \eta + a \exp(-k\eta) \cos k\xi \]

Shape of the water surface: \[ z_b(x) = a \cos kx + \frac{1}{2} + \frac{a^2 k (\cos 2kx - 1)}{2} \]

Mapping over \( \eta \): \[ \eta = 0.5 \left(1 + \frac{\tanh \tilde{\eta}}{\tanh 1.5}\right) \quad -1.5 < \tilde{\eta} < 1.5 \]

Grid of 360 \times 240 \times 180 nodes is employed with mesh sizes:
\[ \Delta x^+ \approx 6 \quad \text{in the horizontal direction} \]
\[ \Delta z_1^+ \approx 0.3 \quad \text{near water surface in the vertical direction} \]
\[ \Delta z_2^+ \approx 3 \quad \text{in the middle of the domain} \]
BOUNDARY CONDITIONS

Air velocity = water velocity in the surface wave:

\[ U(\xi, y, 0) = c(ka \cos kx(\xi, \eta) - 1) \]
\[ V(\xi, y, 0) = 0 \]
\[ W(\xi, y, 0) = cka \sin kx(\xi, \eta) \]

No-slip condition at the upper moving plane:

\[ U(\xi, y, 1) = 1 - c \]
\[ V(\xi, y, 1) = 0 \]
\[ W(\xi, y, 1) = 0 \]

Deviations of temperature and humidity from reference profiles at top and bottom boundaries are put to zero.

All fields are x and y periodic.
Instantaneous vorticity modulus and drops coordinates fields: side view at y=0

- drop terminal velocity; $\kappa = 0.4$; $u_*$ - friction velocity

$V_g = g \frac{d^2 \rho_w}{18 \nu \rho_a}$

$\left( \begin{array}{c} z/\lambda \\ C_m = 0 \\ \omega \end{array} \right)$

$\left( \begin{array}{c} z/\lambda \\ C_m = 0.03 \\ \omega \end{array} \right)$

$\left( \begin{array}{c} z/\lambda \\ ka = 0.2 \\ C_m = 0.03 \end{array} \right)$

$\left( \begin{array}{c} z/\lambda \\ V_g / \kappa u_* \approx 1 \end{array} \right)$
Instantaneous vorticity modulus and drops coordinates fields: front view at $x=3\lambda$

- $k\alpha = 0.2$
- $V_g / k\nu_s \approx 1$

No drops
Instantaneous vorticity modulus and drops coordinates fields: top view at \( z/\lambda = 0.04 \) (\( z^+ = 20 \))

- Vorticity modulus, no drops
- Vorticity modulus, with drops
- Drops locations

\( ka = 0.2 \)

\( V_g / \kappa u^* \approx 1 \)
Drops trajectories

\[ d_0 = 100 \, \mu m, \frac{V_g}{(u_\ast \kappa)} = 0.25 \]  \hspace{1cm} (a)

\[ d_0 = 200 \, \mu m, \frac{V_g}{(u_\ast \kappa)} = 1 \]  \hspace{1cm} (b)

\[ d_0 = 300 \, \mu m, \frac{V_g}{(u_\ast \kappa)} = 2.25 \]  \hspace{1cm} (c)

\[ \text{ka} = 0.2 \]
Drops injection:

Drops falling on the water or reaching the upper plane are re-injected in the vicinity of the wave crests in the range:

$$0.01 < \eta / \lambda < 0.05 \quad (5 < \eta u_*/\nu < 25)$$

$$n\lambda - 0.2 < \xi < n\lambda \quad n = 1, \ldots, 6$$

Drop velocity at injection = water surface velocity (Andreas 2004, Troitskaya et al. 2016); initial diameter is in the range

$$100 \mu m \leq d_0 \leq 300 \mu m$$

Drops temperature at injection = water temperature
Lagrangian dynamics of fluxes:

Rate of change of drop momentum

\[ m_d \frac{dV_x^d}{dt} = -\rho_a f_x^d \]

Rate of change of drop temperature

\[ m_d c_w \frac{dT_d}{dt} = -Q_s^d - Q_L^d \]

\[ f_x^d = 3\pi d \nu \left( V_x^d - U_x (r^d) \right) \left( 1 + 0.15 \text{Re}_d^{0.687} \right) \]

- Momentum flux

\[ Q_s^d = 2\pi \kappa' d \left( T_d - T_a (r^d) \right) \left( 1 + 0.25 \text{Re}_d^{0.5} \right) \]

- Sensible heat flux

\[ Q_L^d = -L_v \frac{dm_d}{dt} \]

- Latent heat flux
Drop trajectory for $d_0=100 \, \mu m$

- **Diameter**
- **Distance from water**
- **Temperature (relative to air)**
- **Momentum flux to the air**
- **Latent and sensible heat flux**
- **Enthalpy flux**

$Q_L^d + Q_S^d \approx 0$ - “Wet bulb” state of the drop
Drop trajectory for $d_0=300$ m$\mu$

- Diameter
- Distance from water
- Temperature (relative to air)
- Momentum flux to the air
- Latent and sensible heat flux
- Enthalpy flux
Distance from drop to water

Drop temperature

Momentum flux

Sensible and latent heat and enthalpy fluxes

\[ \eta_d/\lambda \]

\[ \bar{f}_x^d \]

\[ T_d, ^\circ C \]

\[ Q_{s,L} \]

\[ Q_L, Q_{S+Q_L}, Q_S \]
Phase (ensemble)-averaged fields

- Drops volume fraction (concentration)
- Momentum flux
- Sensible heat flux
- Latent heat flux
Drops volume fraction

Momentum flux

z(η)-profiles

Drops temperature

Sensible and Latent heat fluxes

volume fraction

ka = 0.1
ka = 0.2

[°C]

[T_d]

[Q_{SL}]

Q_L
Q_S
Q_S+Q_L
Turbulent fluxes

- Momentum
- Sensible heat
- Humidity
Mean flow profiles

Air Velocity

Temperature

Humidity
CONCLUSIONS

- Drops impose additional drag on the air and cool and moisten the air;
- the drops contribution to the enthalpy flux increases with diameter;
- smaller drops (d < 150 μm) are mostly in the “wet bulb” state, i.e. do not provide enthalpy to the air;
- the dominant contribution in the enthalpy flux is that of latent heat of evaporation

Remaining problems:
- we need more accurate measurements of spray drops velocity distribution properties at injection in lab experiments;
- increase the bulk Reynolds number (better grid resolution);
- consider “polar low”s, i.e. really strong convection conditions