Автомодельное вырождение дальнего закрученного турбулентного следа

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ИВМ СО РАН

21.12.2021

Reynolds A.J. Similarity in swirling wakes and jets // J. Fluid Mech.1962. Vol. 15. No. 2. P. 241-243.

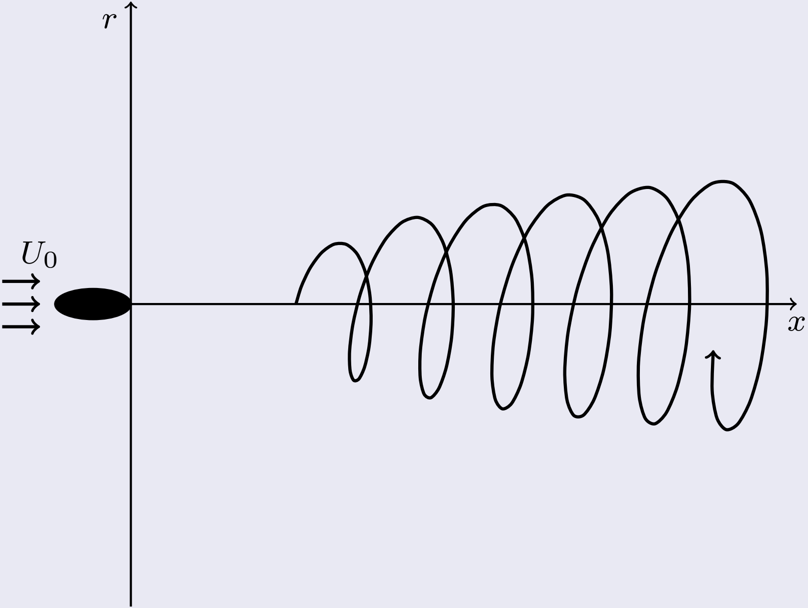
Kostomakha V.A., Lesnova N.V. Turbulent swirling wake behind a sphere with complete or partial drag compensation // Journal of Applied Mechanics and Technical Physics. 1995. Vol. 36. No. 2. P. 226-233.

Rodi W. A new algebraic relation for calculating the Reynolds stresses // ZAMM. 1976. Vol. 56. P. 219-221.

Kaptsov O.V., Fomina A.V., Chernykh G.G., Schmidt A.V. Self-similar decay of the momentumless turbulent wake in a passive stratified medium // Mat. Model. 2015. Vol. 27. No. 1. P. 84-98 (in Russian).

Shmidt A.V. Self-similar solution of the problem of a turbulent flow in a round submerged jet // Journal of Applied Mechanics and Technical Physics. 2015. Vol. 56. No. 3. P. 414-419.

Схема течения



Модель дальнего закрученного турбулентного следа ($J = 0,\ M = 0$)

$$U_0 \frac{\partial U_1}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(C_u r \frac{e^2}{\varepsilon}\frac{\partial U_1}{\partial r}\right),$$

$$U_0 \frac{\partial W}{\partial x} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(C_w r^3 \frac{e^2}{\varepsilon}\frac{\partial (W/r)}{\partial r}\right),$$

$$U_0 \frac{\partial e}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(C_e r \frac{e^2}{\varepsilon}\frac{\partial e}{\partial r}\right) - \varepsilon,$$

$$U_0 \frac{\partial \varepsilon}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(C_\varepsilon r \frac{e^2}{\varepsilon}\frac{\partial \varepsilon}{\partial r}\right) - C_{\varepsilon 2}\frac{\varepsilon^2}{e},$$

$$U_1 = U - U_0, C_u = C_w = 0.25, C_e = 0.147, C_\varepsilon = 0.113, C_{\varepsilon 2} = 1.92.$$

Demenkov A.G., Chernykh G.G., Thermophys. and Aeromech., 2016, 23, no. 5, 667–675.

Автомодельная редукция

$$X_1 = \frac{\partial}{\partial x},\ X_2 = \frac{\partial}{\partial U_1},\ X_3 = U_1\frac{\partial}{\partial U_1},\ X_4 = W\frac{\partial}{\partial W},\ X_5 = r\frac{\partial}{\partial W},$$

$$X_6 = x\frac{\partial}{\partial x} - 2e\frac{\partial}{\partial e} - 3\varepsilon\frac{\partial}{\partial \varepsilon},\ X_7 = r\frac{\partial}{\partial r} + 2e\frac{\partial}{\partial e} + 2\varepsilon\frac{\partial}{\partial \varepsilon}.$$

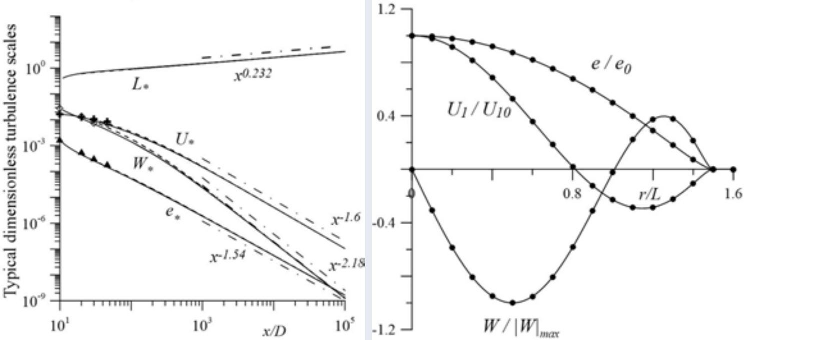
$$U_1(x,r) = x^\beta U_2(r/x^\alpha),\ W(x,r) = x^\gamma W_1(r/x^\alpha),$$

$$e(x,r) = x^{2\alpha-2}K(r/x^\alpha),\ \varepsilon(x,r) = x^{2\alpha-3}E(r/x^\alpha),\ t = r/x^\alpha.$$

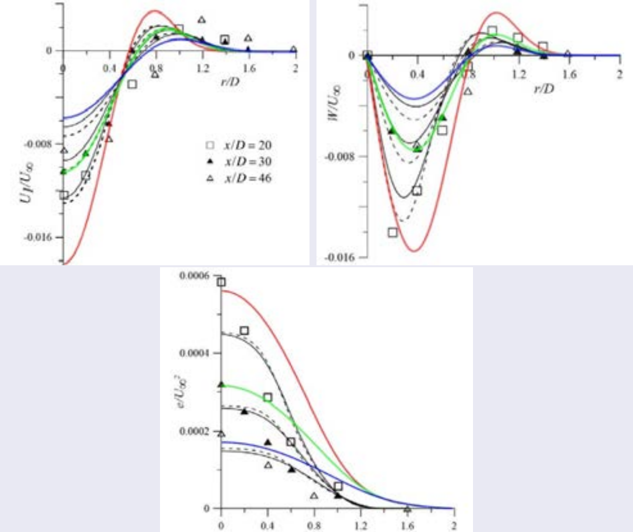
Краевые условия

$$U_2'(0) = W_1(0) = K'(0) = E'(0) = 0, \quad U_2(a) = W_1(a) = K(a) = E(a) = 0.$$

Результаты расчетов



Результаты сопоставления



Модель безымпульсного дальнего закрученного турбулентного следа

$$U_0 \frac{\partial U_1}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r K_U \frac{\partial U_1}{\partial r}\right) + \frac{\partial}{\partial x}\int_r^{\infty} \frac{W^2}{r'} dr',$$

$$U_0 \frac{\partial W}{\partial x} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 K_W \frac{\partial (W/r)}{\partial r}\right), \quad U_0 \frac{\partial e}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r K_e \frac{\partial e}{\partial r}\right) + P - \varepsilon,$$

$$U_0 \frac{\partial \varepsilon}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r K_\varepsilon \frac{\partial \varepsilon}{\partial r}\right) + C_{\varepsilon 1}\frac{\varepsilon P}{e} - C_{\varepsilon 2}\frac{\varepsilon^2}{e},$$

$$U_1 = U - U_0, K_U = K_W = K_e = \frac{2}{3}\Phi\left(1 - \Phi\frac{P}{\varepsilon}\right)\frac{e^2}{\varepsilon}, K_\varepsilon = \frac{K_U}{\sigma}, \Phi = \frac{1 - C_2}{C_1 + P/\varepsilon - 1},$$

$$P = P_W, P_W = K_U r^2 \left(\frac{\partial W/r}{\partial r}\right)^2,$$

$$C_1 = 2.2, C_2 = 0.55, C_{\varepsilon 1} = 1.45, C_{\varepsilon 2} = 1.92, \sigma = 1.3.$$

Chernykh G.G., Demenkov A.G., Kaptsov O.V., Schmidt A.V. On mathematical modeling of swirling turbulent wakes with varied total excess momentum and angular momentum // J. Eng. Thermophys. 2020. V. 29, Iss. 2. P. 222–233

Автомодельная редукция

$$X_1 = \frac{\partial}{\partial x},\ X_2 = \frac{\partial}{\partial U_1},$$
$$X_3 = x\frac{\partial}{\partial x} - 2U_1\frac{\partial}{\partial U_1} - W\frac{\partial}{\partial W} - 2e\frac{\partial}{\partial e} - 3\varepsilon\frac{\partial}{\partial \varepsilon} - 3P\frac{\partial}{\partial P},$$
$$X_4 = r\frac{\partial}{\partial r} + 2U_1\frac{\partial}{\partial U_1} + W\frac{\partial}{\partial W} + 2e\frac{\partial}{\partial e} + 2\varepsilon\frac{\partial}{\partial \varepsilon} + 2P\frac{\partial}{\partial P}.$$

$$U_1(x,r) = x^{2\alpha-2}U_2(r/x^\alpha),\ W(x,r) = x^{\alpha-1}W_1(r/x^\alpha),$$
$$e(x,r) = x^{2\alpha-2}K(r/x^\alpha),\ \varepsilon(x,r) = x^{2\alpha-3}E(r/x^\alpha),$$
$$P(x,r) = x^{2\alpha-3}P_1(r/x^\alpha),\ t = r/x^\alpha.$$

Краевые условия

$$U_2'(0) = W_1(0) = K'(0) = E'(0) = P_1'(0) = 0,$$
$$U_2(a) = W_1(a) = K(a) = E(a) = P_1(a) = 0.$$

Асимптотическое разложение

$$U_2(t) = c_1|t-a|^{\alpha_1} + o(|t-a|^{\alpha_1}),\ W_1(t) = c_2|t-a|^{\alpha_2} + o(|t-a|^{\alpha_2}),$$
$$K(t) = c_3|t-a|^{\alpha_3} + o(|t-a|^{\alpha_3}),\ E(t) = c_4|t-a|^{\alpha_4} + o(|t-a|^{\alpha_4}),$$
$$P_1(t) = c_5|t-a|^{\alpha_5} + o(|t-a|^{\alpha_5}).$$

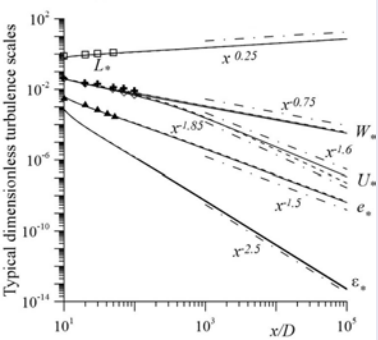
Асимптотическое разложение

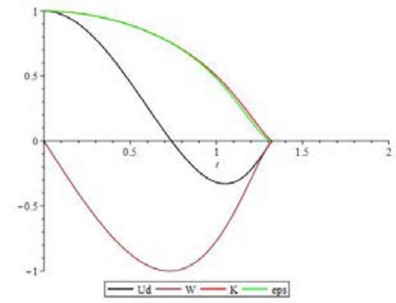
$$U_2(t) = c_1|t-a|^{\alpha_1} + o(|t-a|^{\alpha_1}),\ W_1(t) = c_2|t-a|^{\alpha_2} + o(|t-a|^{\alpha_2}),$$
$$K(t) = c_3|t-a|^{\alpha_3} + o(|t-a|^{\alpha_3}),\ E(t) = c_4|t-a|^{\alpha_4} + o(|t-a|^{\alpha_4}),$$
$$P_1(t) = c_5|t-a|^{\alpha_5} + o(|t-a|^{\alpha_5}).$$

Модифицированный метод стрельбы

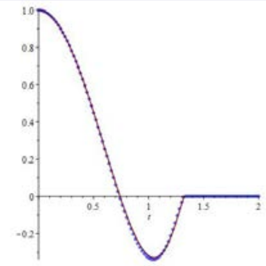
$$U_2''(0.0001) = -8.65, U_2(0.0001) = 0.85, W_1'(0.0001) = -0.803,$$
$$W_1(0.0001) = -0.000013, K(0.0001) = 0.894299, E(0.0001) = 1.04515$$

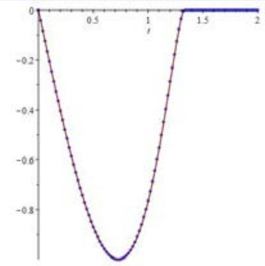
Результаты расчетов

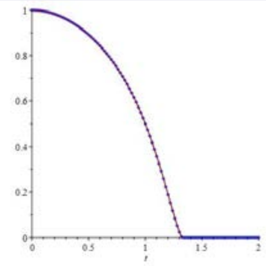




Результаты сопоставления







Модель следа за буксируемой сферой

$$U_0 \frac{\partial U_1}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(rK_U \frac{\partial U_1}{\partial r}\right),$$

$$U_0 \frac{\partial W}{\partial x} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 K_W \frac{\partial (W/r)}{\partial r}\right), \quad U_0 \frac{\partial e}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(rK_e \frac{\partial e}{\partial r}\right) + P - \varepsilon,$$

$$U_0 \frac{\partial \varepsilon}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(rK_\varepsilon \frac{\partial \varepsilon}{\partial r}\right) + C_{\varepsilon 1}\frac{\varepsilon P}{e} - C_{\varepsilon 2}\frac{\varepsilon^2}{e},$$

$$U_1 = U - U_0, K_U = K_W = K_e = \frac{2}{3}\Phi\left(1 - \Phi\frac{P}{\varepsilon}\right)\frac{e^2}{\varepsilon}, K_\varepsilon = \frac{K_U}{\sigma}, \Phi = \frac{1 - C_2}{C_1 + P/\varepsilon - 1},$$

$$P = P_U, P_U = K_U \left(\frac{\partial U}{\partial r}\right)^2,$$

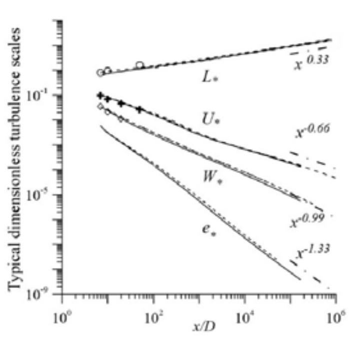
$$C_1 = 2.2, C_2 = 0.55, C_{\varepsilon 1} = 1.45, C_{\varepsilon 2} = 1.92, \sigma = 1.3.$$

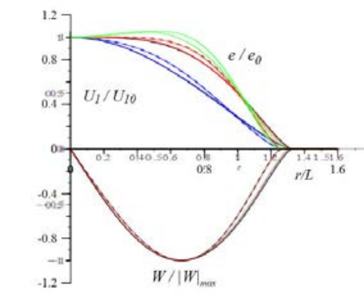
Автомодельная редукция

$$X_1 = \frac{\partial}{\partial x},\ X_2 = W\frac{\partial}{\partial W},\ X_3 = r\frac{\partial}{\partial W},$$
$$X_3 = x\frac{\partial}{\partial x} - 2U_1\frac{\partial}{\partial U_1} - 2e\frac{\partial}{\partial e} - 3\varepsilon\frac{\partial}{\partial \varepsilon} - 3P\frac{\partial}{\partial P},$$
$$X_4 = r\frac{\partial}{\partial r} + 2U_1\frac{\partial}{\partial U_1} + 2e\frac{\partial}{\partial e} + 2\varepsilon\frac{\partial}{\partial \varepsilon} + 2P\frac{\partial}{\partial P}.$$

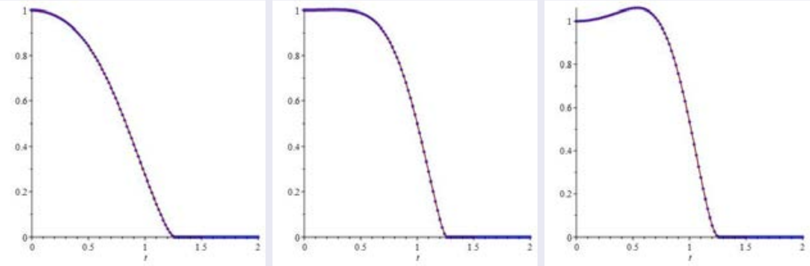
$$U_1(x,r) = x^{\alpha-1}U_2(r/x^{\alpha}),\ W(x,r) = x^{\beta}W_1(r/x^{\alpha}),$$
$$e(x,r) = x^{2\alpha-2}K(r/x^{\alpha}),\ \varepsilon(x,r) = x^{2\alpha-3}E(r/x^{\alpha}),$$
$$P(x,r) = x^{2\alpha-3}P_1(r/x^{\alpha}),\ t = r/x^{\alpha}.$$

Результаты расчетов





Результаты сопоставления



Модель осесимметричной турбулентной струи

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial r}=-\frac{1}{r}\frac{\partial}{\partial r}r\langle u'v'\rangle,\quad \frac{\partial U}{\partial x}+\frac{\partial V}{\partial r}+\frac{V}{r}=0,$$

$$\langle u'v'\rangle=\lambda\langle v'^2\rangle\frac{\partial U}{\partial r},\quad \lambda=\frac{C_2-1}{C_1-1+P/\varepsilon}\frac{e}{\varepsilon},\quad P=-\langle u'v'\rangle\frac{\partial U}{\partial r},$$

$$e=\left(\langle u'^2\rangle+\langle v'^2\rangle+\langle w'^2\rangle\right)/2,\quad \langle w'^2\rangle=\langle v'^2\rangle,$$

$$U\frac{\partial\langle u'^2\rangle}{\partial x}+V\frac{\partial\langle u'^2\rangle}{\partial r}=\frac{C_s}{r}\frac{\partial}{\partial r}\left(\frac{re\langle v'^2\rangle}{\varepsilon}\frac{\partial\langle u'^2\rangle}{\partial r}\right)-2(1-\alpha)\langle u'v'\rangle\frac{\partial U}{\partial r}-$$

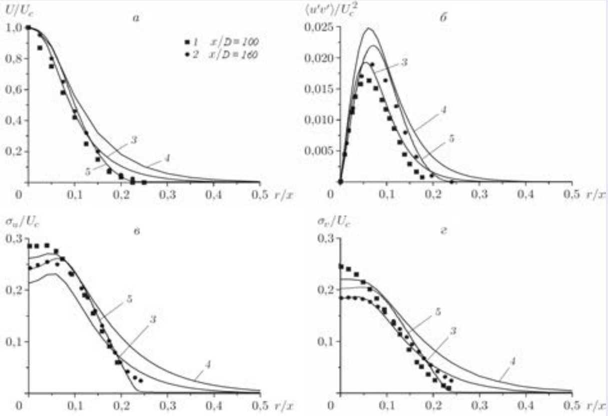
$$-\frac{2}{3}\varepsilon-C_1\frac{\varepsilon}{e}\left(\langle u'^2\rangle-\frac{2}{3}e\right)+\frac{2}{3}\alpha P,$$

$$U\frac{\partial\langle v'^2\rangle}{\partial x}+V\frac{\partial\langle v'^2\rangle}{\partial r}=\frac{C_s}{r}\frac{\partial}{\partial r}\left(\frac{re\langle v'^2\rangle}{\varepsilon}\frac{\partial\langle v'^2\rangle}{\partial r}\right)-\frac{2}{3}\varepsilon-C_1\frac{\varepsilon}{e}\left(\langle v'^2\rangle-\frac{2}{3}e\right)+\frac{2}{3}\alpha P,$$

$$U\frac{\partial\varepsilon}{\partial x}+V\frac{\partial\varepsilon}{\partial r}=\frac{C_\varepsilon}{r}\frac{\partial}{\partial r}\left(\frac{re\langle v'^2\rangle}{\varepsilon}\frac{\partial\varepsilon}{\partial r}\right)+\left(C_{\varepsilon1}\frac{P}{\varepsilon}-C_{\varepsilon2}\right)\frac{\varepsilon^2}{e}.$$

Demenkov A.G., Ilyushin B.B., Chernykh G.G. Numerical model of round turbulent jets // J. Engineering Thermophysics, 2009, 18, 1, 49–56

Результаты расчетов



1) Wygnanski I., Fiedler H. Some measurements in the self-preserving jet // J. Fluid Mech., 1969, 38, 577–612
2) Panchapakesan N. R., Lumley J. L. Turbulence measurements in axisymmetric jets of air and helium. Pt 1. Air jet // J. Fluid Mech., 1993, 246, 197–223
3), 4) - численные расчеты Г.Г. Черных с соавт.

Модель трехмерного следа в пассивно стратифицированной среде

$$U_\infty \frac{\partial u}{\partial x} = \frac{\partial}{\partial y} C_e \frac{e^2}{\varepsilon} \frac{\partial u}{\partial y} + \frac{\partial}{\partial z} C_e \frac{e^2}{\varepsilon} \frac{\partial u}{\partial z},$$

$$U_\infty \frac{\partial e}{\partial x} = \frac{\partial}{\partial y} C_e \frac{e^2}{\varepsilon} \frac{\partial e}{\partial y} + \frac{\partial}{\partial z} C_e \frac{e^2}{\varepsilon} \frac{\partial e}{\partial z} + C_e \frac{e^2}{\varepsilon} \left(\frac{\partial u}{\partial y} \right)^2 + C_e \frac{e^2}{\varepsilon} \left(\frac{\partial u}{\partial z} \right)^2 - \varepsilon,$$

$$U_\infty \frac{\partial \varepsilon}{\partial x} = \frac{\partial}{\partial y} \frac{C_e}{\sigma} \frac{e^2}{\varepsilon} \frac{\partial \varepsilon}{\partial y} + \frac{\partial}{\partial z} \frac{C_e}{\sigma} \frac{e^2}{\varepsilon} \frac{\partial \varepsilon}{\partial z} + C_{\varepsilon 1} e \left(\frac{\partial u}{\partial y} \right)^2 + C_{\varepsilon 1} e \left(\frac{\partial u}{\partial z} \right)^2 - C_{\varepsilon 2} \frac{\varepsilon^2}{e},$$

$$U_\infty \frac{\partial \langle \rho_1 \rangle}{\partial x} = \frac{\partial}{\partial y} C_\rho \frac{e^2}{\varepsilon} \frac{\partial \langle \rho_1 \rangle}{\partial y} + \frac{\partial}{\partial z} C_\rho \frac{e^2}{\varepsilon} \frac{\partial \langle \rho_1 \rangle}{\partial z} - \frac{\partial}{\partial z} C_\rho \frac{e^2}{\varepsilon},$$

$$U_\infty \frac{\partial \langle \rho'^2 \rangle}{\partial x} = \frac{\partial}{\partial y} C_{1\rho} \frac{e^2}{\varepsilon} \frac{\partial \langle \rho'^2 \rangle}{\partial y} + \frac{\partial}{\partial z} C_{1\rho} \frac{e^2}{\varepsilon} \frac{\partial \langle \rho'^2 \rangle}{\partial z} + 2C_\rho \frac{e^2}{\varepsilon} \frac{\partial \langle \rho_1 \rangle}{\partial y}^2 +$$

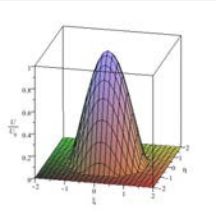
$$+ 2C_\rho \frac{e^2}{\varepsilon} \left(\frac{\partial \langle \rho_1 \rangle}{\partial z} - 1 \right)^2 - C_T \frac{\langle \rho'^2 \rangle \varepsilon}{e}.$$

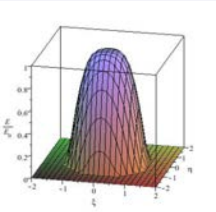
Chernykh G.G., Fomina A.V., Moshkin N.P. Numerical models of turbulent wake dynamics behind a towed body in a linearly stratified medium // Russ. J. Numer. Anal. Math. Model., 2006, 21, 5, 395–424
Chashechkin Yu.D., Chernykh G.G., Voropaeva O.F. The propagation of a passive admixture from a local instantaneous source in a turbulent mixing zone // Int. J. Comp. Fluid Dyn., 2005, 19, 6, 517–529

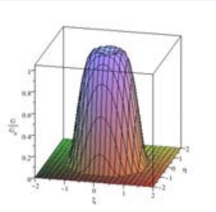
Представление для решения модели

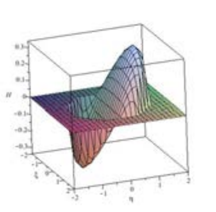
$$u = x^{\alpha-1}U(\tau),\ e = x^{2\alpha-2}E(\tau),\ \varepsilon = x^{2\alpha-3}G(\tau),\ \langle\rho_1\rangle = zH(\tau),$$
$$\langle\rho'^2\rangle = z^2R_1(\tau) + x^{2\alpha}R_2(\tau),\ \tau = \sqrt{y^2+z^2}/x^\alpha.$$

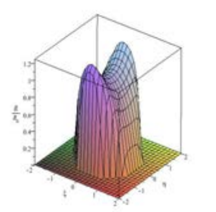
Результаты расчетов











СПАСИБО ЗА ВНИМАНИЕ!