On ST6 Model Assessment and Possible Alternatives

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Overview

Introduction

- Hasselmann Equation $\frac{\partial \varepsilon}{\partial t} + \frac{\partial \omega_k}{\partial \vec{k}} \frac{\partial \varepsilon}{\partial \vec{r}} = S_{nl} + S_{in} + S_{diss}$
- $\varepsilon = \varepsilon(\vec{r}, \vec{k}, t)$
- S_{nl} - nonlinear 4-waves interaction term
- S_{in} - wind input
- S_{diss} - wave-breaking dissipation
- Basis of operational models WaveWatch, WAM
- Several dozens of source terms for last 50 years
- Focus on the latest set of S_{in}^{ST6} and S_{diss}^{ST6}

ST6 model

ST6 model, started by M.Donelan, A.Babanin, M.Banner, Y.Young (1997-2000, Lake George, Australia), and improved for 20+ years to include the effects of:

- wave sheltering
- spectral saturation
- flow separation
- negative wind input
- inherent wave breaking
- induced wave breaking of short waves due to the modulation of longer waves

Energy spectrum $F(k, \theta)$.

$$E = \oint d\theta \int F(k, \theta)\, dk$$

$$F(k, \theta) = \frac{g}{2\omega} \varepsilon(\omega, \theta)$$
$$F(k, \theta) = \omega k N(\mathbf{k})$$

Isotropic spectrum $F(k) = \oint F(k, \theta)\, d\theta$.
Maximum over angle spectrum value for taken k is $F_{\max}(k)$.
Factor of narrowness of the spectrum: $A(k) = \frac{F_{\max}(k)}{F(k)}$

$$W = \frac{U_s}{c} \cos\theta - 1$$

where U_s — wind speed, $c = \frac{\omega}{k} = \frac{g}{\omega}$ – phase wind speed.

$$
\begin{aligned}
B(k) &= k^3 F(k) \\
B_n(k) &= A(k)B(k) \\
G(k,\theta) &= 2.8 - (1 + tanh(10\sqrt{B_n(k)}W^2 - 11))
\end{aligned}
$$

Wind forcing:

$$S_{\mathrm{in}} = \frac{\rho_a}{\rho_w}\omega\gamma(k,\theta)F(k,\theta)$$

where

$$\gamma(k,\theta) = \sigma_{\mathrm{in}} G(k,\theta)\sqrt{B(k)}\, W^2(k,\theta)$$

Here is the very important multiplier σ_{in}:

$$\sigma_{\mathrm{in}} = 1, W > 0$$
$$\sigma_{\mathrm{in}} = -0.05, W < 0$$

so the wind forcing S_{in} can change the sign.
To define the dissipation, introduce the function:

$$F_T(k) = \frac{\beta_T}{k^3} \quad \beta_T = 0.035^2$$

Dissipation contains 2 terms:

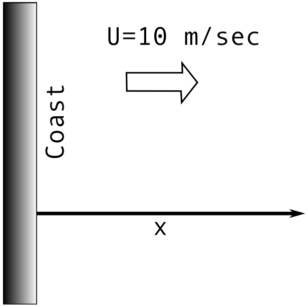
$$S_{\text{diss}} = T_1(k,\theta) + T_2(k,\theta)$$
$$T_1(k,\theta) = -\frac{\alpha_1}{2\pi}\omega\left(\frac{\Delta(k)}{F_T(k)}\right)^4 F(k,\theta)$$
$$T_2(k,\theta) = -\frac{\alpha_2}{2\pi}\int_0^k \left(\frac{\Delta(k)}{F_T(k)}\right)^4 \frac{d\omega}{dk}\,dk$$

where:

- $\Delta(k) = F(k) - F_T(k)$
- $d\omega/dk = g/2\omega$ – the group velocity
- $\alpha_1 = 4.75 \cdot 10^{-6},\ \ \alpha_2 = 7.00 \cdot 10^{-5}$ - empirical constants

Problem statement

- Stationary limited fetch case $\frac{1}{2}\frac{\omega_k}{k}\cos\theta\frac{\partial\varepsilon}{\partial x} = S_{nl} + S_{in} + S_{diss}$
- Waves running only in the wind direction
- Deep water case $\omega = (gk)^{1/2}$
- Exact S_{nl}
- 10° angular resolution, 71 frequencies
- Wind 10 m/sec at 10 m height, blowing orthogonally away from the shore



ZRP model

Limited fetch self-similar solution (Zakharov, Resio, Pushkarev, NPG 2012, hearafter ZRP):

$$\varepsilon = \chi_\star^{p+q} F(\omega \chi_\star^q)$$
$$E \sim x^p$$
$$< \omega > \sim x^{-q}$$

ZRP model:

$$S_{in} = A\omega^{s+1}$$
$$10q - 2p = 1, \quad q = \frac{1}{2+s}$$

In conjunction with experimental regression line (Resio, Long. JPO 2008) ZRP approach yields:

$$p = 1, \;\; q = -0.3, \;\; s = 4/3$$

Normalization and target dependence

Dimensionless wave energy (Liu et al., JPO 2019):

$$E_\star = Eg^2/U_\star^4$$
$$\chi_\star = gX/U_\star^2$$

where $U_\star = U_{10} \cdot \Gamma$, and $\Gamma = 32$
Liu et al., JPO 2019 used target expermental dependence (Kahma, Calcoen, JPO 1992):

$$\varepsilon_* = \frac{H_s^2 g^2}{16 u_*^4} = 2.1 \cdot 10^{-3} \chi_*^{0.79}$$
$$\nu_* = \frac{f_p u_*}{g} = 2.3 \chi_*^{-0.25} / 2\pi$$

which is known not as typical (Badulin, Babanin, Zakharov, Resio, JPO 2007)

$$p = 1, \;\; q = -0.3$$

mST6 model

We developed new mST6 model through ZRP-like approach, which has 2 tunable parameters in the wind input term, using self-similar relations:

$$S_{in} = A\omega^{s+1}$$
$$10q - 2p = 1$$
$$q = \frac{1}{2+s}$$

for the indices p and q from Liu et al., JPO 2019

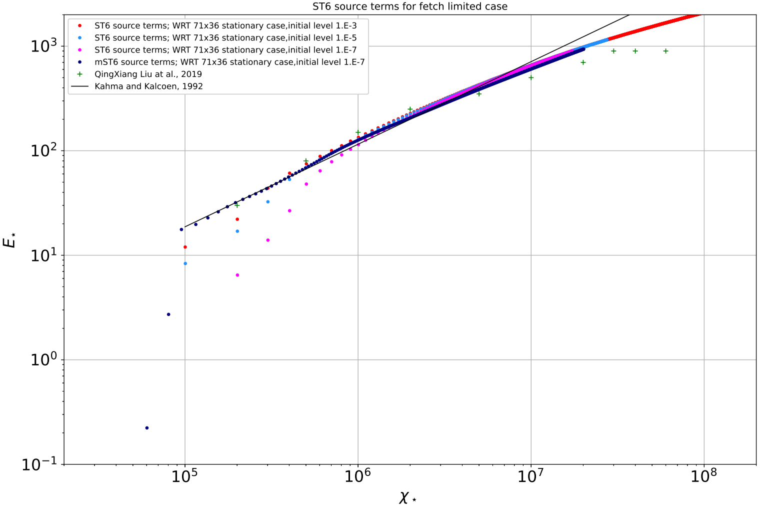


Figure: Dimensionless energy as the function of dimensionless fetch in ST6 case.

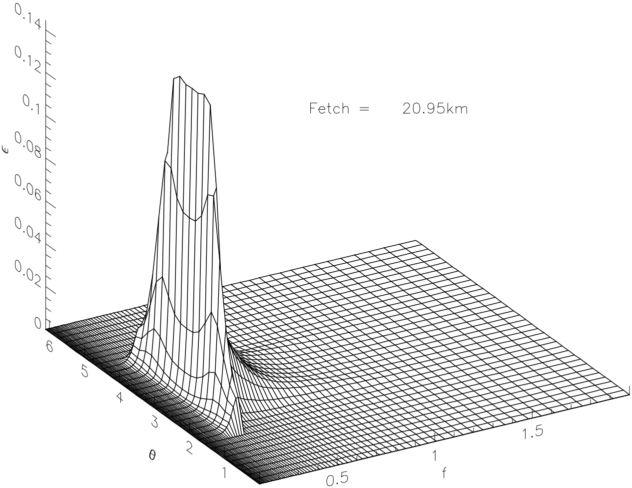


Figure: Wave energy spectrum as the function of frequency and angle in ST6 case

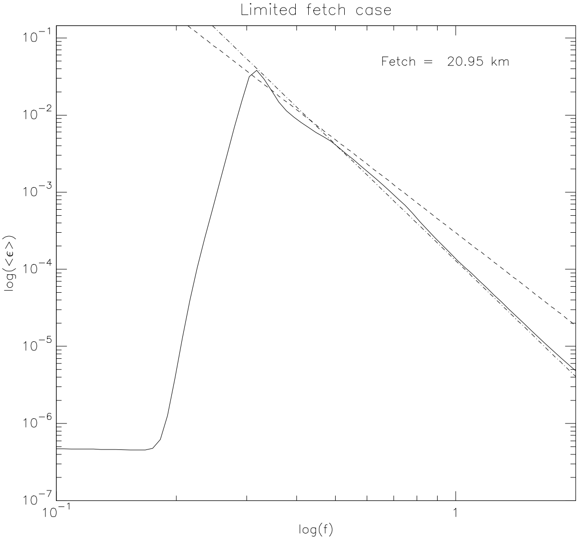


Figure: ST6 case. Decimal logarithm of the angle averaged wave energy spectral density $\frac{1}{2\pi}\int_0^{2\pi}\varepsilon(\omega,\theta)d\theta$ as the function of the decimal logarithm of frequency f for ST6 case. Dashed line – the KZ spectral fit $\sim \omega^{-4}$; dash-dotted line – the Phillips spectral fit $\sim \omega^{-5}$.

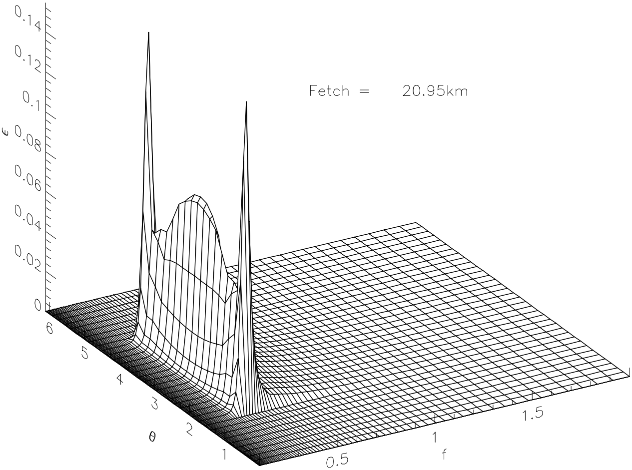
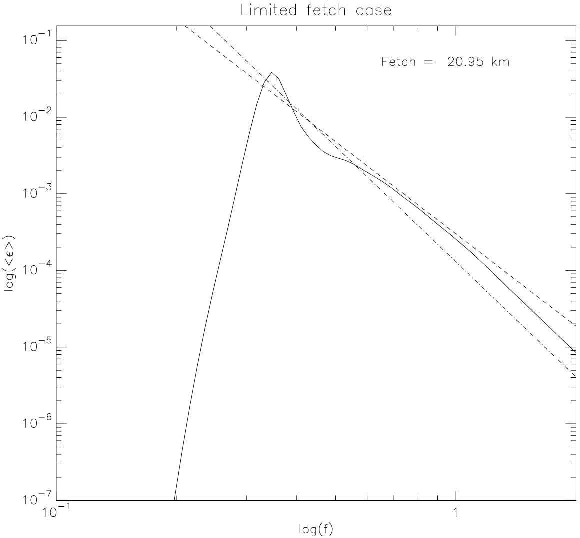


Figure: Wave energy spectrum as the function of frequency and angle for mST6 case.



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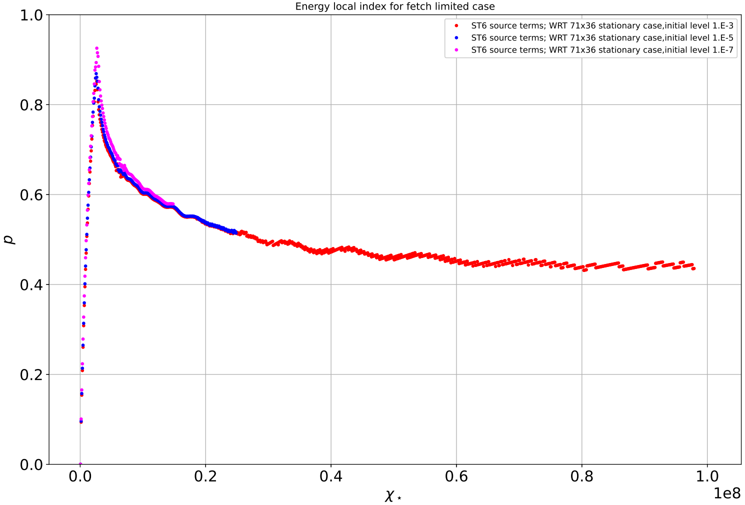

Figure: ST6 case. Energy local index p as the function of dimensionless fetch.

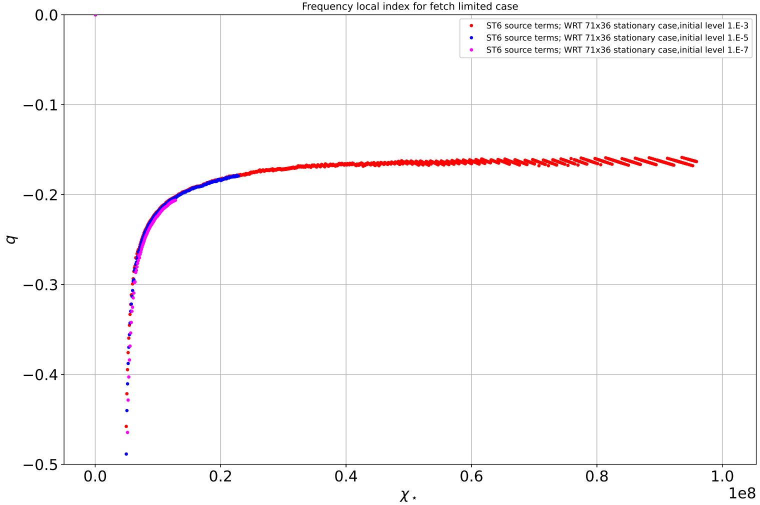

Figure: ST6 case. Frequency local index q as the function of dimensionless fetch.

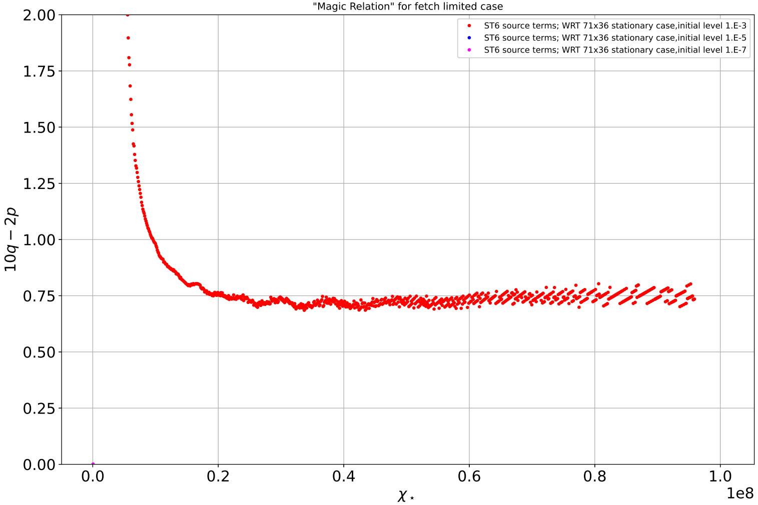

Figure: ST6 case. Magic relations $10q - 2p$ as the function of dimensionless fetch.

Conclusions

1. ST6 model provides reasonable correspondence for the experimental, numerical and theoretical data in the range of fetches between 10 and 80 km
2. ST6 model strongly depends on the turbulence level at the shore line, and fails to reproduce theoretical and experimental total wave energy growth for suffuciently low-level wave energy boundary conditions for the fetches shorter than 10 km
3. While ST6 model exhibits partial asymptotic quasi self-similar behavior, its indices never have never been observed in the experiments
4. Alternative model mST6 exhibut self-similarity in the full range of the experimental target data from 1km
5. mST6 is the demo model should not be construed as an advice for use applications, due to not good KC1992 experimental target. The correct approach is realized in ZRP2012