Двухпотоковая неустойчивость одного класса сферически симметричных состояний динамического равновесия плазмы Власова-Пуассона

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Постановка точной задачи

$$\frac{\partial f^+}{\partial t} + \bar{v}\nabla_{\bar{x}} f^+ - \frac{qn}{m_+}\nabla_{\bar{x}}\varphi\nabla_{\bar{v}} f^+ = 0$$

$$\frac{\partial f^-}{\partial t} + \bar{v}\nabla_{\bar{x}} f^- + \frac{q}{m_-}\nabla_{\bar{x}}\varphi\nabla_{\bar{v}} f^- = 0$$

$$\Delta_{\bar{x}}\varphi = -4\pi\mathrm{q}\int\limits_{R^3}\left(n\, f^+(\bar{x},\bar{v},t) - f^-(\bar{x},\bar{v},t)\right) d\bar{v}$$

$$f^{\pm} = f^{\pm}(\bar{x},\bar{v},t) \geq 0; f^{\pm}(\bar{x},\bar{v},0) = f_0{}^{\pm}(\bar{x},\bar{v})$$

$$f^{\pm} \to 0 \text{ при } |\bar{v}| \to \infty$$

Сферически симметричный случай

$$\frac{\partial f^+}{\partial t} + v\frac{\partial f^+}{\partial r} - \frac{qn}{m_+}\frac{\partial \varphi}{\partial r}\frac{\partial f^+}{\partial v} = 0$$

$$\frac{\partial f^-}{\partial t} + v\frac{\partial f^-}{\partial r} + \frac{q}{m_-}\frac{\partial \varphi}{\partial r}\frac{\partial f^-}{\partial v} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \varphi}{\partial r}\right) = -16\pi^2 q\int_0^\infty \left(nf^+(r,v,t) - f^-(r,v,t)\right)v^2\,dv$$

$$f^\pm = f^\pm(r,v,t) \geq 0;\ f^\pm(r,v,0) = {f_0}^\pm(r,v)$$

$$f^\pm \to 0 \text{ при } v \to \infty$$

Закон сохранения энергии

$$E \equiv 8\pi^2 \int_0^\infty \int_0^\infty (m_+ f^+ + m_- f^-) v^4 r^2 \, dr dv +$$

$$+\frac{1}{2}\int_0^\infty \left(\frac{\partial \varphi}{\partial r}\right)^2 r^2 dr = const$$

$$f^\pm, \frac{\partial \varphi}{\partial r} \to 0 \text{ при } r \to \infty \text{ или периодичны по } r$$

Интегралы движения

$$C^\pm \equiv 16\pi^2 \int_0^\infty \int_0^\infty \Phi^\pm(f^\pm) \, v^2 r^2 dv dr = const$$

Стационарные решения

$$f^{\pm} = f^{\pm 0}(v), \varphi = \varphi^{0} \equiv const$$

$$n \int_{0}^{\infty} f^{+0} v^{2} dv = \int_{0}^{\infty} f^{-0}\, v^{2} dv$$

$$f^{\pm 0} \to 0 \text{ при } v \to \infty$$

Формулировка линеаризованной задачи

$$\frac{\partial f^{+\prime}}{\partial t} + v\frac{\partial f^{+\prime}}{\partial r} - \frac{qn}{m_+}\frac{\partial \varphi'}{\partial r}\frac{\partial f^{+0}}{\partial v} = 0$$

$$\frac{\partial f^{-\prime}}{\partial t} + v\frac{\partial f^{-\prime}}{\partial r} + \frac{q}{m_-}\frac{\partial \varphi'}{\partial r}\frac{\partial f^{-0}}{\partial v} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \varphi'}{\partial r}\right) = -16\pi^2 q\int_0^\infty \left(nf^{+\prime} - f^{-\prime}\right)v^2 dv$$

$$f^{\pm\prime}(r,v,0) = f_0^{\pm\prime}(r,v)$$

$$f^{\pm\prime} \to 0 \text{ при } v \to \infty$$

Линейный аналог функционала энергии

$$E_1 \equiv 8\pi^2 \int_0^\infty \int_0^\infty \left(\frac{d^2\Phi^+}{df^{+2}}(f^{+0}) f^{+\prime 2} + \frac{d^2\Phi^-}{df^{-2}}(f^{-0}) f^{-\prime 2} \right) v^2 r^2 dv dr +$$

$$+\frac{1}{2}\int_0^\infty \left(\frac{\partial \varphi'}{\partial r}\right)^2 r^2 dr = const$$

$$f^{\pm\prime}, \frac{\partial \varphi'}{\partial r} \to 0 \text{ при } r \to \infty \text{ или периодичны по } r$$

$$\frac{d\Phi^+}{df^+}(f^{+0}) = -\frac{q\varphi^0 n}{2} - \frac{v^2 m_+}{2} \qquad \frac{d\Phi^-}{df^-}(f^{-0}) = \frac{q\varphi^0}{2} - \frac{v^2 m_-}{2}$$

Достаточные условия линейной устойчивости (теорема Ньюкомба-Гарднера-Розенблюта)

$$\frac{d^2\Phi^+}{df^{+2}}(f^{+0}) \geq 0 \Rightarrow v\frac{df^{+0}}{dv} \leq 0$$

$$\frac{d^2\Phi^-}{df^{-2}}(f^{-0}) \geq 0 \Rightarrow v\frac{df^{-0}}{dv} \leq 0$$

Стационарные решения

$$f^{+0}(v) = \frac{1+v^2}{e^{v^2}}, f^{-0}(v) = n\frac{1+v^2}{e^{v^2}}$$

$$\varphi = \varphi^0 \equiv const$$

Контрпример к теореме Ньюкомба-Гарднера и критерию Пенроуза

$$f^{+\prime} = f_1^{+\prime}(r,v)e^{\alpha t}$$

$$f^{-\prime} = f_1^{-\prime}(r,v)e^{\alpha t}$$

$$\varphi' = ce^{\alpha t},\ \ c \equiv const$$

$$f^{+\prime} = e^{\alpha\left(t-\frac{r}{v}\right)-v}$$

$$f^{-\prime} = ne^{\alpha\left(t-\frac{r}{v}\right)-v}$$

$$\varphi' = ce^{\alpha t},\ \ c \equiv const$$

Гидродинамическая подстановка

$$g^{+} \equiv v^2 r^2 f^{+}, \quad v = u^{+}(r,\lambda,t); \quad g^{-} \equiv v^2 r^2 f^{-}, \quad v = u^{-}(r,\lambda,t)$$

$$g^{\pm}(r,v,t) = \rho^{\pm}(r,\lambda,t)\left(\frac{\partial u^{\pm}}{\partial \lambda}(r,\lambda,t)\right)^{-1}$$

$$\frac{\partial u^{\pm}}{\partial \lambda}(r,\lambda,t) \neq 0; \quad \frac{d\lambda}{dt} = 0; \quad \lambda \in [0,\infty)$$

Постановка точной задачи в новых переменных

$$\frac{\partial u^+}{\partial t} + u^+ \frac{\partial u^+}{\partial r} = -\frac{qn}{m_+}\frac{\partial \varphi}{\partial r} \qquad \frac{\partial \rho^+}{\partial t} + \frac{\partial (u^+ \rho^+)}{\partial r} = 0$$

$$\frac{\partial u^-}{\partial t} + u^- \frac{\partial u^-}{\partial r} = \frac{q}{m_-}\frac{\partial \varphi}{\partial r} \qquad \frac{\partial \rho^-}{\partial t} + \frac{\partial (u^- \rho^-)}{\partial r} = 0$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi}{\partial r}\right) = -16\pi^2 q \int_0^\infty (n\rho^+ - \rho^-) d\lambda$$

$$u^\pm(r,\lambda,0) = u_0^\pm(r,\lambda), \quad \rho^\pm(r,\lambda,0) = \rho_0^\pm(r,\lambda)$$

$$\rho^\pm \to 0 \text{ при } \lambda \to \infty$$

Вспомогательные уравнения

$$\frac{\partial^2 u^\pm}{\partial\lambda\partial t} + \frac{\partial u^\pm}{\partial\lambda}\frac{\partial u^\pm}{\partial r} + u^\pm \frac{\partial^2 u^\pm}{\partial\lambda\partial r} = 0$$

$$\kappa^\pm \equiv \rho^\pm \left(\frac{\partial u^\pm}{\partial\lambda}\right)^{-1} \geq 0: \;\; \frac{\partial\kappa^\pm}{\partial t} + u^\pm \frac{\partial\kappa^\pm}{\partial r} = 0$$

Закон сохранения энергии

$$E_2 \equiv 8\pi^2 \int_0^\infty \int_0^\infty \left(m_+\rho^+ u^{+2} + m_-\rho^- u^{-2}\right) d\lambda \mathrm{dr} +$$

$$+\frac{1}{2}\int_0^\infty \left(\frac{\partial\varphi}{\partial r}\right)^2 r^2 dr = const$$

$$\rho^\pm, u^\pm, \frac{\partial\varphi}{\partial r} \to 0 \text{ при } r \to \infty \text{ или периодичны по } r$$

Интегралы движения

$$C_1^\pm \equiv 16\pi^2 \int_0^\infty \int_0^\infty \Phi_1^\pm(\kappa^\pm) \frac{\partial u^\pm}{\partial \lambda} d\lambda dr = const$$

Стационарные решения

$$u^{\pm} = u^{\pm 0}(\lambda),\ \rho^{\pm} = \rho^{\pm 0}(\lambda) \geq 0,\ \varphi = \varphi^{0} \equiv const$$

$$n \int_0^{\infty} \rho^{+0} d\lambda = \int_0^{\infty} \rho^{-0} d\lambda$$

$$\rho^{\pm 0} \to 0 \text{ при } \lambda \to \infty$$

Стационарные вспомогательные функции

$$\kappa^{\pm 0} \equiv \rho^{\pm 0} \left(\frac{du^{\pm 0}}{d\lambda} \right)^{-1} \geq 0$$

Формулировка линеаризованной задачи

$$\frac{\partial u^{+\prime}}{\partial t} + u^{+0}\frac{\partial u^{+\prime}}{\partial r} = -\frac{qn}{m_+}\frac{\partial \varphi'}{\partial r} \qquad \frac{\partial \rho^{+\prime}}{\partial t} + u^{+0}\frac{\partial \rho^{+\prime}}{\partial r} + \rho^{+0}\frac{\partial u^{+\prime}}{\partial r} = 0$$

$$\frac{\partial u^{-\prime}}{\partial t} + u^{-0}\frac{\partial u^{-\prime}}{\partial r} = \frac{q}{m_-}\frac{\partial \varphi'}{\partial r} \qquad \frac{\partial \rho^{-\prime}}{\partial t} + u^{-0}\frac{\partial \rho^{-\prime}}{\partial r} + \rho^{-0}\frac{\partial u^{-\prime}}{\partial r} = 0$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \varphi'}{\partial r}\right) = -16\pi^2 q\int_0^\infty \left(n\rho^{+\prime}(r,\lambda,t) - \rho^{-\prime}(r,\lambda,t)\right)d\lambda$$

$$u^{\pm\prime}(r,\lambda,0) = u_0^{\pm\prime}(r,\lambda),\ \rho^{\pm\prime}(r,\lambda,0) = \rho_0^{\pm\prime}(r,\lambda)$$

$$\rho^{\pm\prime} \to 0 \text{ при } \lambda \to \infty$$

Линеаризованные вспомогательные уравнения

$$\frac{\partial^2 u^{\pm\prime}}{\partial\lambda\partial t} + \frac{\partial}{\partial\lambda}\left(u^{\pm0}\frac{\partial u^{\pm\prime}}{\partial r}\right) = 0$$

$$\kappa^{\pm\prime} = \frac{\rho^{\pm\prime}}{\frac{du^{\pm0}}{d\lambda}} - \frac{\rho^{\pm0}}{\left(\frac{du^{\pm0}}{d\lambda}\right)^2}\frac{\partial u^{\pm\prime}}{\partial\lambda}: \quad \frac{\partial\kappa^{\pm\prime}}{\partial t} + u^{\pm0}\frac{\partial\kappa^{\pm\prime}}{\partial r} = 0$$

Линейный аналог функционала энергии

$$E_3 \equiv 8\pi^2 \int_0^\infty \int_0^\infty \left[\begin{gathered} m_+ \rho^{+0} u^{+\prime 2} + 2m_+ u^{+0} \rho^{+\prime} u^{+\prime} + m_- \rho^{-0} u^{-\prime 2} + 2m_- u^{-0} \rho^{-\prime} u^{-\prime} + \\ + \frac{du^{+0}}{d\lambda} \frac{d^2\Phi_1^+}{d\kappa^{+2}} (\kappa^{+0}) \kappa^{+\prime 2} + \frac{du^{-0}}{d\lambda} \frac{d^2\Phi_1^-}{d\kappa^{-2}} (\kappa^{-0}) \kappa^{-\prime 2} \end{gathered} \right] d\lambda dr +$$

$$+ \frac{1}{2} \int_0^\infty \left(\frac{\partial \varphi'}{\partial r} \right)^2 r^2 dr = const$$

$$\rho^{\pm\prime}, u^{\pm\prime}, \frac{\partial \varphi'}{\partial r} \to 0 \text{ при } r \to \infty \text{ или периодичны по } r$$

$$\frac{d\Phi_1^+}{d\kappa^+}(\kappa^{+0}) = -\frac{q\varphi^0 n}{2} - \frac{(u^{+0})^2 m_+}{2}, \quad \frac{d\Phi_1^-}{d\kappa^-}(\kappa^{-0}) = \frac{q\varphi^0}{2} - \frac{(u^{-0})^2 m_-}{2}$$

Достаточные условия линейной устойчивости

$$\kappa^{\pm\prime} = 0 \Longrightarrow \rho^{\pm\prime} = \kappa^{\pm 0} \frac{\partial u^{\pm\prime}}{\partial \lambda}$$

$$\int_0^\infty \left[\left(u^{\pm 0} \kappa^{\pm 0} u^{\pm\prime 2}\right)\Big|_{\lambda \to +\infty} - \left(u^{\pm 0} \kappa^{\pm 0} u^{\pm\prime 2}\right)\Big|_{\lambda = 0} \right] dr \to 0$$

$$E_3 = -8\pi^2 \int_0^\infty \int_0^\infty \left(m_+ u^{+0} \frac{d\kappa^{+0}}{d\lambda} u^{+\prime 2} + m_- u^{-0} \frac{d\kappa^{-0}}{d\lambda} u^{-\prime 2} \right) d\lambda dr + \frac{1}{2} \int_0^\infty \left(\frac{\partial \varphi'}{\partial r} \right)^2 r^2 dr = const$$

$$u^{+0} \frac{d\kappa^{+0}}{du^{+0}} \leq 0,\ u^{-0} \frac{d\kappa^{-0}}{du^{-0}} \leq 0$$

Поля лагранжевых смещений

$$\xi^{+} = \xi^{+}(r, \lambda, t)\colon\ \frac{\partial \xi^{+}}{\partial t} = u^{+\prime} - u^{+0}\frac{\partial \xi^{+}}{\partial r}$$

$$\xi^{-} = \xi^{-}(r, \lambda, t)\colon\ \frac{\partial \xi^{-}}{\partial t} = u^{-\prime} - u^{-0}\frac{\partial \xi^{-}}{\partial r}$$

Преобразованная линеаризованная задача

$$\frac{\partial^2 \xi^{+}}{\partial t^2} + 2u^{+0}\frac{\partial^2 \xi^{+}}{\partial r \partial t} + (u^{+0})^2\frac{\partial^2 \xi^{+}}{\partial r^2} = -\frac{qn}{m_{+}}\frac{\partial \varphi'}{\partial r}$$

$$\frac{\partial^2 \xi^{-}}{\partial t^2} + 2u^{-0}\frac{\partial^2 \xi^{-}}{\partial r \partial t} + (u^{-0})^2\frac{\partial^2 \xi^{-}}{\partial r^2} = \frac{q}{m_{-}}\frac{\partial \varphi'}{\partial r}$$

$$\rho^{+\prime} = -\rho^{+0}\frac{\partial \xi^{+}}{\partial r},\ \ \rho^{-\prime} = -\rho^{-0}\frac{\partial \xi^{-}}{\partial r}$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi'}{\partial r}\right) = -16\pi^2 q\int_0^\infty\left(-n\rho^{+0}\frac{\partial\xi^+}{\partial r} + \rho^{-0}\frac{\partial\xi^-}{\partial r}\right)d\lambda$$

$$\xi^\pm(r,\lambda,0) = \xi_0^\pm(r,\lambda),\quad \frac{\partial\xi^\pm}{\partial t}(r,\lambda,0) = \left(\frac{\partial\xi^\pm}{\partial t}\right)_0(r,\lambda)$$

$$\xi^\pm \to 0 \text{ при } \lambda \to \infty$$

Вспомогательное соотношение

$$r^2\frac{\partial\varphi'}{\partial r} = -16\pi^2 q\int_0^\infty(-n\rho^{+0}\xi^+ + \rho^{-0}\xi^-)d\lambda$$

Преобразованный линейный аналог интеграла энергии

$$E_3 \equiv 8\pi^2 \int_0^\infty \int_0^\infty m_+ \rho^{+0} \left[\left(\frac{\partial \xi^+}{\partial t} \right)^2 - (u^{+0})^2 \left(\frac{\partial \xi^+}{\partial r} \right)^2 \right] d\lambda dr +$$

$$+8\pi^2 \int_0^\infty \int_0^\infty m_- \rho^{-0} \left[\left(\frac{\partial \xi^-}{\partial t} \right)^2 - (u^{-0})^2 \left(\frac{\partial \xi^-}{\partial r} \right)^2 \right] d\lambda dr +$$

$$+\frac{1}{2} \int_0^\infty \left(\frac{\partial \varphi'}{\partial r} \right)^2 r^2 dr = const$$

$$\xi^\pm, \frac{\partial \varphi'}{\partial r} \to 0 \text{ при } r \to \infty \text{ или периодичны по } r$$

Функционал Ляпунова

$$\mathrm{M} \equiv 16\pi^2 \int_0^\infty \int_0^\infty \{m_+\rho^{+0}\xi^{+2} + m_-\rho^{-0}\xi^{-2}\} d\lambda dr$$

$$\frac{dM}{dt} = 32\pi^2 \int_0^\infty \int_0^\infty \left\{m_+\rho^{+0}\xi^+ \frac{\partial \xi^+}{\partial t} + m_-\rho^{-0}\xi^- \frac{\partial \xi^-}{\partial t}\right\} d\lambda dr$$

$$\frac{d^2M}{dt^2} = 32\pi^2 \int_0^\infty \int_0^\infty \left[\begin{matrix} m_+\rho^{+0}\left(\left(\frac{\partial \xi^+}{\partial t}\right)^2 + \xi^+ \frac{\partial^2 \xi^+}{\partial t^2}\right) + \\ +m_-\rho^{-0}\left(\left(\frac{\partial \xi^-}{\partial t}\right)^2 + \xi^- \frac{\partial^2 \xi^-}{\partial t^2}\right) \end{matrix} \right] d\lambda dr$$

$$\frac{d^2M}{dt^2} - 2\nu\frac{dM}{dt} + 2\nu^2 M =$$

$$= 32\pi^2 \int_0^\infty \int_0^\infty \left[\begin{array}{l} m_+\rho^{+0}\left(\left(\frac{\partial\xi^+}{\partial t}\right)^2 + \xi^+\frac{\partial^2\xi^+}{\partial t^2}\right) + \\ +m_-\rho^{-0}\left(\left(\frac{\partial\xi^-}{\partial t}\right)^2 + \xi^-\frac{\partial^2\xi^-}{\partial t^2}\right) \end{array} \right] dr d\lambda -$$

$$- 64\nu\pi^2 \int_0^\infty \int_0^\infty \left\{ m_+\rho^{+0}\xi^+\frac{\partial\xi^+}{\partial t} + m_-\rho^{-0}\xi^-\frac{\partial\xi^-}{\partial t} \right\} d\lambda dr +$$

$$+32\pi^2\nu^2 \int_0^\infty \int_0^\infty \{m_+\rho^{+0}\xi^{+2} + m_-\rho^{-0}\xi^{-2}\} d\lambda dr \geq$$

$$\geq -2\int_0^\infty \left(\frac{\partial\varphi'}{\partial r}\right)^2 r^2 dr$$

$$\left(\frac{\partial\varphi'}{\partial r} r^2\right)^2 = 256\pi^4 q^2 \left[\left(\int_0^\infty n\xi^+\rho^{+0}\, d\lambda\right)^2 - \right.$$

$$\left. -2\int_0^\infty n\xi^+\rho^{+0}\, d\lambda \int_0^\infty n\xi^-\rho^{-0}\, d\lambda + \left(\int_0^\infty \xi^-\rho^{-0}\, d\lambda\right)^2\right] \leq$$

$$\leq \frac{512\pi^4 q^2 n^2}{m_-}\left[\int_0^\infty \rho^{+0} d\lambda \int_0^\infty \{m_+\rho^{+0}\xi^{+2} + m_-\rho^{-0}\xi^{-2}\}\, d\lambda\right]$$

$$\left(\frac{\partial \varphi'}{\partial r}\right)^2 r^2 \leq \frac{512\pi^4 q^2 n^2}{m_-}\left[\int_0^\infty f^{+0} v^2 dv \ \int_0^\infty \left\{m_+ \rho^{+0} \xi^{+2} + m_- \rho^{-0} \xi^{-2}\right\} d\lambda\right]$$

$$\frac{d^2 M}{dt^2} - 2\nu \frac{dM}{dt} + 2\nu^2 M \geq -2\alpha_1' M$$

$$\alpha_1' \equiv \frac{32\pi^2 q^2 n^2}{m_-} \int_0^\infty f^{+0} v^2 dv$$

Основное дифференциальное неравенство

$$\frac{d^2M}{dt^2} - 2\nu\frac{dM}{dt} + 2(\nu^2 + \alpha_1')M \geq 0$$

Дополнительные условия

$$M\left(\frac{2\pi n}{\sqrt{\nu^2 + \alpha_1'}}\right) > 0; n = 0,1,2\ldots$$

$$\frac{dM}{dt}\left(\frac{2\pi n}{\sqrt{\nu^2 + \alpha_1'}}\right) \geq 2\left(\nu + \frac{\alpha_1'}{\nu}\right)\mathrm{M}\left(\frac{2\pi n}{\sqrt{\nu^2 + \alpha_1'}}\right)$$

$$M\left(\frac{2\pi n}{\sqrt{\nu^2+\alpha_1'}}\right) = M(0)\exp\left(\frac{2\pi n}{\sqrt{\nu^2+\alpha_1'}}\right)$$

$$\frac{dM}{dt}\left(\frac{2\pi n}{\sqrt{\nu^2+\alpha_1'}}\right) = \frac{dM}{dt}(0)\exp\left(\frac{2\pi n}{\sqrt{\nu^2+\alpha_1'}}\right)$$

$$M(0) > 0,\ \frac{dM}{dt}(0) \geq 2\left(\nu + \frac{\alpha_1'}{\nu}\right)\mathrm{M}(0)$$

Априорная оценка снизу

$$M(t) \geq C\exp(\nu t), C \equiv const > 0$$

Стационарные решения

$$u^{\pm 0}(\lambda) = \lambda, \rho^{+0}(\lambda) = \frac{1+\lambda^2}{e^{\lambda^2}}, \rho^{-0}(\lambda) = n\frac{1+\lambda^2}{e^{\lambda^2}}$$

$$\varphi^0 \equiv const$$

Контрпример к теореме Ньюкомба–Гарднера и критерию Пенроуза

$$\xi^+ = \xi_1^+(r,\lambda)e^{\alpha t}$$

$$\xi^- = \xi_1^-(r,\lambda)e^{\alpha t}$$

$$\varphi' = \varphi_1 e^{\alpha t},\ \varphi_1 \equiv const$$

$$\xi^+ = e^{\alpha(t-\frac{r}{\lambda})}e^{-\lambda^2}(1+r)$$

$$\xi^- = e^{\alpha(t-\frac{r}{\lambda})}e^{-\lambda^2}(1+r)$$

$$\varphi' = \varphi_1 e^{\alpha t},\ \varphi_1 \equiv const$$

Условия линейной практической неустойчивости

$$M\left(\frac{2\pi n}{\sqrt{\nu^2 + \alpha_1'}}\right) > 0; n = 0,1,2 \ldots$$

$$\frac{dM}{dt}\left(\frac{2\pi n}{\sqrt{\nu^2 + \alpha_1'}}\right) \geq 2\left(\nu + \frac{\alpha_1'}{\nu}\right) \mathrm{M}\left(\frac{2\pi n}{\sqrt{\nu^2 + \alpha_1'}}\right)$$

Теорема Ирншоу

Всякая равновесная конфигурация точечных зарядов неустойчива, если на них, кроме кулоновских сил притяжения и отталкивания, никакие другие силы не действуют

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