Formation of magnetic filaments in the convective zone of the Sun

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OUTLINE

- Collapse in hydrodynamics and MHD: motivation
- Coherent structures for frozen-in-fluid fields and their amplification due to compressibility
- Formation of magnetic filaments in convective zone of the Sun

Papers

- E.A. Kuznetsov, T. Passot and P.L. Sulem *Compressible dynamics of magnetic field lines for incompressible MHD flows*, Physics of Plasmas, **11**, 1410-1415 (2004).
- E.A. Kuznetsov, E.A. Mikhailov, *Notes about collapse in magnetohydrodynamics*, ZhETF, **158**, 1 (2020); arXiv:3276237 [physics.geo-ph].

Collapse in hydrodynamics and MHD: motivation

In our papers with Agafontsev and Mailybaev it was shown that In 3D Euler hydrodynamics the vorticity ω in the pancake-type vortex structures grows exponentially in time. The formation of such structures due to the frozenness of the vorticity results in maximal values of ω proportional to their widths as $\ell^{-2/3}$. This process can be considered as *folding* similar to the wave breaking in gases. In MHD at high values of magnetic Reynolds numbers, $Re_m \gg 1$, the magnetic field is also a frozen-in field. Therefore one can expect, that the exponential in time growth of the magnetic field also should be observed due to compressibility of magnetic field lines. Such situation is realized in the convective zone of the Sun that leads to the magnetic field filamentation. This was first addressed attention by Parker in his pioneer work in 1963.

Frozenness

Consider the MHD equation of motion for the magnetic field $\mathbf{B}$ in the ideal case,

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{rot}(\mathbf{v} \times \mathbf{B}), \ \mathrm{div}\, \mathbf{v} = 0.$$

Frozenness means that any Lagrangian particle is pasted to its own $\mathbf{B}$-line and can not leave it. The $\mathbf{B}$-field can be changed only by the velocity component $\mathbf{v}_n$ normal to $\mathbf{B}$ that allows one to introduce a new type of trajectories given by $\mathbf{v}_n$ as

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}_n(\mathbf{x}, t), \ \ \mathbf{x}|_{t=0} = \mathbf{a}.$$

Solution $\mathbf{x} = \mathbf{x}(\mathbf{a}, t)$ describes the motion of the field lines.

Frozenness

In terms of mapping $\mathbf{x} = \mathbf{x}(\mathbf{a}, t)$, Eq. for $\mathbf{B}$ admits explicit integration (K., Ruban, 1998)

$$\mathbf{B}(\mathbf{x}, t) = \frac{\widehat{J}\,\mathbf{B}_0(\mathbf{a})}{J}, \quad \widehat{J}(\mathbf{a}, t) = \left[\frac{\partial x_i}{\partial a_j}\right], \quad J = \det \widehat{J},$$

where $\widehat{J}$ is the Jacobi matrix of the mapping, $\mathbf{B}_0(\mathbf{a})$ is the initial field analogous to the Cauchy invariant (Cauchy 1815, the Kelvin theorem 1869).

The Liouville formula for Jacobian J reads

$$\frac{dJ}{dt} = \text{div}\,\mathbf{v_n} \cdot J.$$

In the general situation, $\text{div}\,\mathbf{v_n} \neq 0$. By this reason, the Jacobian can take arbitrary values, in particular, can vanish. This is the origin of compressibility of the magnetic lines

Vortex line representation for B field

The inverse Jacobian, $n = 1/J$, has the meaning of density of B-lines and satisfies the continuity equation

$$\partial_t n + \mathrm{div}(n\mathbf{v}_n) = 0.$$

Thus, the frozen-in-fluid fields are compressible, in spite of the fluid incompressibility!

One more useful relation. The maximal value B_{max} is defined from

$$\frac{dB_{max}}{dt} = B_{max}\,\tau_i(\nabla_i v_j)\tau_j \approx B_{max}\mathrm{div}\,\mathbf{v}_\tau = -B_{max}\mathrm{div}\,\mathbf{v}_\mathbf{n}.$$

where $\tau = \mathbf{B}/|B|$. As the result, at the maximal point this gives: $B_{max}J_{min} \approx \mathrm{const}$.

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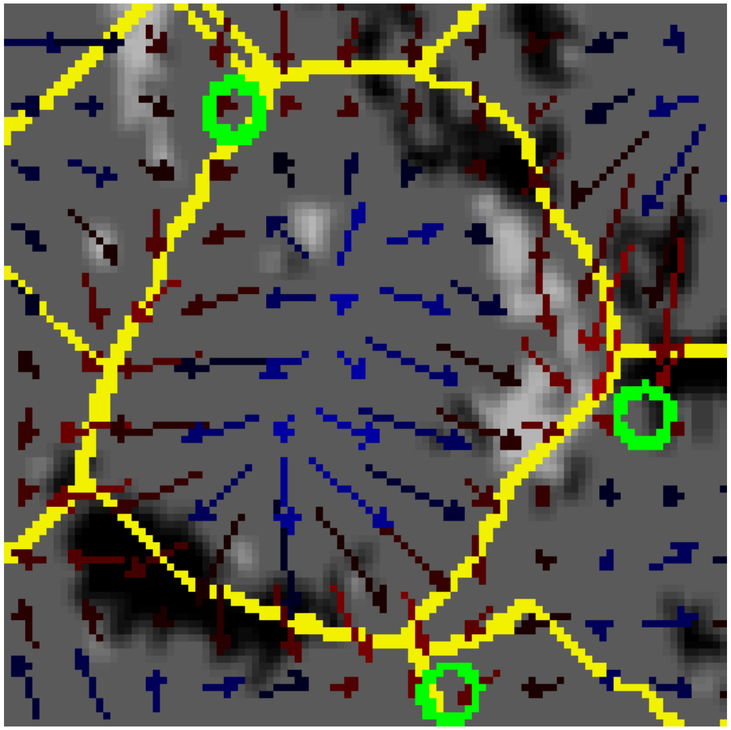
Now we consider the problem of generation of strong magnetic fields in MHD due to the folding mechanism predicted in K., T. Passot and P.L. Sulem (Physics of Plasmas, **11**, 1410-1415 (2004)). On our opinion, the formation of magnetic filaments in the convective zone of the Sun can be explained by this mechanism.

Formation of magnetic filaments in convective zone of the Sun

SOHO magnetogram overlaid with lines of convergence of the horizontal flow and with green dots showing the convergence points. The measured flow is shown as colored arrows, red for inferred downflow and blue for inferred upflow. The field is shown light grey for positive fields and dark for negative fields. Only field above the background noise is shown.



SOHO magnetogram overlaid with lines of convergence of

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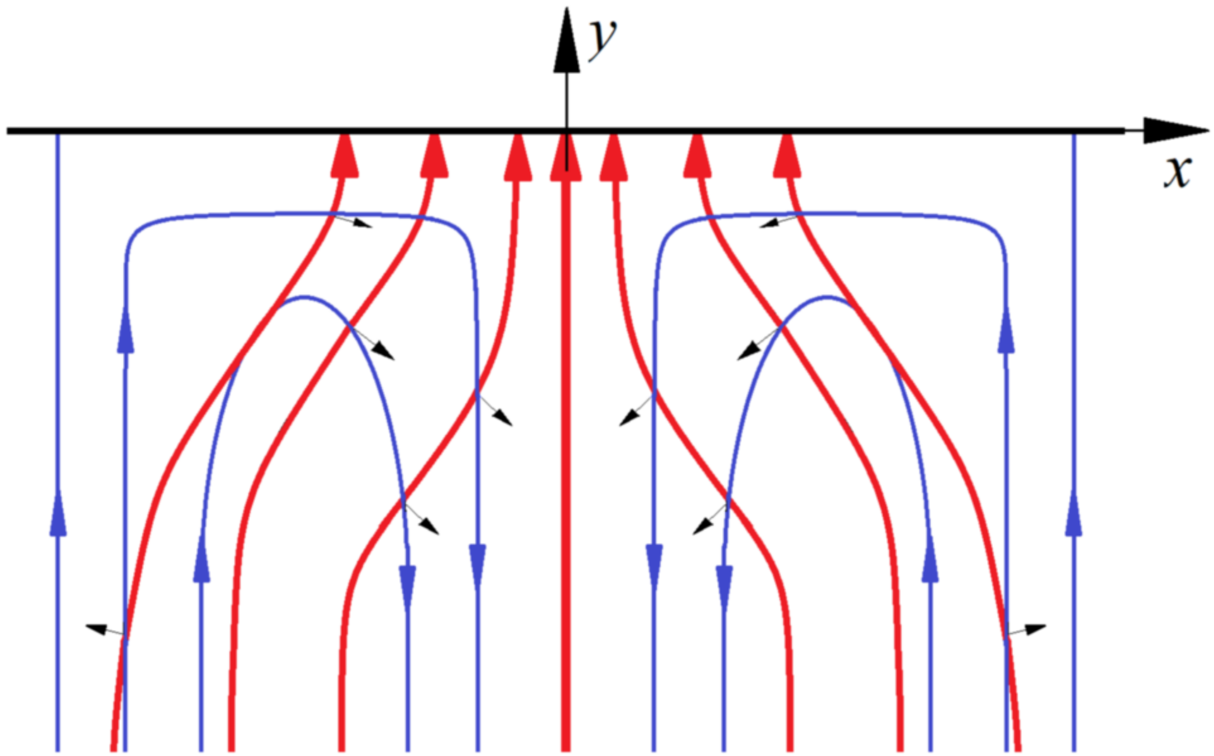

Streamlines (blue), magnetic lines (red), normal velocity (black)

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- For the convection zone $\rho\mathbf{v}^2/2 \gg \mathbf{B}^2/(8\pi)$ (their ratio is about $10^4 - 10^5$) where density $\rho = \text{const}$ and $\text{div}\,\mathbf{v} = 0$. Therefore the fluid velocity can be considered as a given vector field. Besides, the magnetic Reynolds number $Re_m \sim 10^6$ that allows one to neglect magnetic viscosity.

- **Parameters:**

  Near the boundary with photosphere (the beginning of the Sun atmosphere) density in convection zone $\rho \sim 10^{-5}\, g/sm^3$. The characteristic velocity is about $500\, m/s$. The mean solar magnetic field is about a few gauss.

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- The induction equation for $\mathbf{B}$ in terms of magnetic potential A is written as
  $\frac{\partial}{\partial t}A + (\mathbf{v}\cdot\nabla)A = 0$.
  The isolines of A coincide with magnetic field lines. Further we will assume that at $t = 0$ the magnetic field is const: $\mathbf{B} = (0, B_0)$.
- Consider simplest 2D convective cell given by stream function $\psi = \sin x \cdot \sin y$ (rolls), which gives velocity components
  $v_x = -\sin x \cdot \cos y$, $v_y = \cos x \cdot \sin y$.
  For such flows the equation for A admits application of the method of characteristics.

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The equations for characteristics are the Hamiltonian ones with ψ as the Hamiltonian:

$$\frac{dx}{dt} = -\frac{\partial\psi}{\partial y}, \quad \frac{dy}{dt} = \frac{\partial\psi}{\partial x}$$

with the initial conditions $\mathbf{r}|_{t=0} = \mathbf{a}$.
If one considers a small vicinity near the point $x = y = 0$ where we should expect the magnetic field amplification and the formation of the magnetic filament then the stream function should be written as $\psi = xy$ with the initial conditions $\psi = a_x a_y$. Then the obtained equations are turned out linear that give:

$$x = a_x e^{-t} \text{ and } y = a_y e^t.$$

For the initial condition for $A = -B_0 a_x$ it is easily to get that

$$B_x = 0, \;\; B_y = B_0 e^t.$$

These analytical results are confirmed by simulations.

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In the general situation for 2D flows filamentation takes place in the hyperbolic points where

$$\psi_{xx}\psi_{yy} - \psi_{xy}^2 < 0,$$

respectively, a region where this criterion holds is called hyperbolic. In the opposite case,

$$\psi_{xx}\psi_{yy} - \psi_{xy}^2 > 0,$$

such points and regions are called elliptic. In the latter case the magnetic field rotates around such points.

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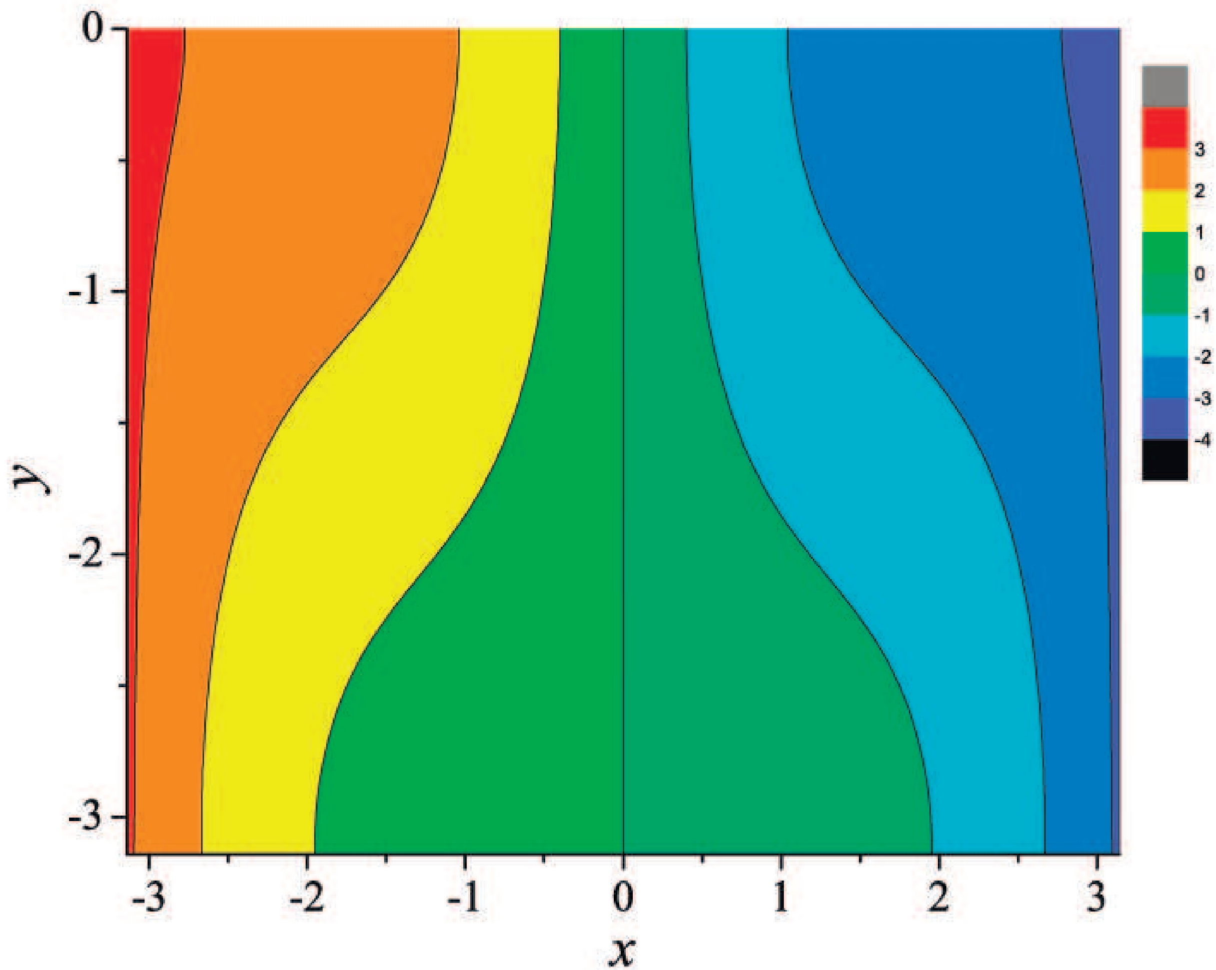
The profile of magnetic potential A in the region $-\pi \leq x \leq \pi$, $-\pi \leq y \leq 0$ at $t = 1$ for magnetic viscosity $\eta = 0$.



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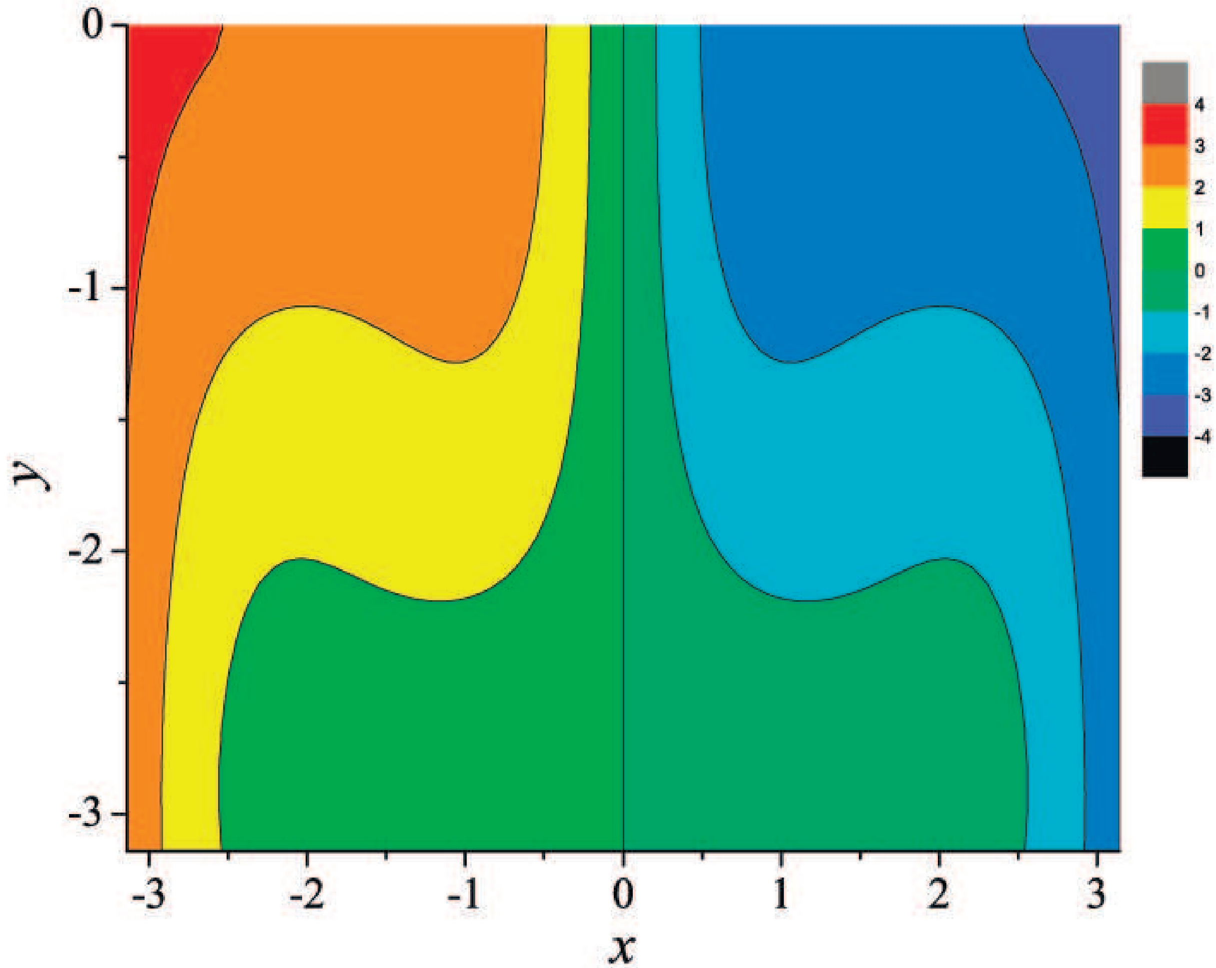
The profile of magnetic potential A at $t = 2$ for $\eta = 0$.



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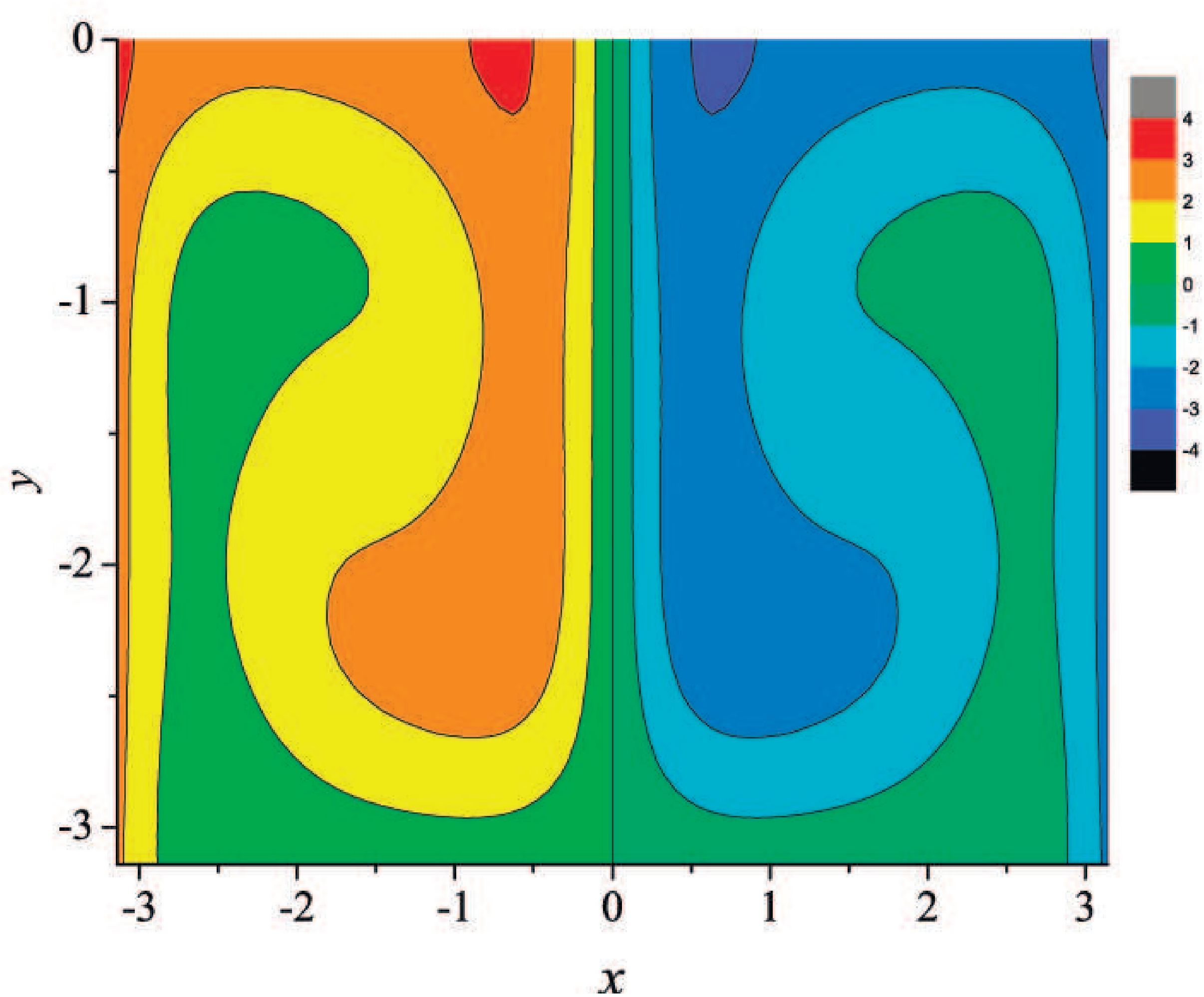
The profile of magnetic potential A at $t = 5$ for $\eta = 0$.



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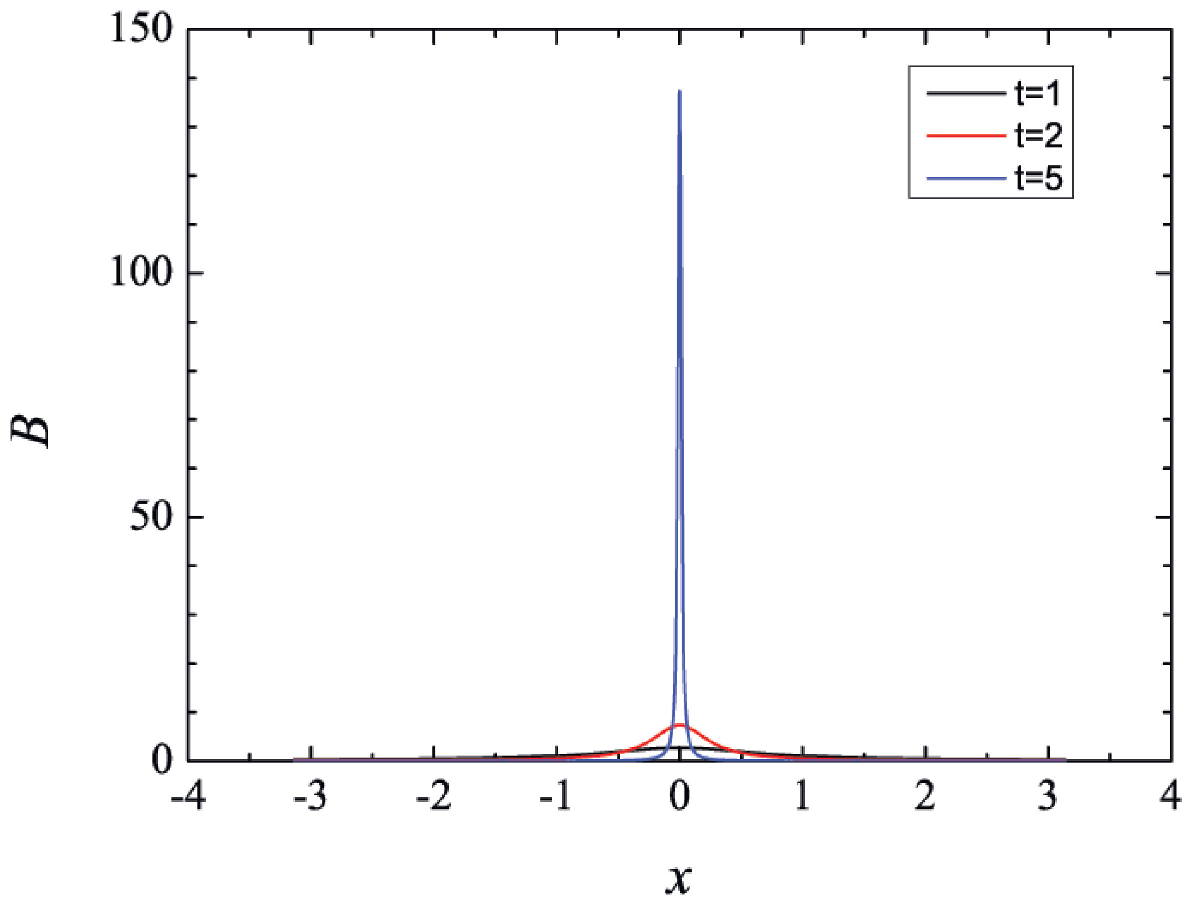
The behavior of B_y along free surface $y = 0$ for magnetic viscosity $\eta = 0$.



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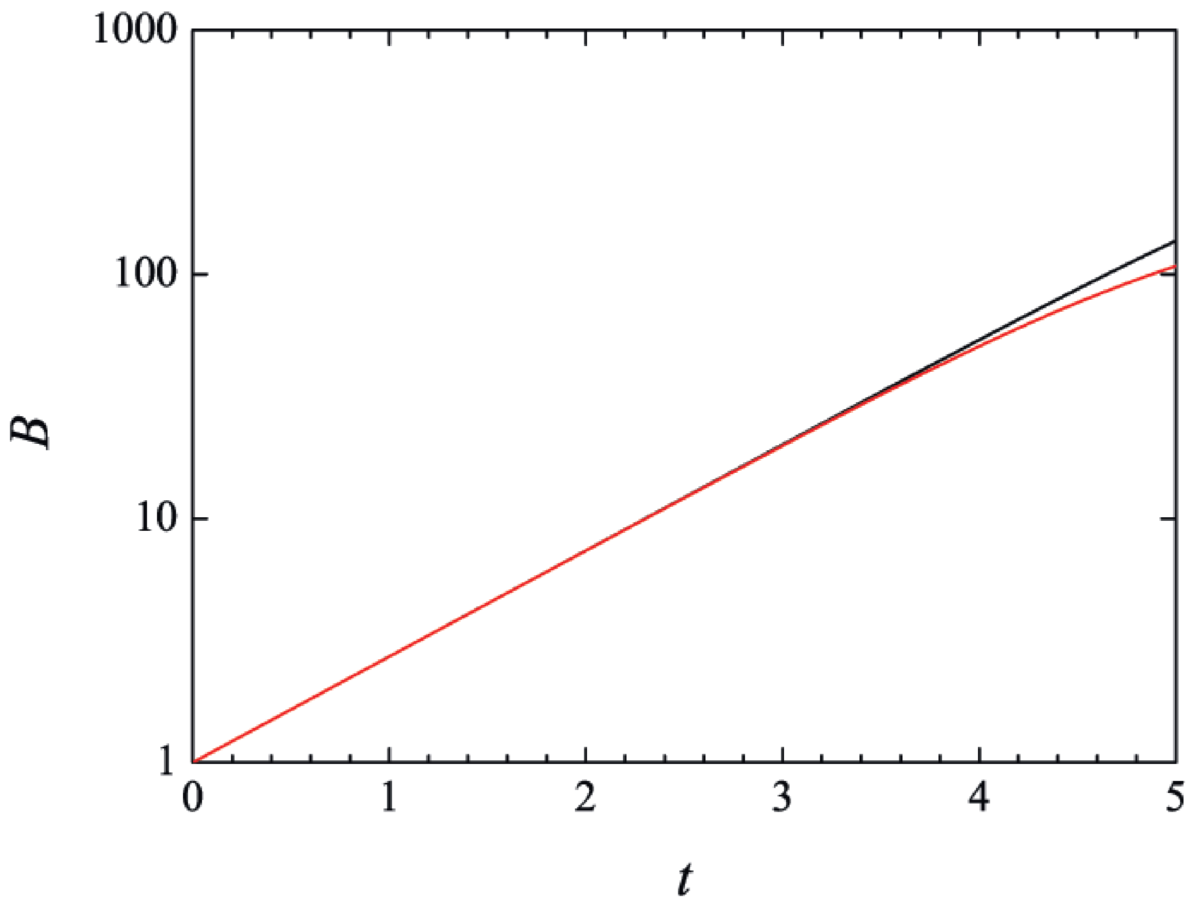
The behavior of maximal B at $x = y = 0$: black line for zero magnetic viscosity $\eta = 0$, red line for $\eta = 0.0001$.



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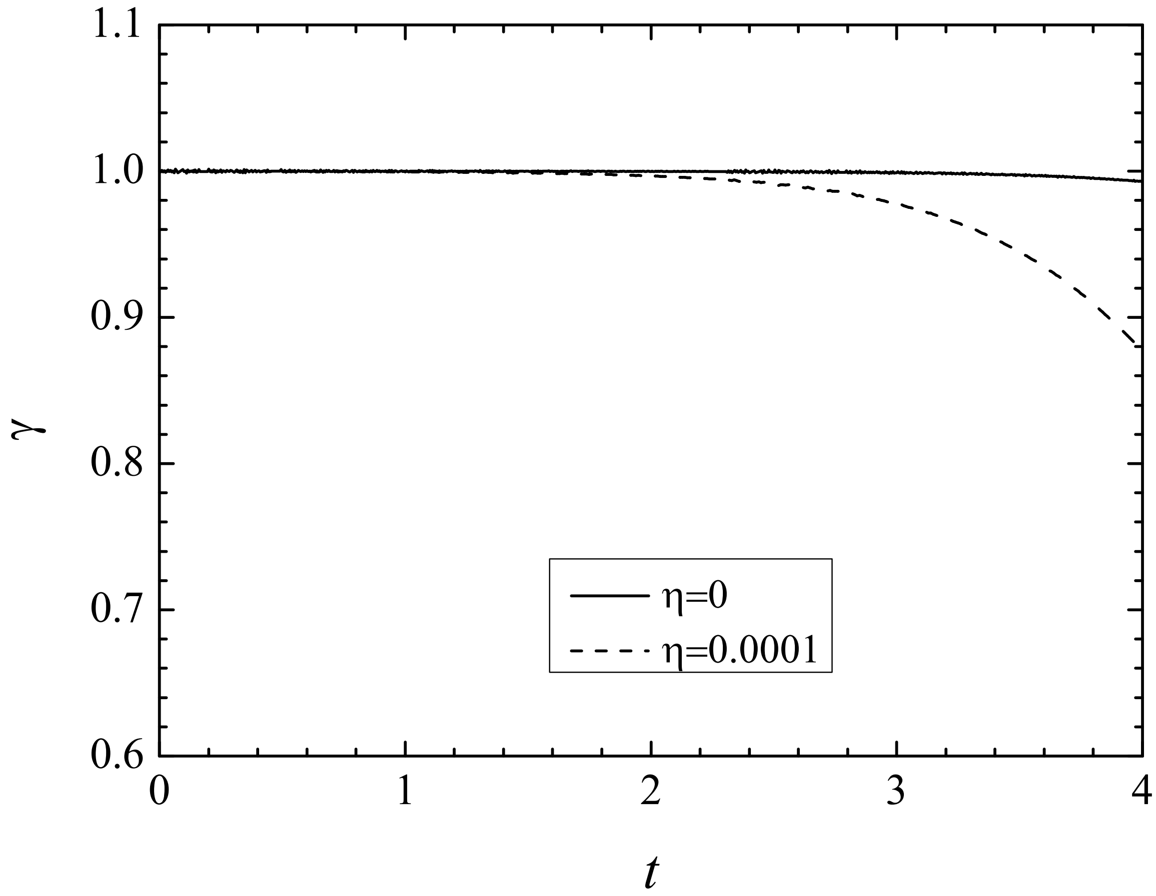
Growth rate $\gamma = d\ln B_{max}/dt$ at $x = y = 0$.



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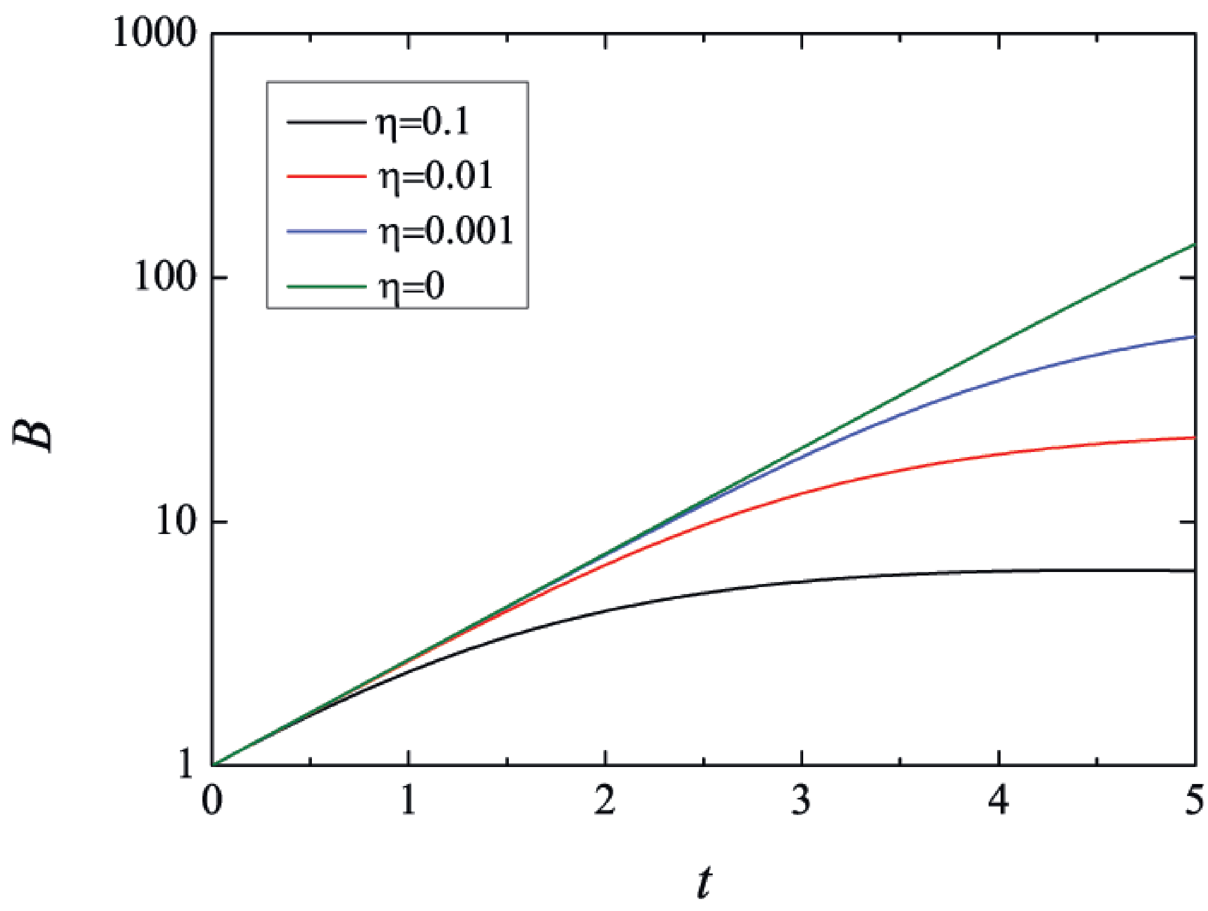
The temporal behavior of maximal B for different η . Viscosity destroys the frozenness of magnetic field that results in saturation.



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Based on the magnetic field flux conservation it is possible to get the saturated field estimate:

$$B_{sat} \sim Re_m^{1/2} B_0$$

For $B_0 = 10\,G$ and $Re_m = 10^6$ we have $B_{sat} \sim 10^4\,G$. For such large values the feedback influence of the growing magnetic field on the convection velocity is seemed large, but however because the magnetic field in filament is perpendicular to its gradient this influence occurs not too essential.

In the 3D case, the mechanism of filamentation remains the same like in 2D. The difference with 2D is connected with magnetic filaments become flatten relative to the interface between cells.

Conclusion

- Convective cells are responsible for the formation of magnetic filaments in convective zone of the Sun.
- The formation of magnetic filaments is connected with compressibility of the magnetic field. This process in time has exponential character.
- Exponential increase of the magnetic field is observed in neighborhood of the flow hyperbolicity, which is the main criterion of the filamentation.
- Directions of magnetic filaments are correlated with convective downflow. This downflow plays a role of a specific attractor for magnetic field. Filaments are concentrated around interfaces of convective cells.

THANKS FOR YOUR ATTENTION