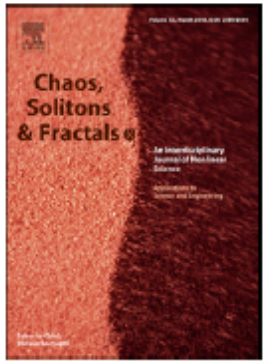
Chaos, Solitons and Fractals 154 (2022) 111699

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos



Gompertz model in COVID-19 spreading simulation

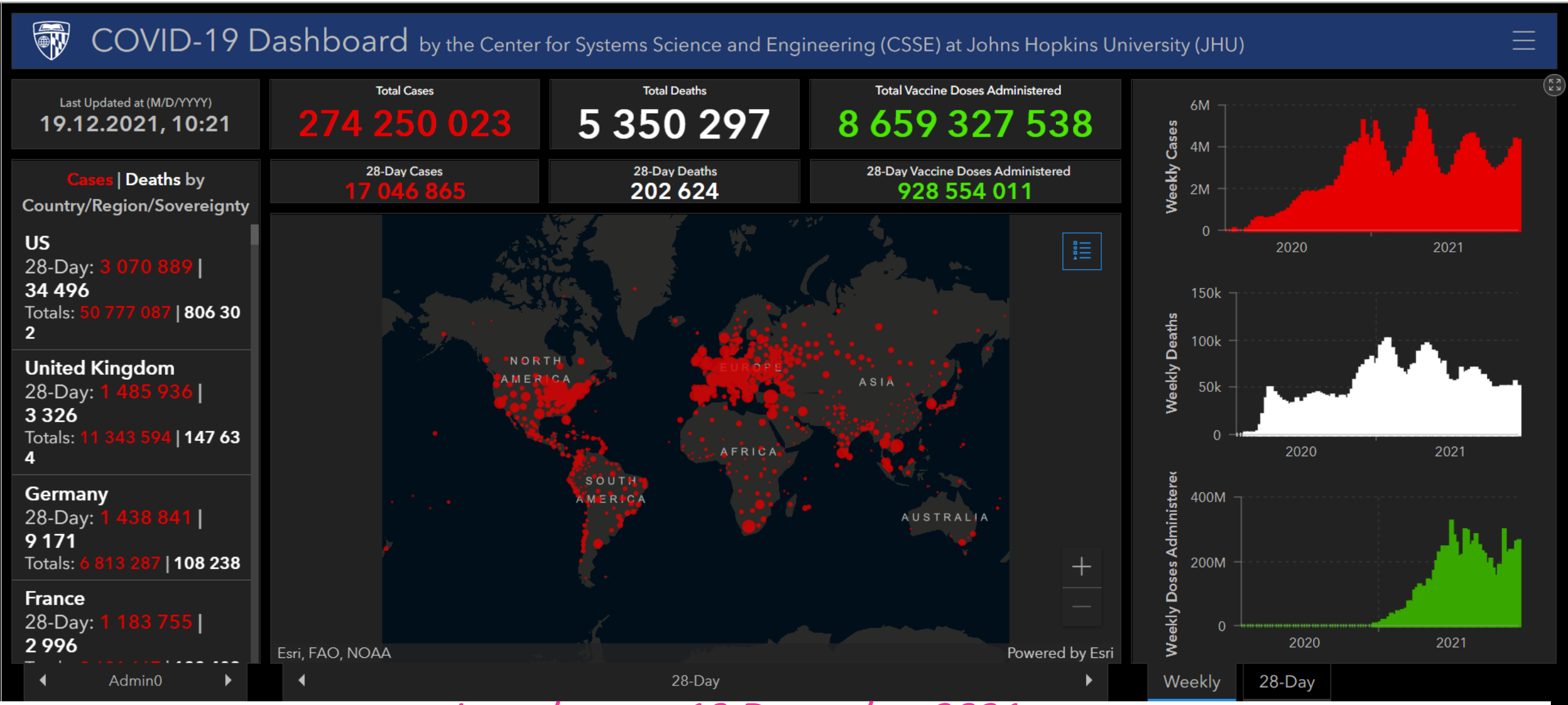
E. Pelinovsky[a,b], M. Kokoulina[c], A. Epifanova[c], A. Kurkin[c,*], O. Kurkina[c], M. Tang[d], E. Macau[e], M. Kirillin[b]

[a] National Research University – Higher School of Economics, Myasnitskaya st., 20, Moscow 101000, Russian Federation
[b] Institute of Applied Physics RAS, Ul'yanov st., 46, Nizhny Novgorod 603950, Russian Federation
[c] Nizhny Novgorod State Technical University n.a. R.E. Alekseev, Minin st., 24, Nizhny Novgorod 603950, Russian Federation
[d] East China Normal University, Shanghai 200241, China
[e] Universidade Federal de São Paulo, São Paulo 04021-001, Brazil

The study is supported by joint RFBR – BRICS project no. 20–51–80 0 04)

COVID-19

https://www.arcgis.com/apps/dashboards/bda7594740fd40299423467b48e9ecf6



Last data on 19 December 2021

Pierre Verhulst was born into a wealthy family who spared no expense to give their son a top quality education. Verhulst excelled in science but had other talents too, twice winning a Latin poetry prize.

While Verhulst had been in Rome there had been a rebellion in Belgium with much fighting between revolutionaries and Dutch troops. In fact it was news of this rebellion that had motivated him to write a constitution for the Papal States. Verhulst was keen to get involved and, despite his illness and against the advice of his friends, in the middle of 1831 he enlisted in the army set up to oppose the Dutch forces. He also wanted to influence the political situation and wrote *Mémoire sur les abus dans l'enseignement supérieur actuel* which criticised the way university professors were chosen through political favouritism. He also criticised the standards of teaching in the universities and suggested reforms that the National Congress could implement to improve the situation.

Pierre François Verhulst
28.10.1804 – 15.02.1849,
Brussels, Belgium

On 28 September 1835 Verhulst was appointed professor of mathematics at the Université Libre of Brussels. There he gave courses on astronomy, celestial mechanics, the differential and integral calculus, the theory of probability, geometry and trigonometry.

Logistic Equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{N_\infty}\right)$$

Verhulst P.F. Notice sur la loi que la population poursuit dans son accroissement. Correspondance Mathematique et Physique. 1838. V. 10. P. 113 – 121.

Logistic Equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{N_\infty}\right)$$

$$\frac{dx}{d\tau} = x(1-x)$$

$$N(t) = \frac{N_0 N_\infty \exp(rt)}{N_\infty + N_0\left[\exp(rt) - 1\right]}$$

$$x(t) = \frac{x_0 \exp(\tau)}{1 + x_0\left[\exp(\tau) - 1\right]}$$

N(t) – полное число заболевших со временем

$$K = \frac{\Delta N}{\Delta t} \simeq \frac{dN}{dt}$$

- число заболевших в день

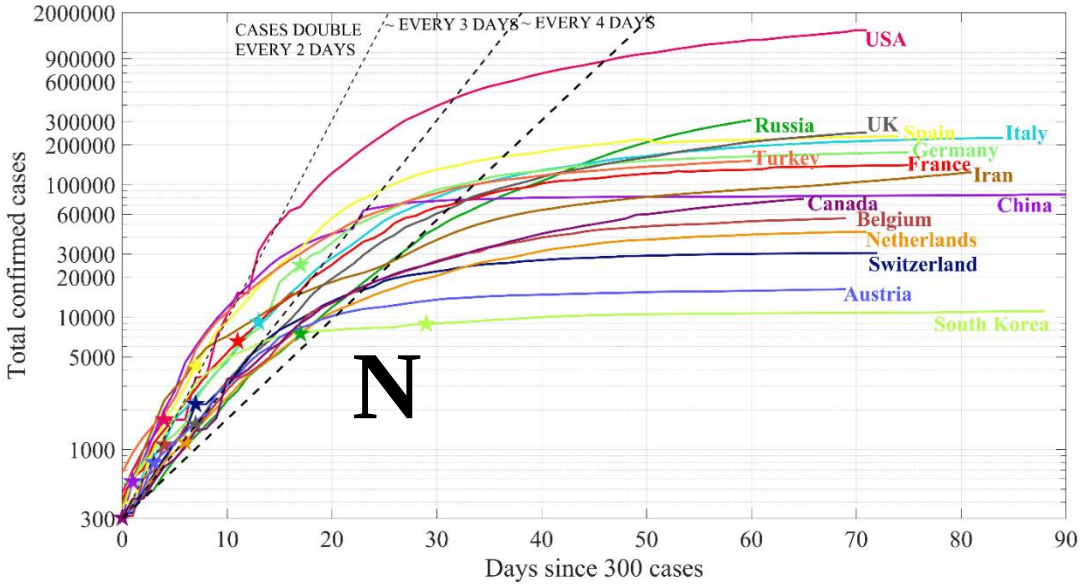
“Practical” Logistic Curve

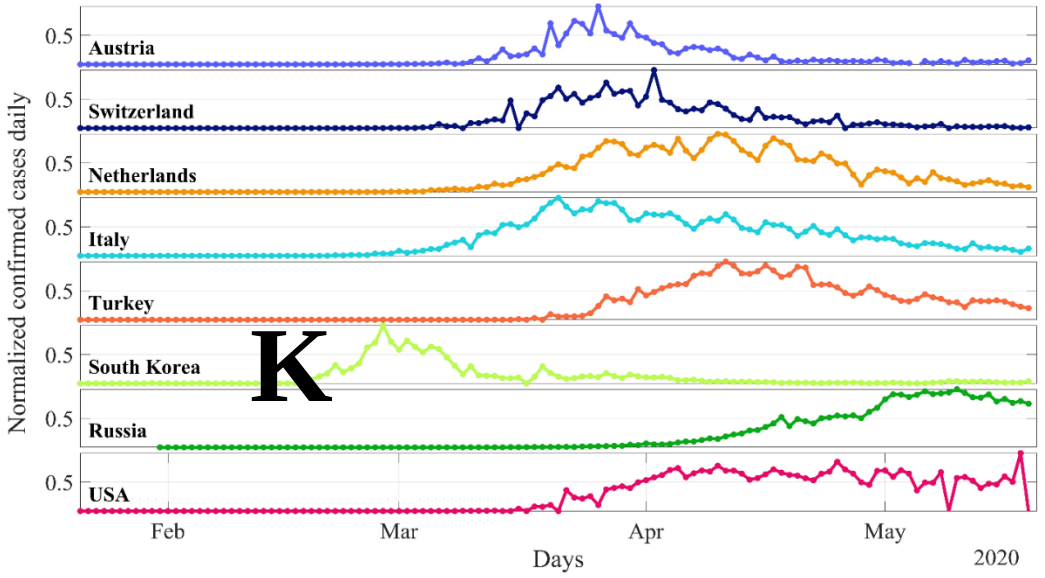
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{N_\infty}\right)$$

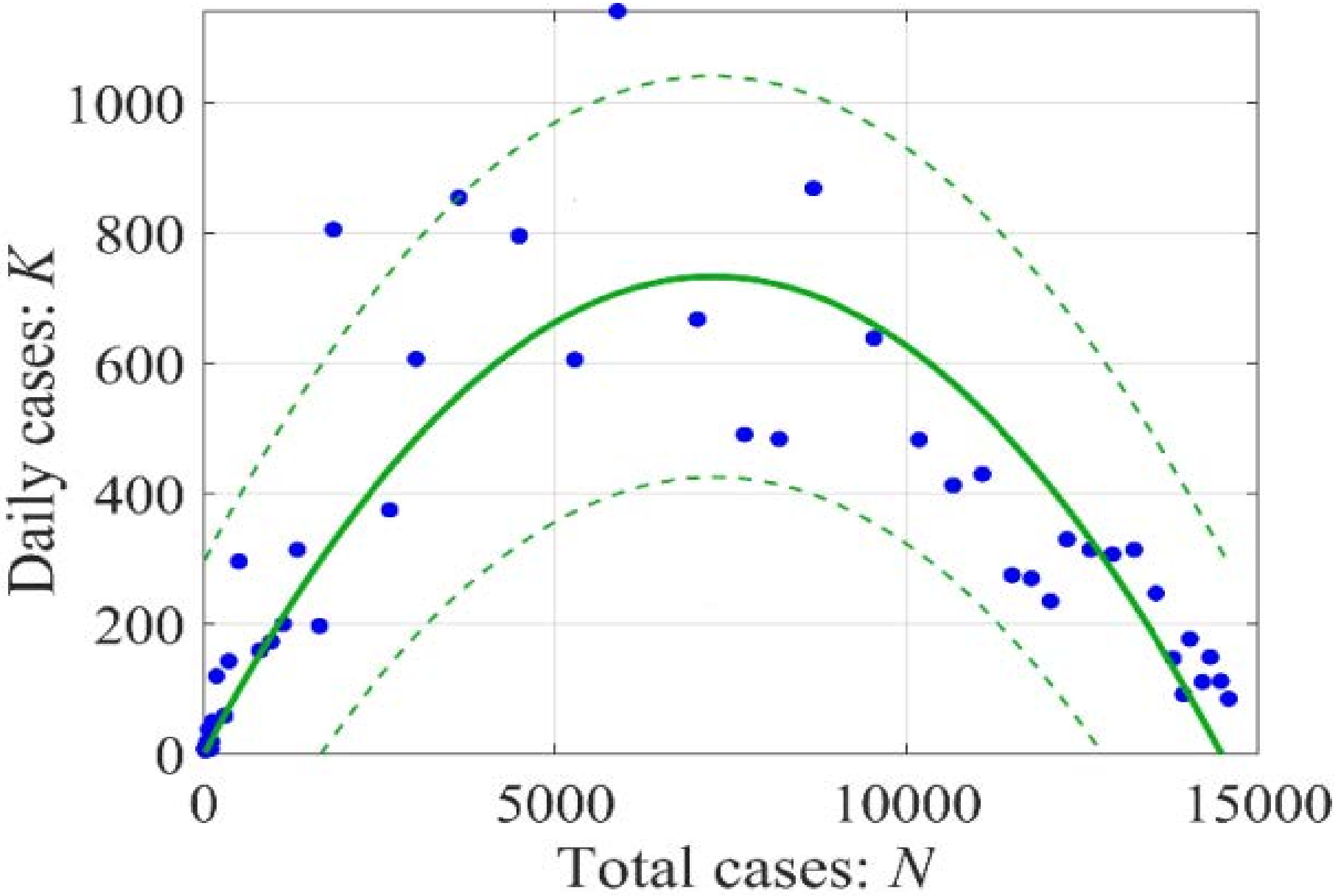
Taking into account $K \cong \frac{dN}{dt}$

$$K = rN\left(1 - \frac{N}{N_\infty}\right)$$

Algebraic problem – using parabola approximation to find coefficients







Austria

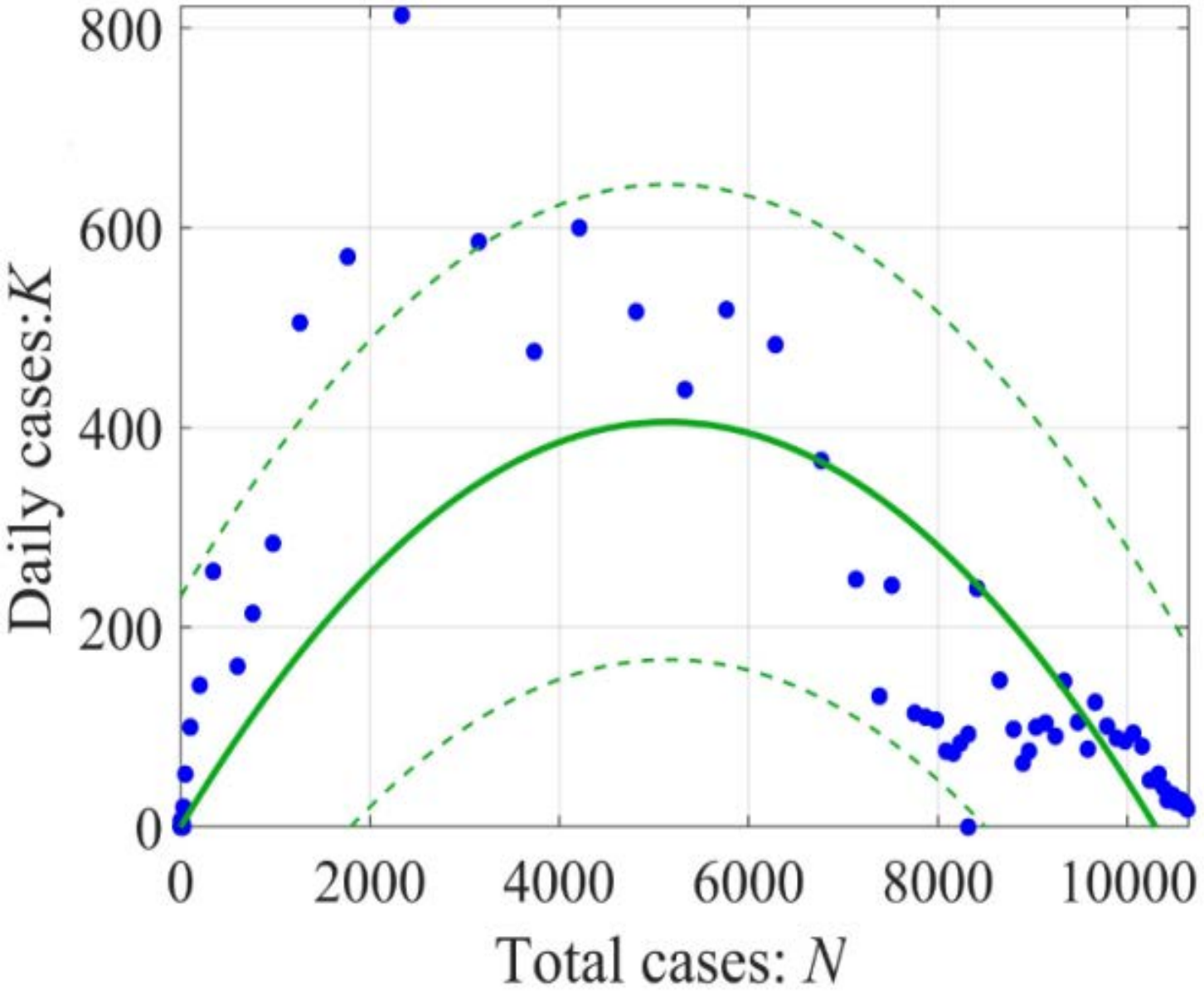
$N_\infty = 14700$

$r = 0.195$

$R^2 = 0.81$

Good

solid green line is the regression, dashed lines give 95% prediction bounds



South Korea

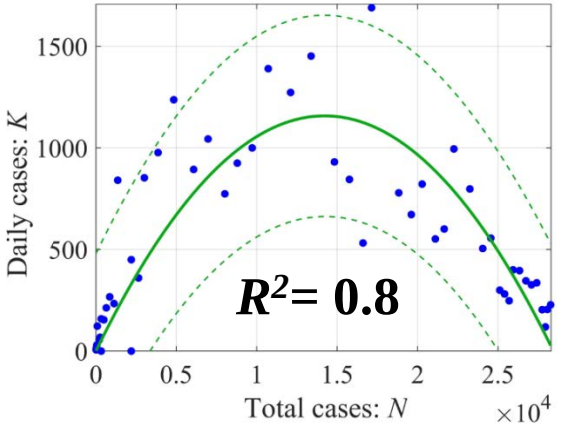
$$N_{\infty} = 10300$$

$r = 0.16$

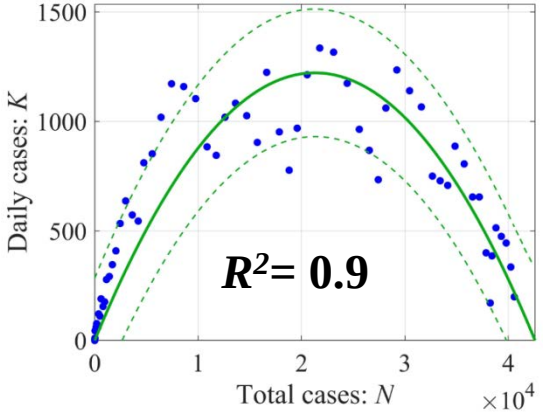
$R^2 = 0.55$

Bad

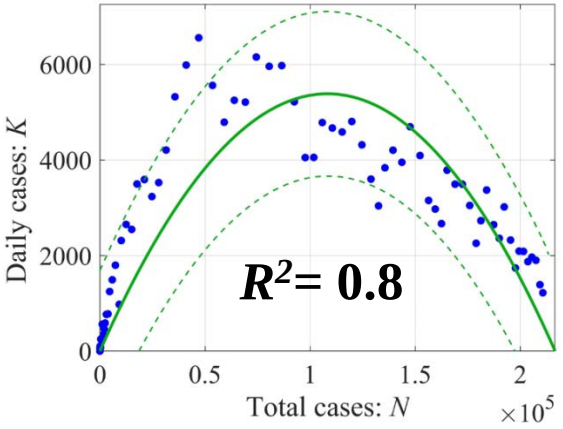
solid green line is the regression, dashed lines give 95% prediction bounds



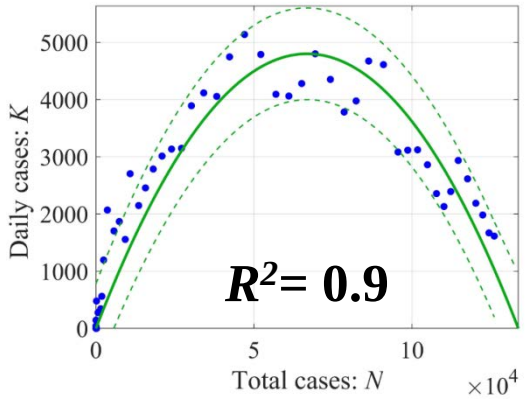
Switzerland



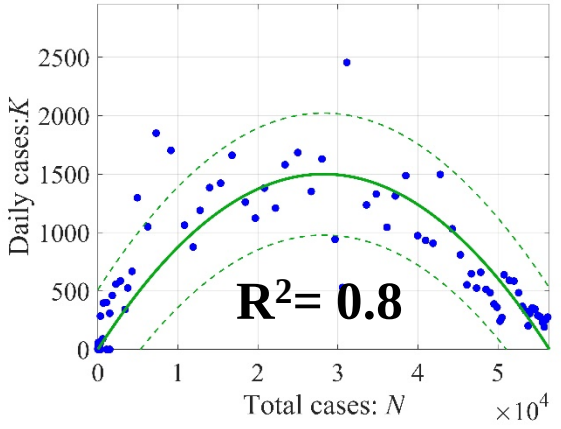
Netherlands



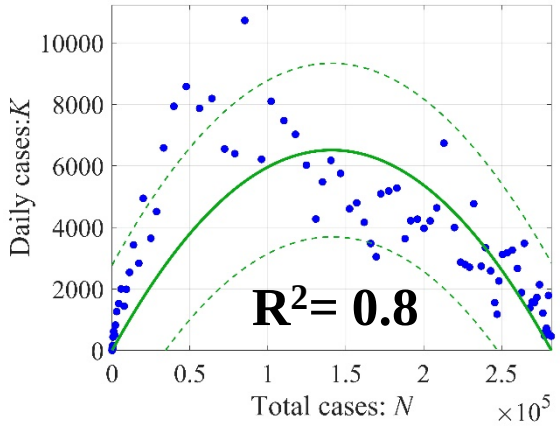
Italy



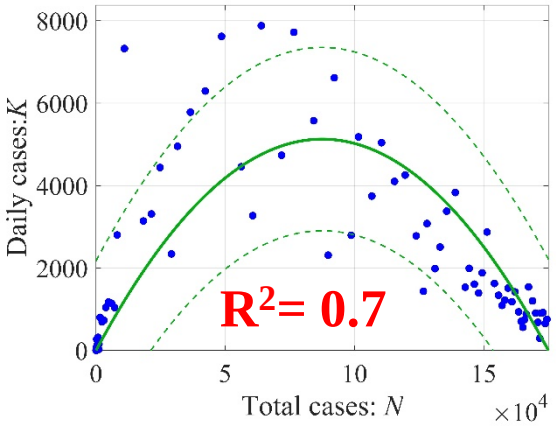
Turkey



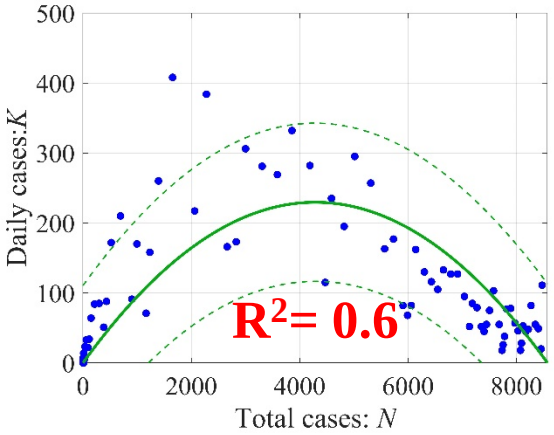
Belgium



Spain



Germany



Czech

Australia, Norway R^2= 0.7

Logistic Models

The logistic-like model as a whole describes well both the change in the world population, the biological population, and the number of sick people during the epidemic. We've also cited ad effectiveness of advertise as an example. Therefore, the logistic equation can be considered as basic in many areas of natural science, where one can expect a monotonous growth of the temporal function and its saturation at some level.

Pelinovsky, E., Kurkin, A., Kurkina, O., Kokoulina, M., and Epifanova, A. Logistic equation and COVID-19. *Chaos, Solitons and Fractals* (*Nonlinear Science, and Nonequilibrium and Complex Phenomena)*, 2020, vol. 140, 110241.

Consolini C., Materassi M. A stretched logistic equation for pandemic spreading. *Chaos, Solitons and Fractals*. 2020. Vol. 140. Art. No. 110113

Wu K., Darcet D., Wang Q., Sornette D. Generalized logistic growth modeling of the COVID-19 outbreak: comparing the dynamics in the 29 provinces in China and in the rest of the world. *Nonlinear Dynamics*, 2020, vol. 101, 1561–1581

Carletti T., Fanelli D., Piazza F. COVID-19: The unreasonable effectiveness of simple models. *Chaos, Solitons and Fractals*. 2020. Vol. 140. Art. No. 100034.

Benjamin Gompertz (5 March 1779 – 14 July 1865) was a British self-educated mathematician and actuary, who became a Fellow of the Royal Society. Gompertz is now best known for his Gompertz law of mortality, a demographic model published in 1825.

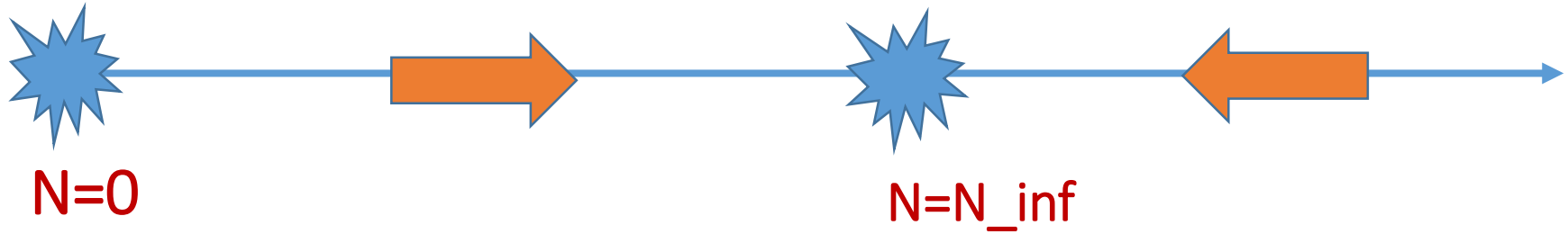
Of the German Jewish family of Gompertz, he was born in London, where his father and grandfather had been successful diamond merchants. In line with his father's wishes, he entered the London Stock Exchange. He became a member of the Mathematical Society of Spitalfields, and served as its president when it was merged with the Astronomical Society of London. In 1819 he was elected a F.R.S., and in 1832 became a member of the council. The Astronomical Society was founded in 1820, and he was elected a member of the council in 1821

$$\frac{dN}{dt} = rN\left(1 - \frac{\ln N}{\ln N_\infty}\right)$$

Also two parameters in model.

System with two equilibrium points

Gompertz, Benjamin (1825). "On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies". *Philosophical Transactions of the Royal Society of London*. **115**: 513–585. doi:10.1098/rstl.1825.0026. S2CID 145157003

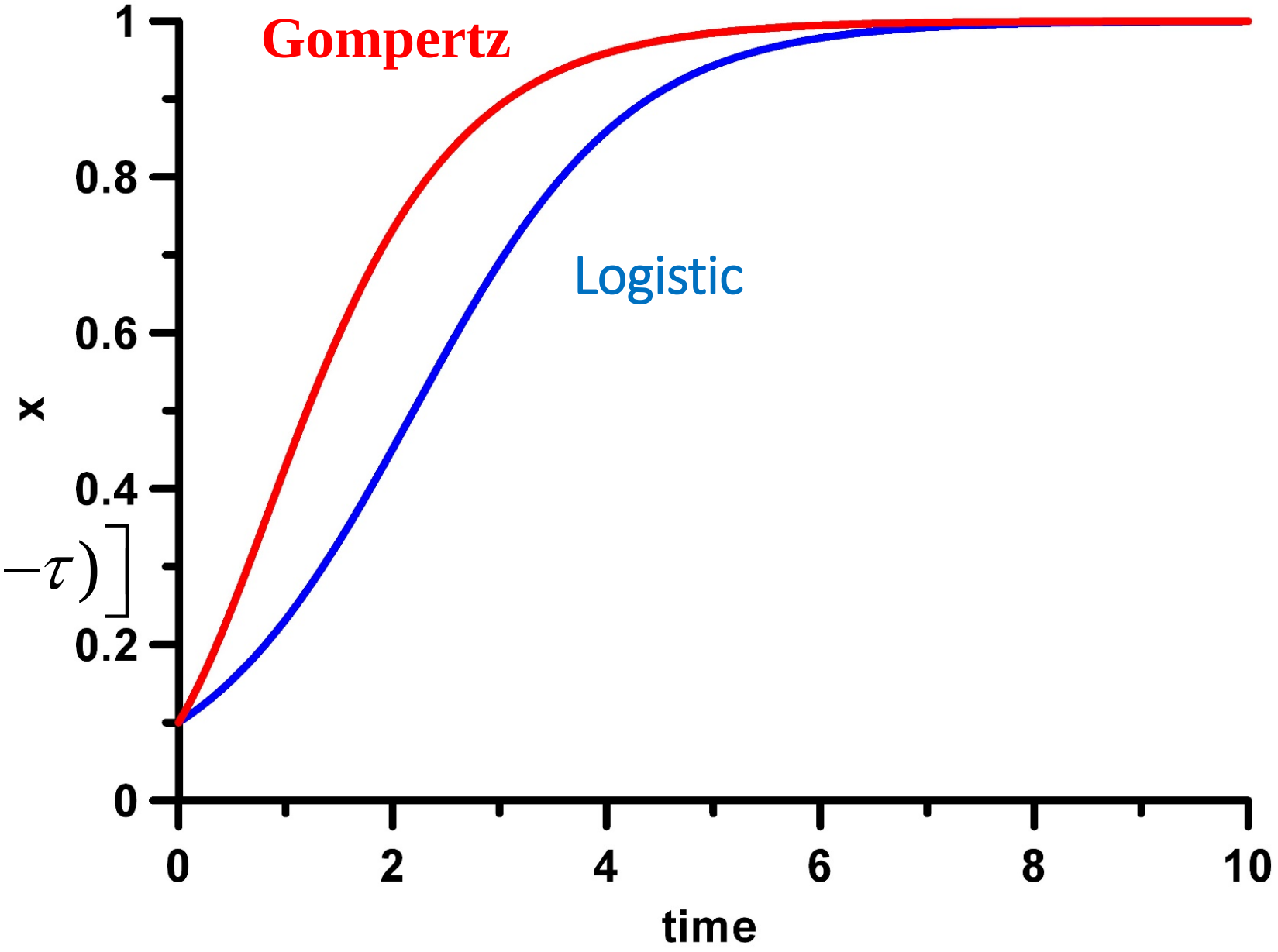


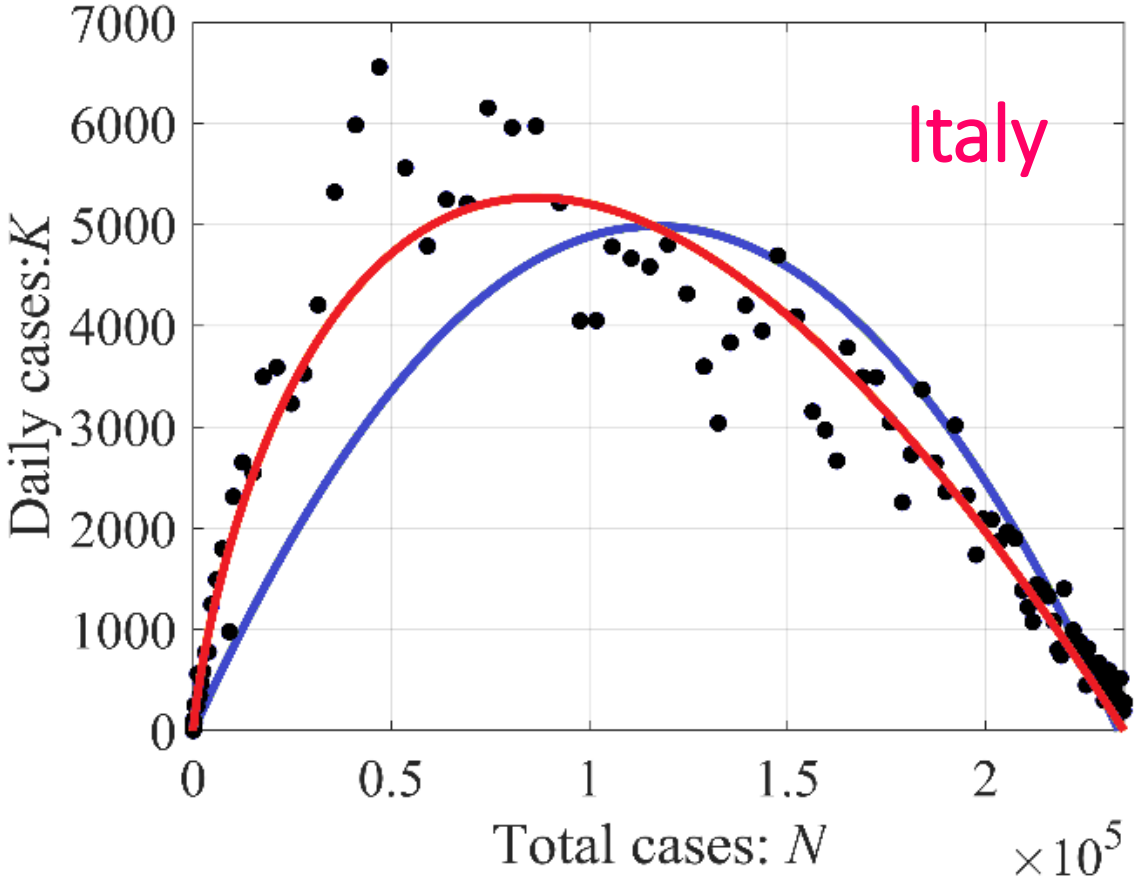
Logistic curve

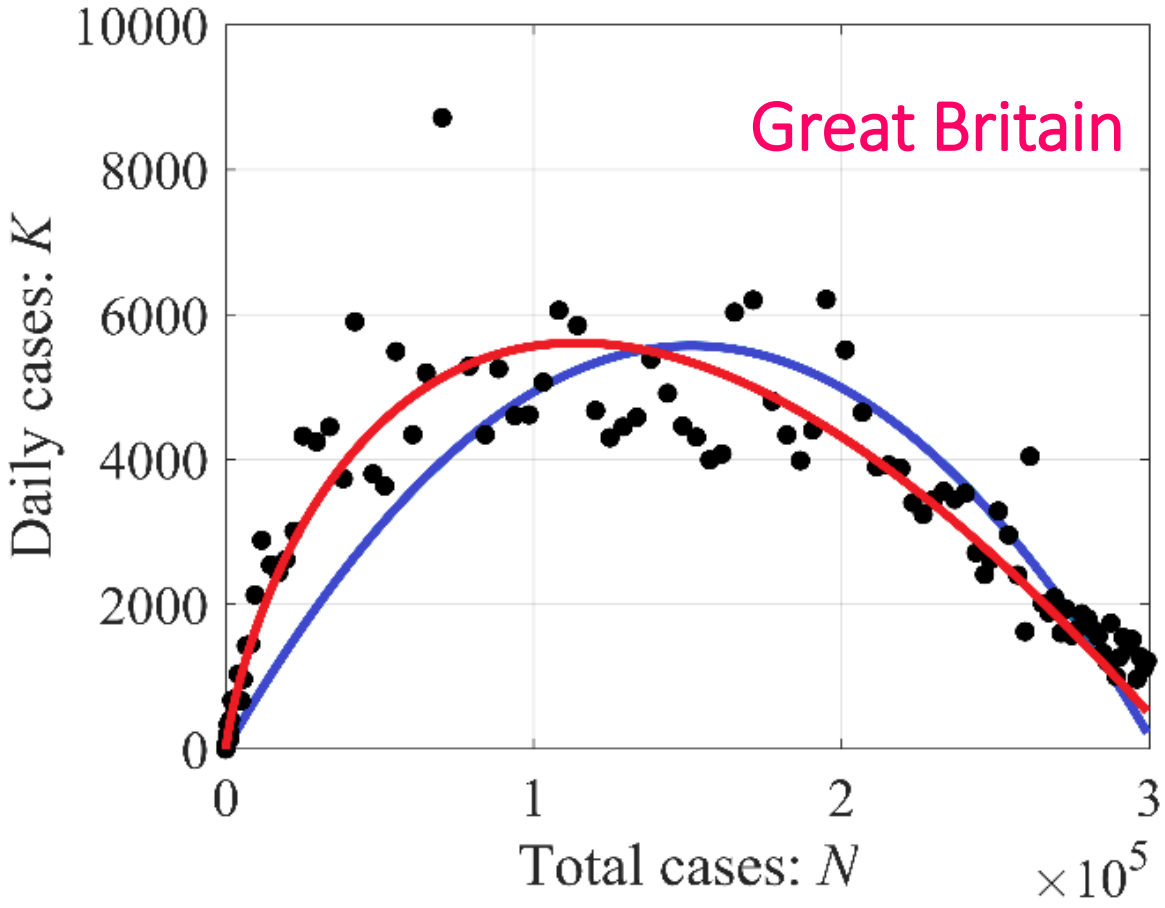
$$x(\tau) = \frac{x_0 \exp(\tau)}{1 + x_0 \left[\exp(\tau) - 1\right]}$$

Benjamin Gompertz

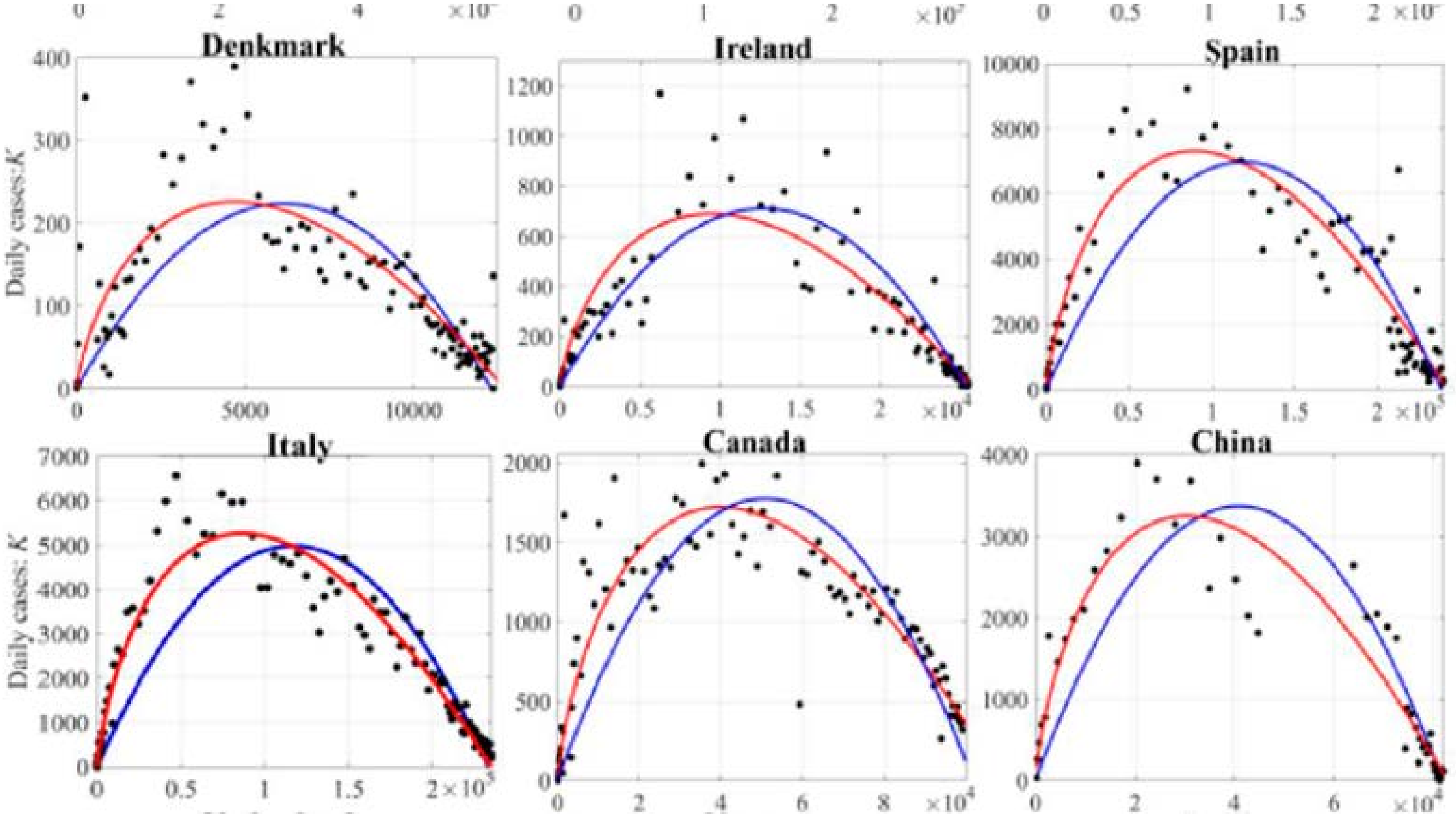
$$x(\tau) = \exp\left[\left(\ln x_0\right) \cdot \exp(-\tau)\right]$$







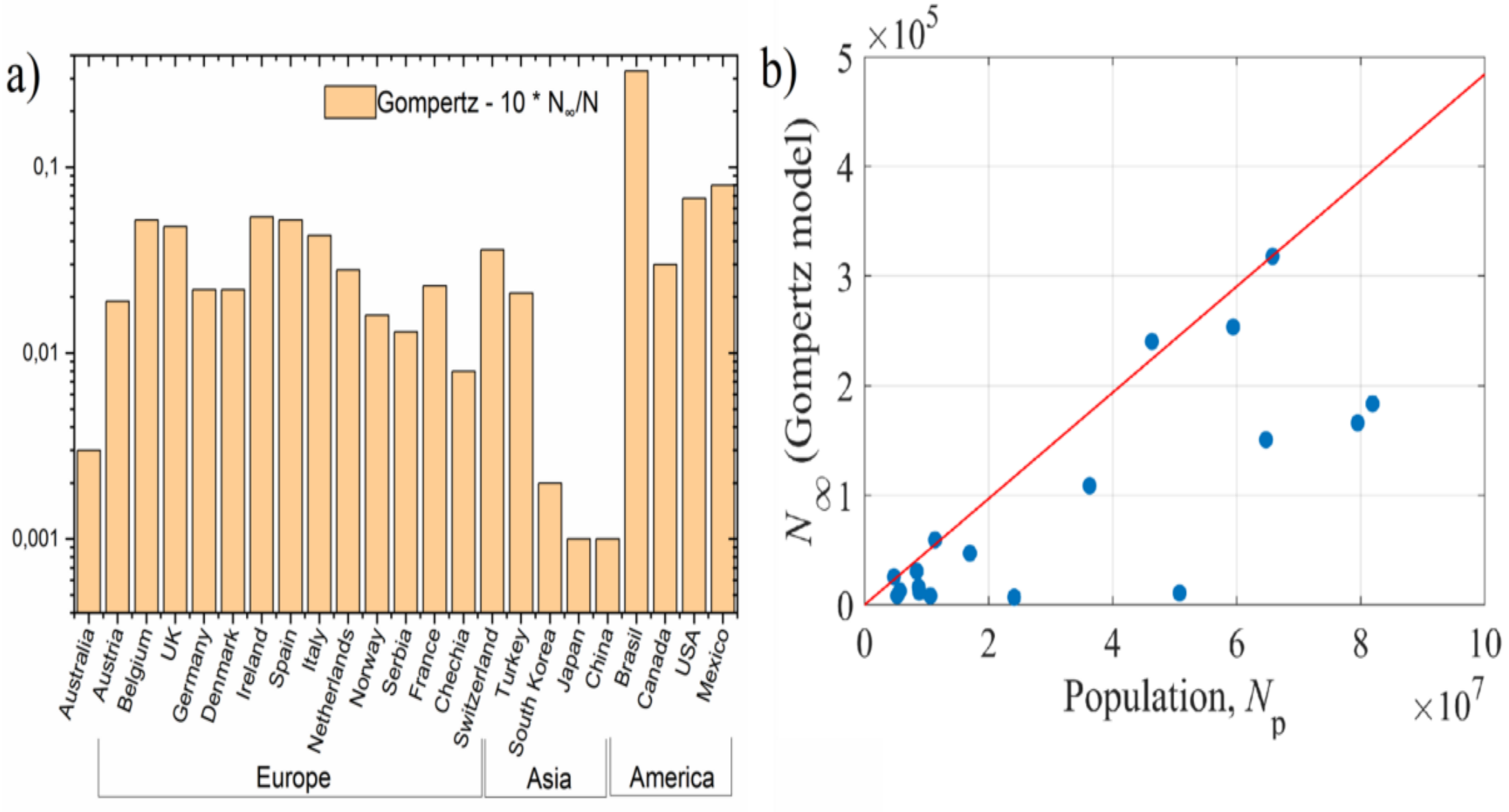
Blue – logistic model, red - Gompertz model



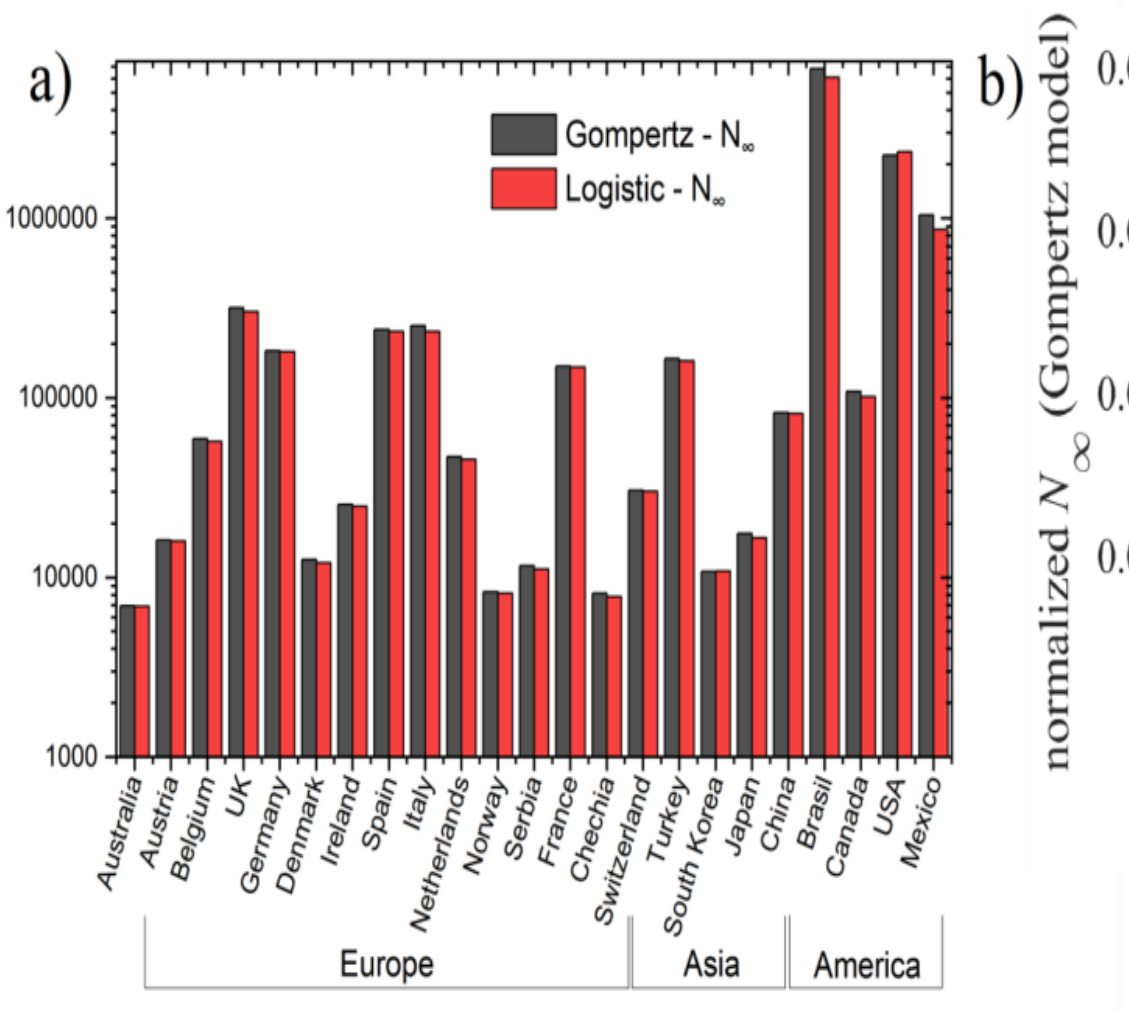
Blue – logistic model, red - Gompertz model

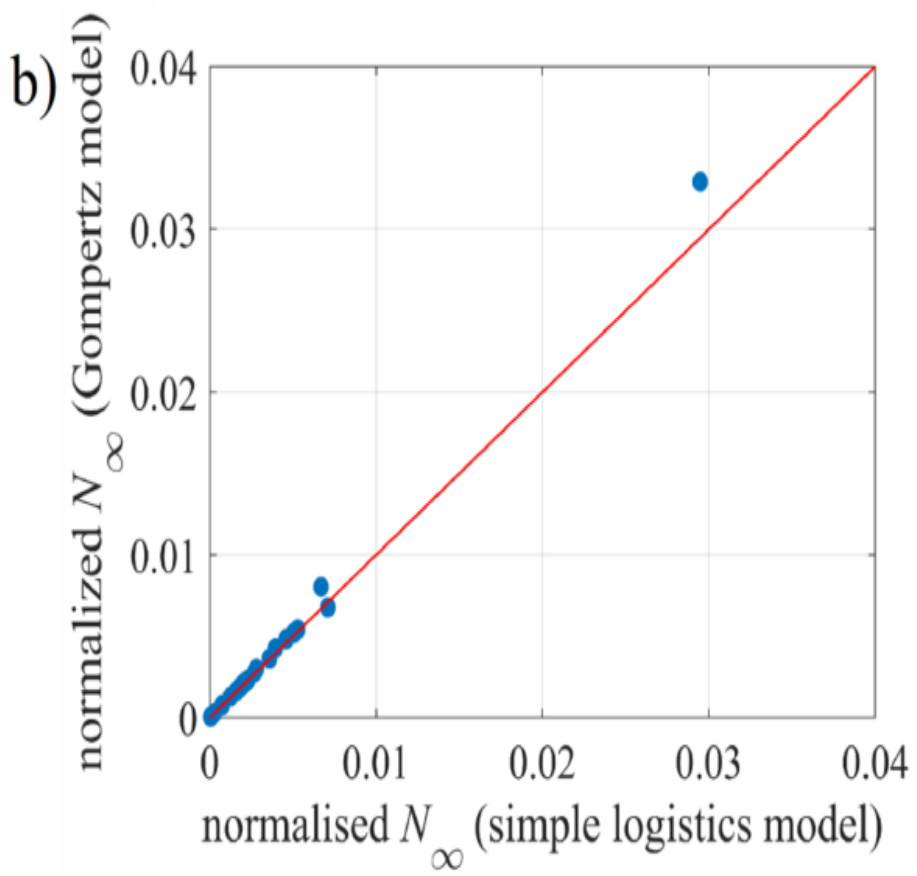
Model		Gompertz equation				Simple logistic equation			
№	Country	r	N_∞	R^2	N_∞/N_p ×10	r	N_∞	R^2	N_∞/N_p ×10
1	Australia	1.297	6974	**0.81**	0.003	0.21	6901	**0.73**	0.003
2	Austria	1.079	16155	**0.82**	0.019	0.16	15950	**0.67**	0.018
3	Belgium	0.743	59278	**0.79**	0.052	0.1	57350	**0.66**	0.050
4	Brazil	0.26	6834000	**0.67**	0.329	0.03	6120000	**0.58**	0.295
5	Great Britain	0.6	318100	**0.89**	0.048	0.074	302300	**0.75**	0.046
6	Germany	0.9	183500	**0.83**	0.022	0.1	181300	**0.67**	0.022
7	Denmark	0.5	12580	**0.74**	0.022	0.075	12110	**0.64**	0.021

Better correlation analysis for Gompertz model

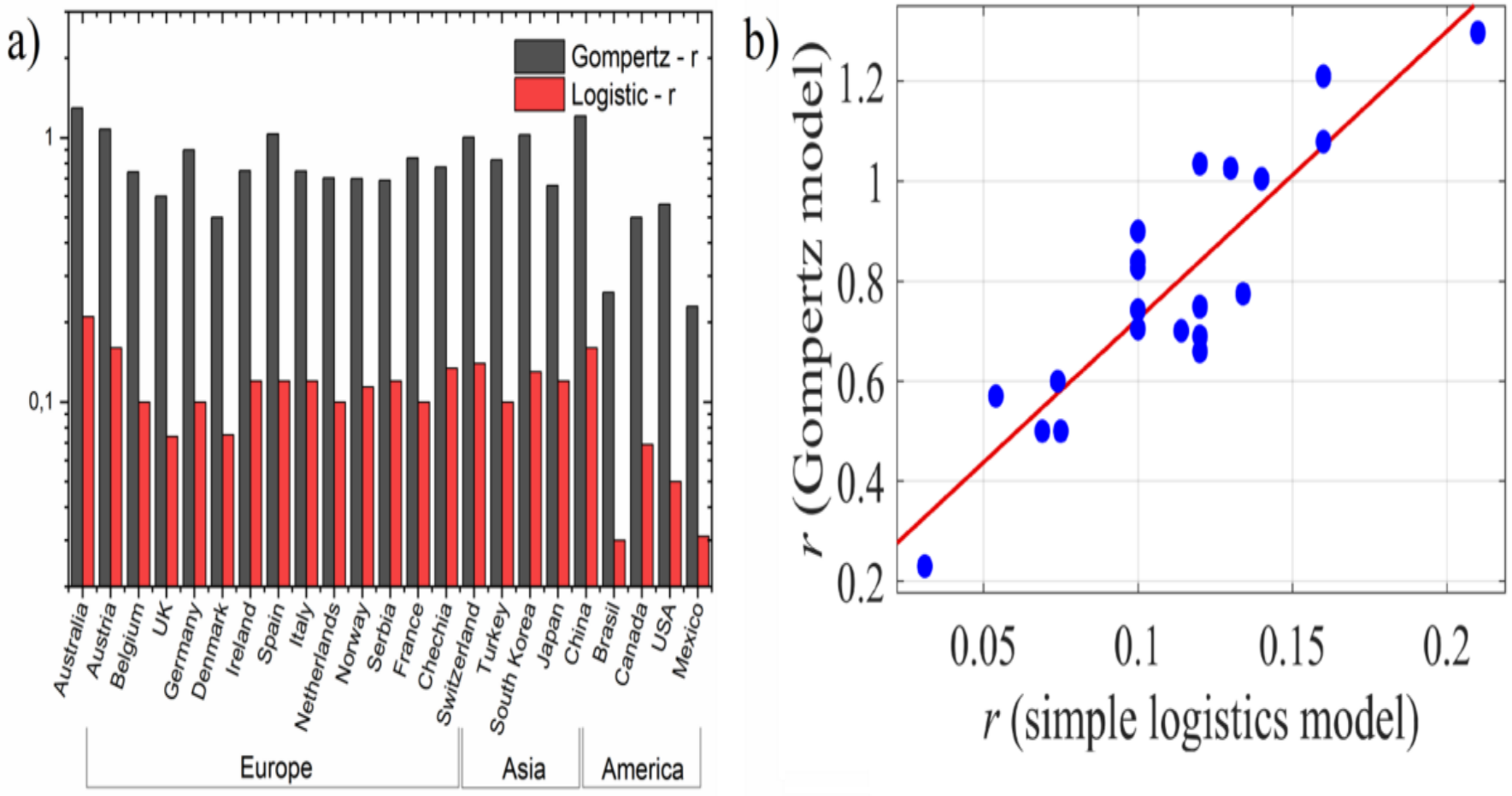


Maximal number of infected persons N_∞ normalized for country population N_p (a) and maximal number of infected persons N_∞ versus N_p (b) in Gompertz model.

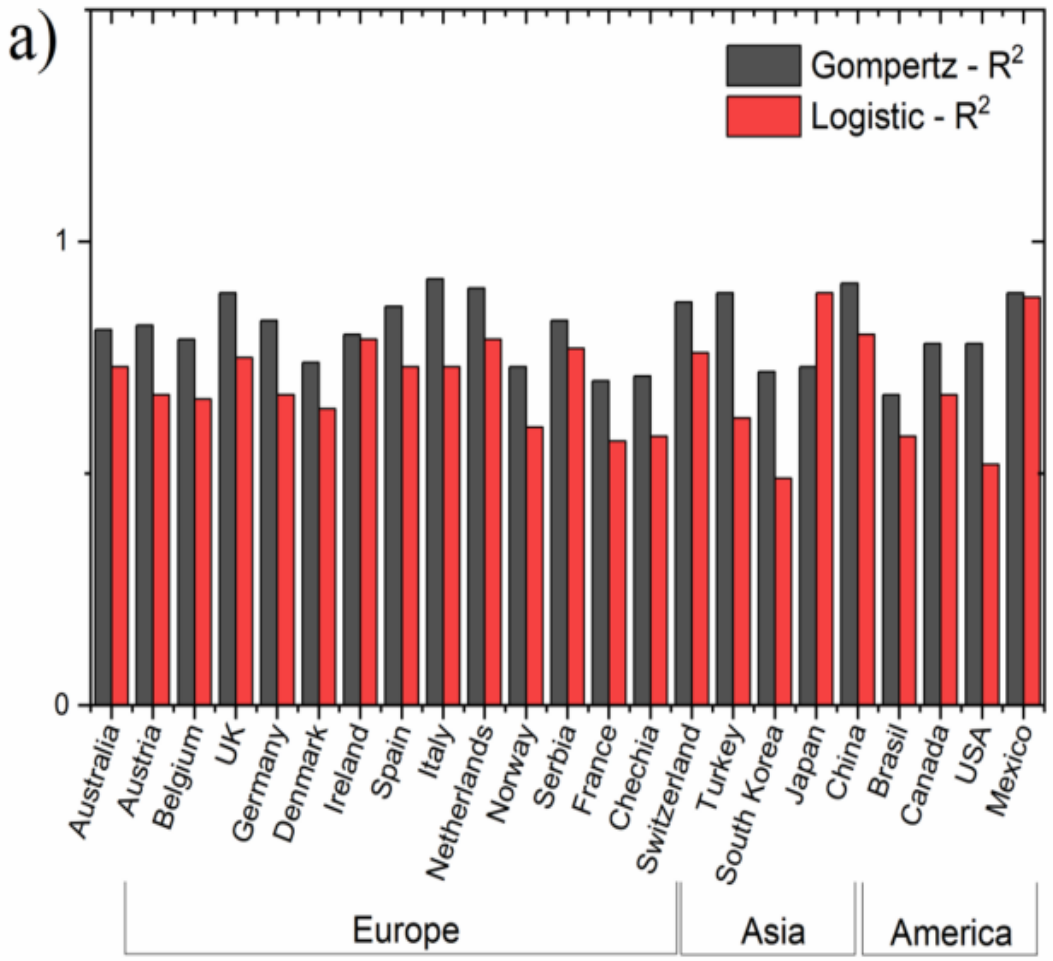


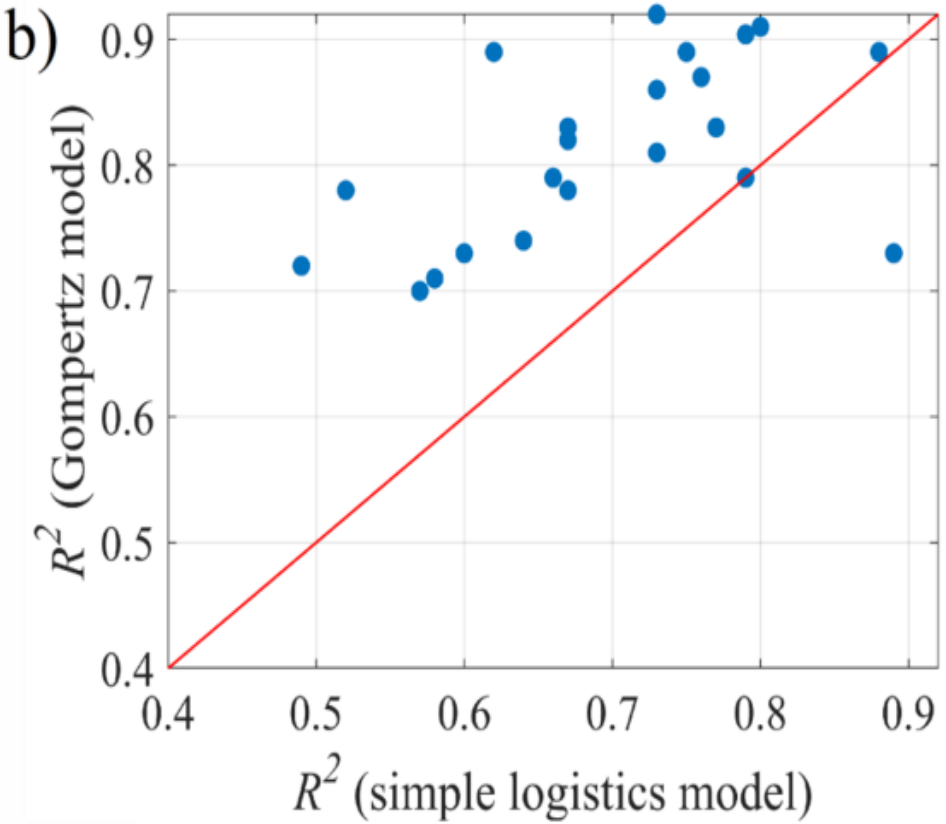


Maximal number of infected persons N_∞ in simple logistic model and Gompertz equation for all considered countries (a) and correlation of the N_∞ normalized for total population N_p for the two considered models (b).



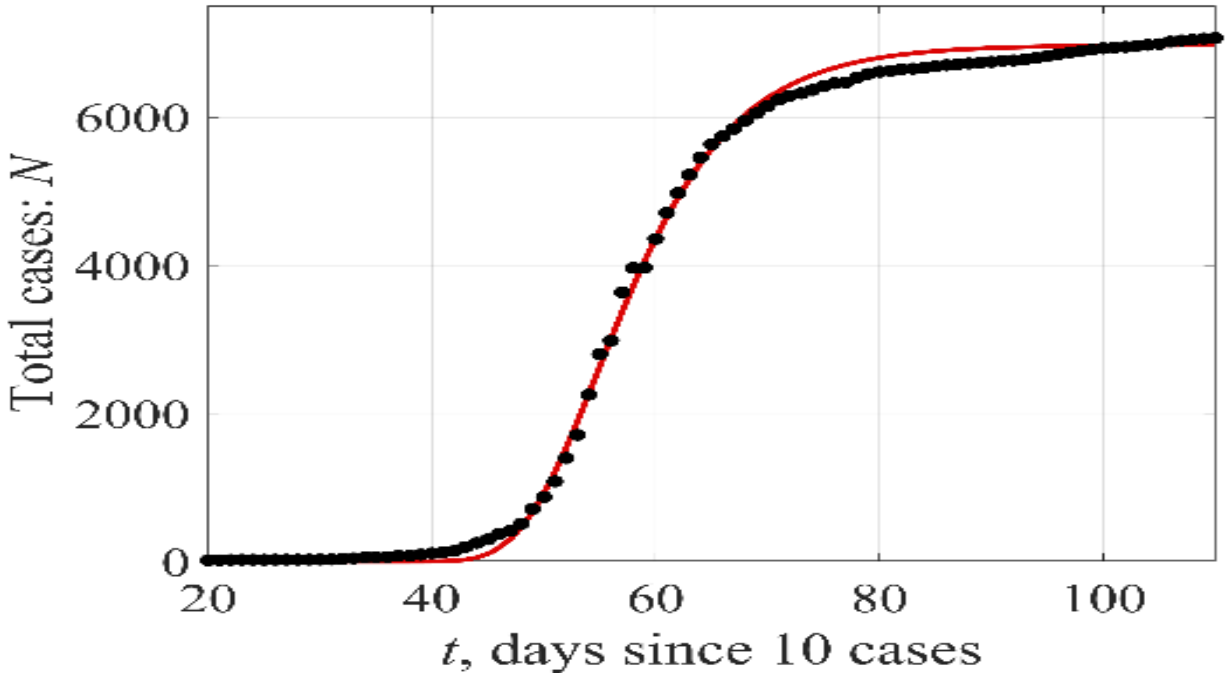
Infection rate *r* in simple logistic model and Gompertz equation (a) and *r* values acquired with Gompertz models versus logistic equation (b) for all considered countries. The red line shows regression line with coefficient of 5.9.





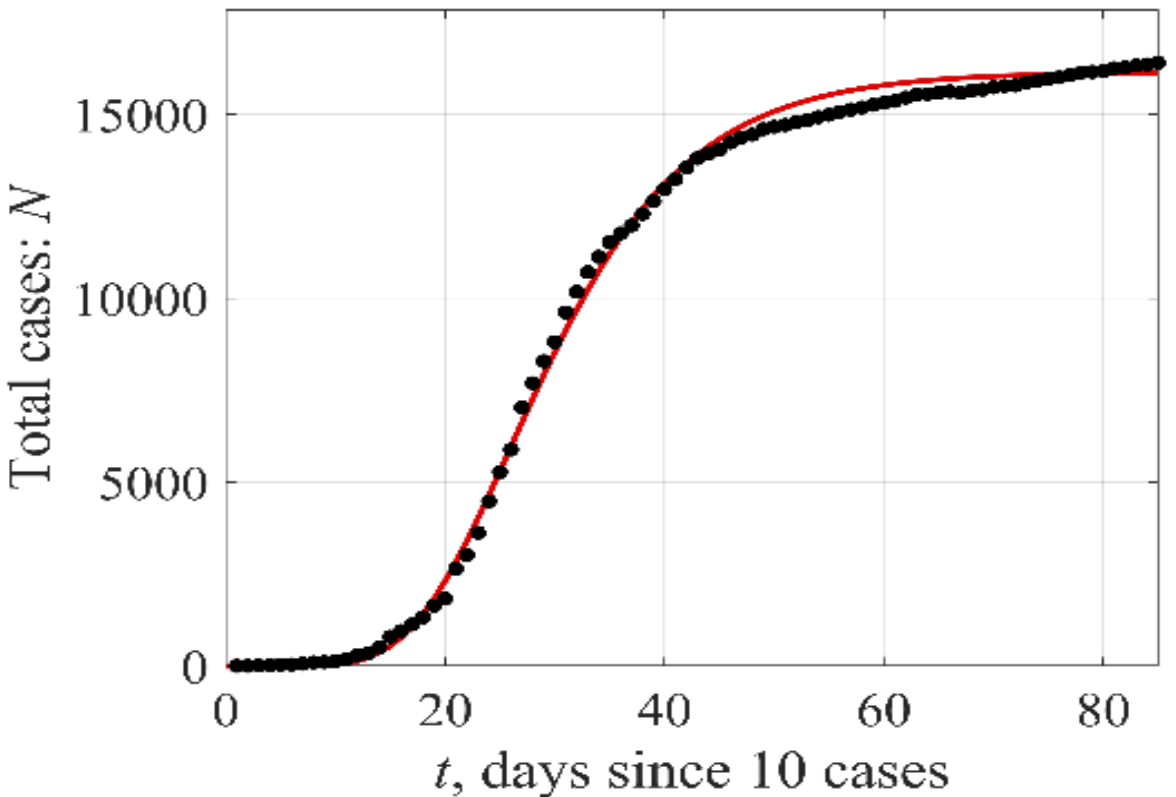
Better correlation analysis for Gompertz model

Comparison of the determination coefficients R^2 for the logistic equation and the Gompertz model.



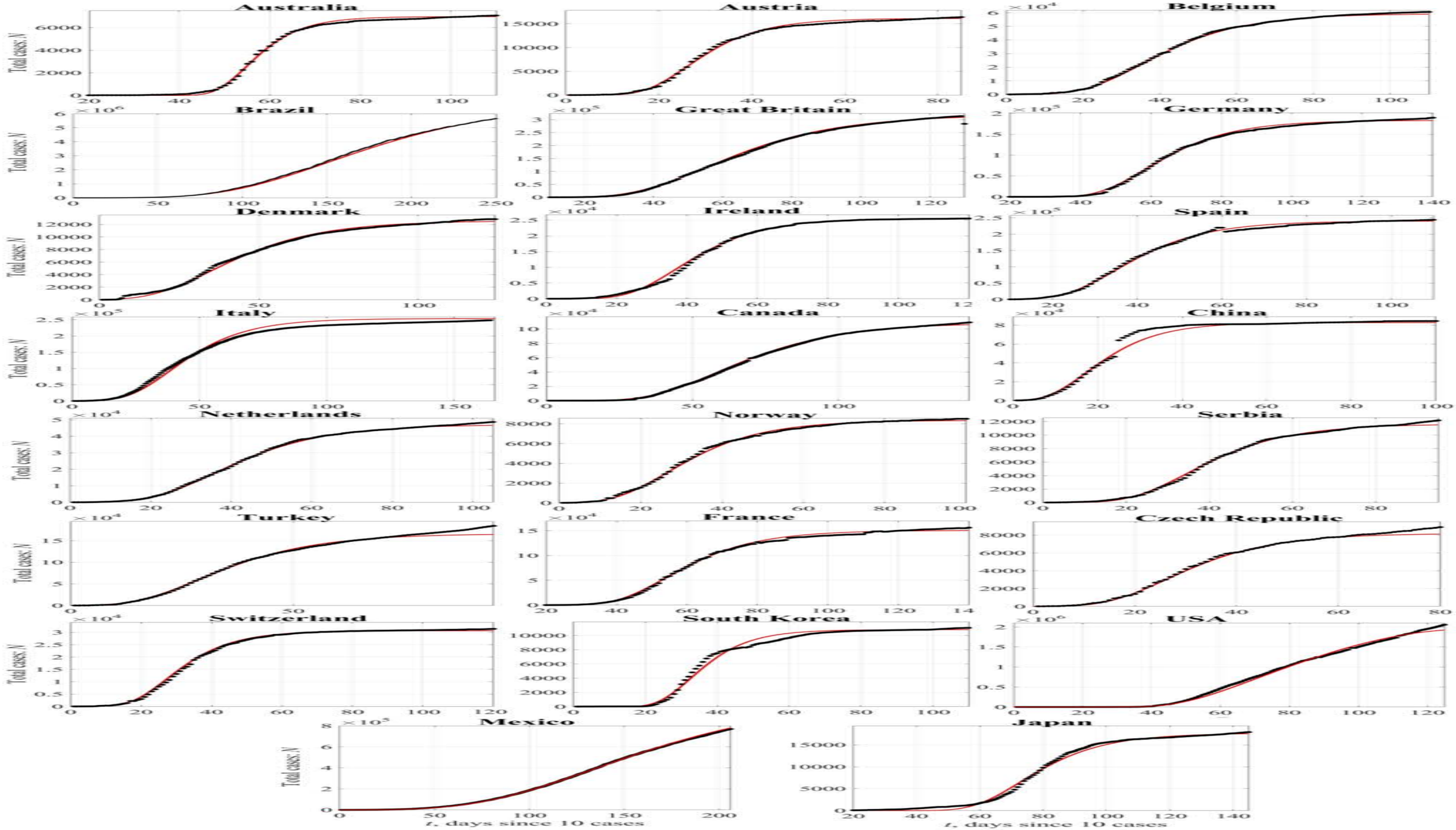
Australia

Austria



Black points - data

Red line – Gompertz model


Australia
Austria
Belgium
Brazil
Great Britain
Germany
Denmark
Ireland
Spain
Italy
Canada
China
Netherlands
Norway
Serbia
Turkey
France
Czech Republic
Switzerland
South Korea
USA
Mexico
Japan
Total cases, N
t, days since 10 cases

Title: Gompertz model in COVID-19 spreading simulation

Chaos, Solitons and Fractals: the interdisciplinary journal of Nonlinear Science, and Nonequilibrium and Complex Phenomena 1st December 2021

Dear Professor Pelinovsky,

many thanks for having submitted your work to our attention.

Together with the help of our two Managing Editors, we deeply studied your Manuscript. In particular, I read carefully your paper, and found it of potential interest for the wide readership of our Journal. I also sought advice from one colleague Referee, who confirmed my judgment that the results you are offering are scientifically sound, and that your Manuscript does not contain scientific flaws. Moreover, I found that your presentation is clear, and the paper is well written. Your results and conclusions are therefore accessible and of potential interest also to non-specialized readers. For all what said above, my conclusions are that your results deserve to be disseminated in the community, and am therefore glad to announce you that your Manuscript Gompertz model in COVID-19 spreading simulation has been accepted for publication in Chaos, Solitons and Fractals: the interdisciplinary journal of Nonlinear Science, and Nonequilibrium and Complex Phenomena.

Thank you for submitting your work to Chaos, Solitons & Fractals.

With kind regards,

Stefano Boccaletti, Ph. D

Editor-in-Chief

Chaos, Solitons and Fractals 154 (2022) 111699

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Gompertz model in COVID-19 spreading simulation

E. Pelinovsky[a,b], M. Kokoulina[c], A. Epifanova[c], A. Kurkin[c,*], O. Kurkina[c], M. Tang[d], E. Macau[e], M. Kirillin[b]

[a] National Research University – Higher School of Economics, Myasnitskaya st., 20, Moscow 101000, Russian Federation
[b] Institute of Applied Physics RAS, Ul'yanov st., 46, Nizhny Novgorod 603950, Russian Federation
[c] Nizhny Novgorod State Technical University n.a. R.E. Alekseev, Minin st., 24, Nizhny Novgorod 603950, Russian Federation
[d] East China Normal University, Shanghai 200241, China
[e] Universidade Federal de São Paulo, São Paulo 04021-001, Brazil