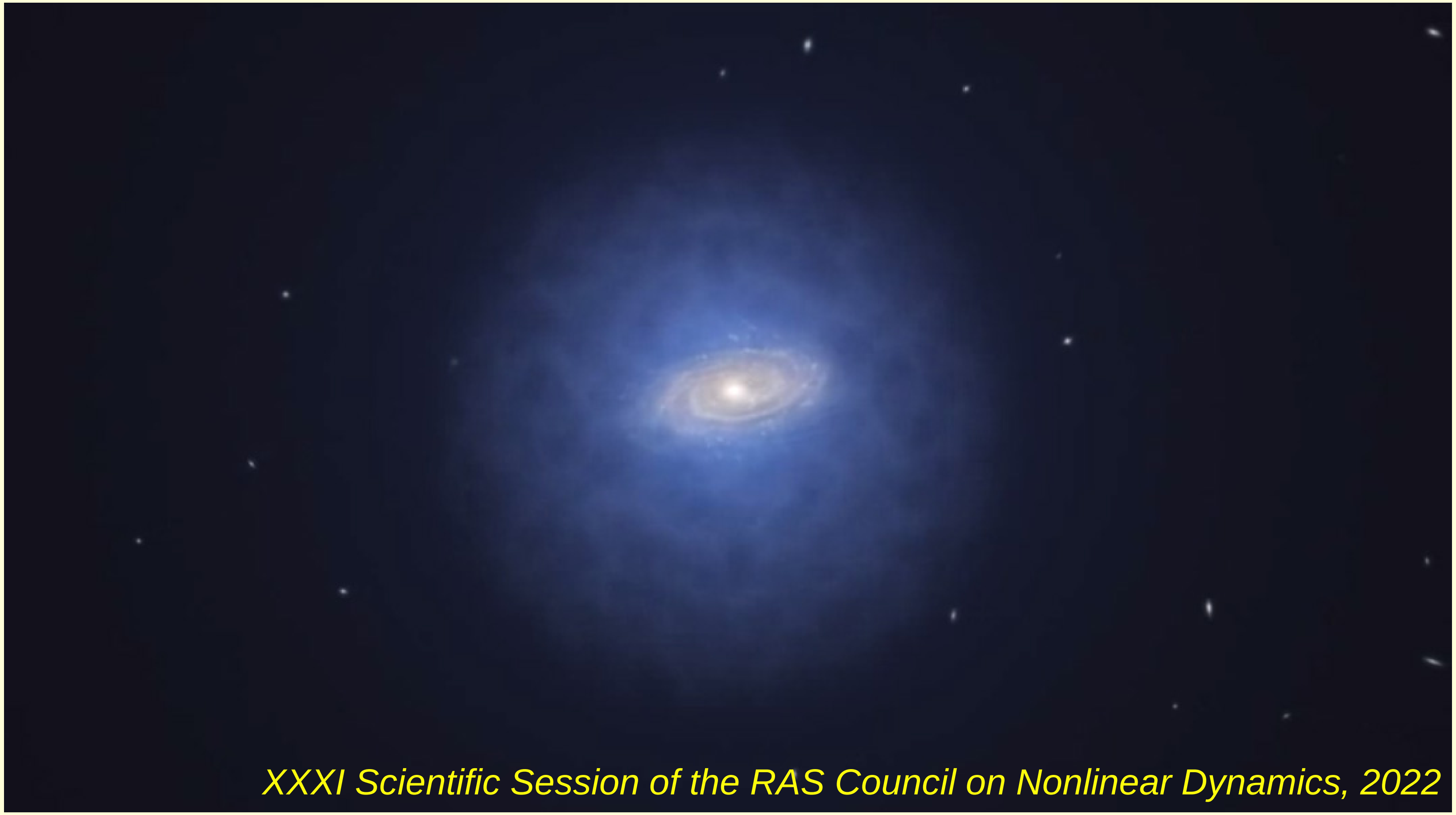
Resonant phenomena in the motion of test particles in spherically symmetric lumps of oscillating dark matter

Eugene Maslov and Vladimir Koutvitsky

IZMIRAN, Russia



1 Oscillating dark matter

1.1 ΛCDM

Problems at galactic and subgalactic scales: the cusp profile of central densities in galactic halos, the overpopulation of substructures predicted by N-body simulations

- J. R. Primack (2009)

1.2 SFDM

Fundamental nonlinear scalar field describing coherent state of ultra light particles, e.g., axions, with mass $m \sim 10^{-21} \div 10^{-23}$ eV and $\omega \sim m$ ($T \sim 0.1 \div 10$ years).

- M. S. Turner (1983)
- E. Seidel and W.-M. Suen (1991, 1994)
- P. J. E. Peebles (1999, 2000)
- D. J. E. Marsh (2016)

Primordial fluctuations

$\Downarrow$

Inflation

$\Downarrow$

Homogeneous oscillating background

$\Downarrow$

Parametric instability

$\Downarrow$

Oscillons (pulsons)

$\Downarrow$

Selfgravity

$\Downarrow$

Oscillatons (gravipulsons)

1.3 Possible effects of the dark matter oscillations

● Periodic variations in pulsar timing array (A. Khmelnitsky and V.Rubakov (2014))

●Detecting axion dark matter wind with laser interferometers (A.Aoki and J. Soda (2017))

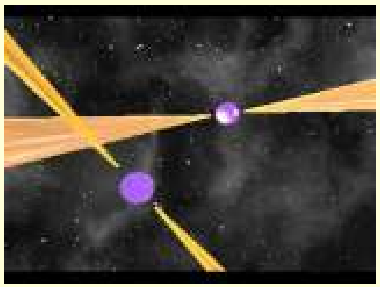
● Secular variation of the orbital period in binary pulsar systems (D. Blas, D. L. Nacir, and S. Sibiryakov (2017)

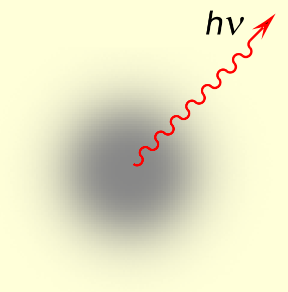
●Resonance effects in circular motion of stars at galactic center (M. Bosković et al (2018))

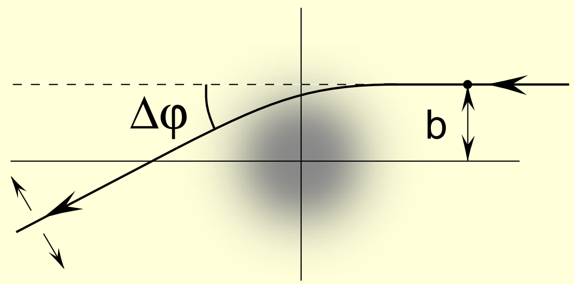
● Periodic variations of spectroscopic emission lines from the stars at the halo center caused by the gravitational frequency shift (M.Bosković et al (2018), V. Koutvitsky and E. Maslov (2019))

● Periodic variations of intensity of images when lensing the distant sources (V. Koutvitsky and E. Maslov (2020))

● Periodic variations of deflection angle of test particles (V.Koutvitsky and E. Maslov (2021))







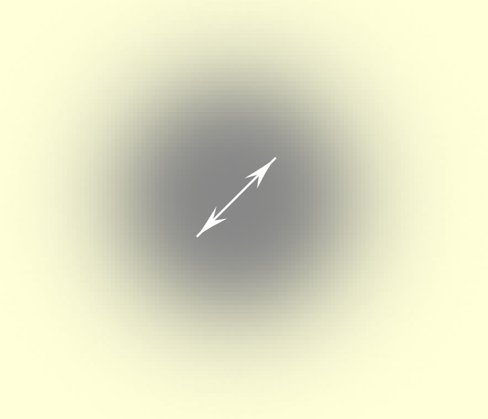
2 Finite motions of test particles in time-dependent spherically symmetric gravitational fields

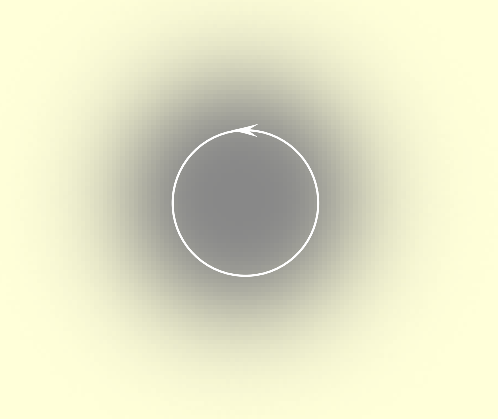
According to the basic concepts of General Relativity, massive particles in a spherically symmetric spacetime move along the geodesics $x^\mu = x^\mu(s)$ satisfying the equation

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0,$$

where ds is a proper time interval,

$$ds^2 = B\,dt^2 - A\,dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2).$$





Consider a spherically symmetric **nonstatic** metric of the form

$$ds^2 = B(t,r)\,dt^2 - A(t,r)\,dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2).$$

For the trajectories lying in the plane $\vartheta = \pi/2$, the geodesic equation reduces to

$$\frac{d}{ds}\ln\left(B\frac{dt}{ds}\right) = \frac{\dot{B}}{2B}\frac{dt}{ds} - \frac{\dot{A}}{2B}\left(\frac{dr}{ds}\right)^2\left(\frac{dt}{ds}\right)^{-1},$$

$$\frac{d^2r}{ds^2} + \frac{B'}{2A}\left(\frac{dt}{ds}\right)^2 + \frac{\dot{A}}{A}\frac{dt}{ds}\frac{dr}{ds} + \frac{A'}{2A}\left(\frac{dr}{ds}\right)^2 - \frac{r}{A}\left(\frac{d\varphi}{ds}\right)^2 = 0,$$

$$\frac{d^2\varphi}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\varphi}{ds} = 0,$$

where $(\cdot) = \partial/\partial t$, $(') = \partial/\partial r$. For a particle performing a finite motion with angular momentum J we obtain

$$\frac{d\varphi}{ds} = \frac{J}{r^2}, \qquad B\left(\frac{dt}{ds}\right)^2 - A\left(\frac{dr}{ds}\right)^2 = 1 + \frac{J^2}{r^2}.$$

Denote

$$Y(t) = B(t, r(t))\frac{dt}{ds}.$$

Then

$$\left(\frac{dr}{dt}\right)^2 = \frac{B}{A}\left[1 - \frac{B}{Y^2}\left(1 + \frac{J^2}{r^2}\right)\right]$$

The function $Y^2(t)$ satisfies the equation

$$\frac{dY^2}{dt} = f(t, r(t))Y^2 + g(t, r(t)),$$

where

$$f(t, r) = \frac{\partial}{\partial t}\ln\frac{B}{A}, \qquad g(t, r) = B\left(1 + \frac{J^2}{r^2}\right)\frac{\partial}{\partial t}\ln A.$$

The solution is

$$Y^2 = e^{\int f\,dt}\left(1 + \int g e^{-\int f\,dt} dt\right)$$

Using these results we finally obtain

$$\frac{d^2r}{dt^2} + \frac{B'}{2A} - \frac{\gamma(r)B}{rA} + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{2B}\right)\frac{dr}{dt}$$

$$+ \left(\frac{\gamma(r)}{r} + \frac{A'}{2A} - \frac{B'}{B}\right)\left(\frac{dr}{dt}\right)^2 - \frac{\dot{A}}{2B}\left(\frac{dr}{dt}\right)^3 = 0,$$

where $A = A(t,r)$, $B = B(t,r)$, $(\cdot) = \partial/\partial t$, $(') = \partial/\partial r$, and

$$\gamma(r) = \frac{J^2/r^2}{1 + J^2/r^2}, \quad 0 \leqslant \gamma(r) < 1.$$

2.1 Weak field approximation

In the weak field approximation

$$A = 1 - 2\psi + O(\varkappa^2),$$

$$B = 1 + 2\chi + O(\varkappa^2),$$

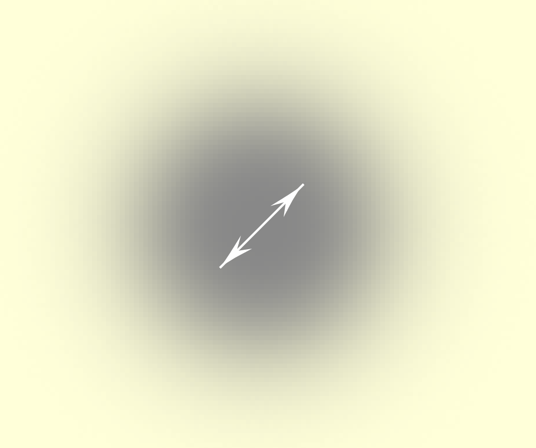
where $\psi(t,r)$ and $\chi(t,r)$ are time-periodic functions of order $\varkappa \ll 1$, and $\varkappa \sim G$. In this case, $f(t,r) \sim O(\varkappa)$, $g(t,r) \sim O(\varkappa)$, so that for finite motions $Y(t) \approx 1 + O(\varkappa)$, and, hence,

$$\frac{dr}{dt} \sim O(\varkappa^{1/2}).$$

Thus, assuming $\gamma \ll 1$ and neglecting the terms $O(\varkappa^2)$, $O(\varkappa^{5/2})$, we obtain

$$\frac{d^2r}{dt^2} - \left[2\dot{\psi}(t,r) + \dot{\chi}(t,r)\right]\frac{dr}{dt} + \chi'(t,r) - \gamma(r)/r = 0.$$

2.1.1 Radial motion



In this case the angular momentum $J = 0$, so that

$$\gamma = \frac{J^2/r^2}{1 + J^2/r^2} = 0.$$

Considering small oscillations of a particle arround the center, we expand $\psi(t, r)$ and $\chi(t, r)$ at $r = 0$. Neglecting the terms $O(\varkappa^{3/2} r^2)$ and $O(\varkappa r^3)$, we obtain

$$\frac{d^2 r}{dt^2} - \left[2\dot{\psi}(t,0) + \dot{\chi}(t,0)\right] \frac{dr}{dt} + \chi''(t,0) r = 0.$$

Finally, substitution

$$r(t) = u(t) \exp\left(\psi(t,0) + \frac{1}{2}\chi(t,0)\right)$$

results in the Hill equation

$$\frac{d^2 u}{dt^2} + \left(\ddot{\psi}(t,0) + \frac{1}{2}\ddot{\chi}(t,0) + \chi''(t,0)\right) u = 0.$$

2.1.2 Circular motion

In this case we set

$$r(t) = r_0(1 + \eta(t)),$$

where $|\eta(t)| \ll 1$, and expand $\psi(t, r)$, $\chi(t, r) = \bar{\chi}(r) + \tilde{\chi}(t, r)$, and $\gamma(r)/r$ at $r = r_0$. For a circular background trajectory, the constant terms in the equation must be cancelled. This gives the equation

$$r_0 \bar{\chi}'(r_0) = \gamma(r_0)$$

relating r_0 and angular momentum J. Using this relation and neglecting the terms $O(\varkappa^{3/2}\eta,\ \varkappa\eta^2)$, we obtain

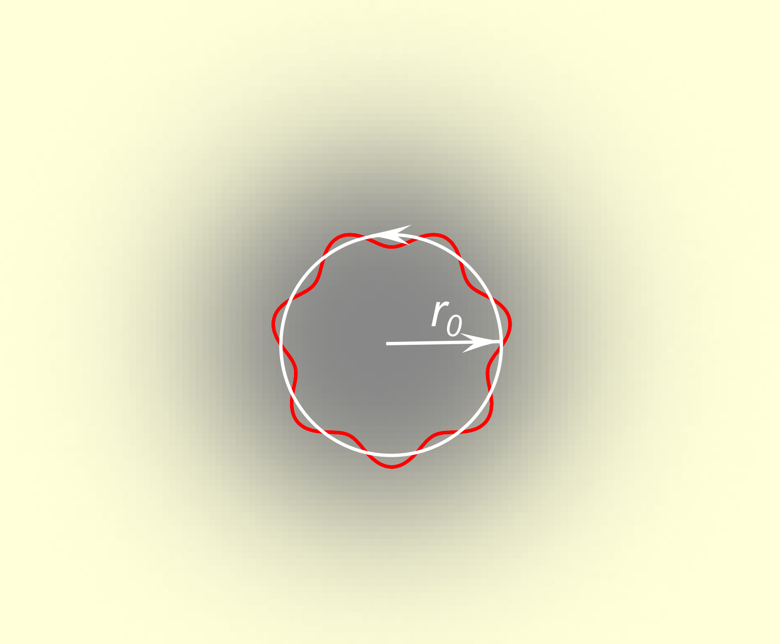
$$\frac{d^2\eta}{dt^2} - \left[2\dot{\psi}(t, r_0) + \dot{\chi}(t, r_0)\right] \frac{d\eta}{dt} + \left(\frac{3\gamma(r_0)}{r_0^2} + \chi''(t, r_0)\right) \eta = -\frac{1}{r_0} \tilde{\chi}'(t, r_0).$$

Finally, substitution

$$\eta(t) = u(t) \exp\left(\psi(t, r_0) + \frac{1}{2}\chi(t, r_0)\right)$$

results in the inhomogeneous Hill equation

$$\frac{d^2u}{dt^2} + \left(\frac{3\gamma(r_0)}{r_0^2} + \ddot{\psi}(t, r_0) + \frac{1}{2}\ddot{\chi}(t, r_0) + \chi''(t, r_0)\right) u = -\frac{1}{r_0}\tilde{\chi}'(t, r_0).$$

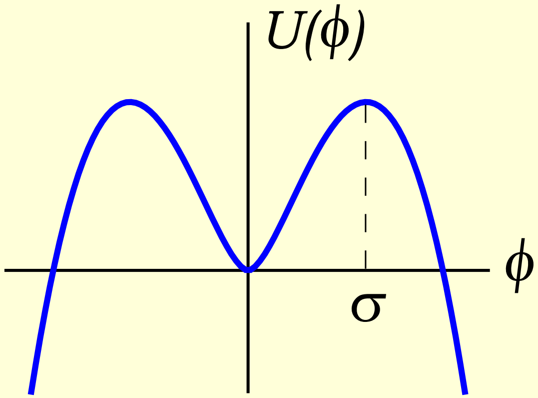


Circular motion of a test particle with small oscillations about r_0.

3 Finite motions of a test particle in the time-periodic spherically symmetric scalar field

As a gravitating mass, we consider a pulsating dark matter halo made from the self-gravitating real scalar field with the potential

$$U(\phi) = \frac{1}{2}m^2\phi^2\left(1 - \ln\frac{\phi^2}{\sigma^2}\right)$$



● quantum field theory [G. Rosen (1969), Bialynicki-Birula & Mycielski (1975)]

● inflationary cosmology [Linde (1982, 1992), Albrecht & Steinhardt (1982), Barrow & Parsons (1995)]

● supersymmetric extensions of the Standard Model (flat direction potentials in the gravity mediated supersymmetric breaking scenario) [Enqvist & McDonald (1998)]

Einstein-Klein-Gordon system

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \left[\phi_{,\mu}\phi_{,\nu} - \left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} - U(\phi) \right) g_{\mu\nu} \right],$$

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x_\mu} \left(\sqrt{-g} \frac{\partial \phi}{\partial x^\mu} \right) + \frac{dU(\phi)}{d\phi} = 0.$$

The case of spherical symmetry: $ds^2 = B dt^2 - A dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2)$:

$$\begin{aligned} \frac{A_r}{A} + \frac{A-1}{r} &= 4\pi G r A \left[\frac{1}{B}\phi_t^2 + \frac{1}{A}\phi_r^2 + m^2\phi^2 \left(1 - \ln \frac{\phi^2}{\sigma^2} \right) \right], \\ \frac{B_r}{B} - \frac{A-1}{r} &= 4\pi G r A \left[\frac{1}{B}\phi_t^2 + \frac{1}{A}\phi_r^2 - m^2\phi^2 \left(1 - \ln \frac{\phi^2}{\sigma^2} \right) \right], \end{aligned}$$

$$\frac{1}{B}\phi_{tt} - \frac{1}{A} \left(\phi_{rr} + \frac{2}{r}\phi_r \right) + \frac{1}{2B} \left(\frac{A_t}{A} - \frac{B_t}{B} \right) \phi_t + \frac{1}{2A} \left(\frac{A_r}{A} - \frac{B_r}{B} \right) \phi_r = m^2 \phi \ln \frac{\phi^2}{\sigma^2}.$$

Boundary conditions

$$\phi(t,\infty) = 0,\ A(t,\infty) = 1,\ B(t,\infty) = 1,\ \phi_r(t,0) = 0,\ A(t,0) = 1.$$

3.1 Weak field approximation

In the weak field approximation this system has a pulsating solution of the form

$$\phi(t,r) = \sigma[a(\tau) + \varkappa Q(\tau,\rho) + O(\varkappa^2)]e^{(3-\rho^2)/2},$$
$$A(t,r) = 1 - 2\psi(t,r) + O(\varkappa^2), \quad B(t,r) = 1 + 2\chi(t,r) + O(\varkappa^2),$$

where

$$\psi(t,r) = \frac{\varkappa}{2}\left[V_{\max}\left(1 - \frac{\sqrt{\pi}\,\mathrm{erf}\rho}{2\rho}e^{\rho^2}\right) + a^2\rho^2\right]e^{3-\rho^2},$$
$$\chi(t,r) = -\frac{\varkappa}{2}\left[V_{\max}\left(1 + \frac{\sqrt{\pi}\,\mathrm{erf}\rho}{2\rho}e^{\rho^2}\right) + a^2\ln a^2\right]e^{3-\rho^2},$$

$\tau = mt$, $\rho = mr$, $\varkappa = 4\pi G\sigma^2 \ll 1$ (G is the gravitational constant). The function $a(\tau)$ oscillates in the range $-a_{\max} \leqslant a(\theta) \leqslant a_{\max}$ in the local minimum of the potential $V(a)$,

$$a_{\tau\tau} = -dV/da, \qquad V(a) = (a^2/2)\left(1 - \ln a^2\right) \leqslant V_{\max} = V(a_{\max}),$$

with the period

$$T = 4\int_0^1 [(1 - \ln a_{\max}^2)(1 - z^2) + z^2\ln z^2]^{-1/2}\,dz.$$

3.1.1 Radial motion

$$\frac{d^2u}{dt^2} + \left(\ddot{\psi}(t,0) + \frac{1}{2}\ddot{\chi}(t,0) + \chi''(t,0)\right) u = 0.$$

$$\Downarrow$$

$$\frac{d^2u}{d\tau^2} + \varkappa e^3 \left[\frac{3}{2}a^2 - \frac{1}{2}a^2 \ln a^2 \left(1 + 2\ln a^2\right) - V_{\max}\left(\frac{5}{3} + \ln a^2\right)\right] u = 0.$$

This is a **singular** Hill equation! For $\mu >> 1$ its solution has the form

$$u(\tau) \sim F(\tau) e^{\mu\tau},$$

where $F(\tau)$ is a $T/2$–periodic or $T/2$–antiperiodic function, and μ is the Floquet exponent.

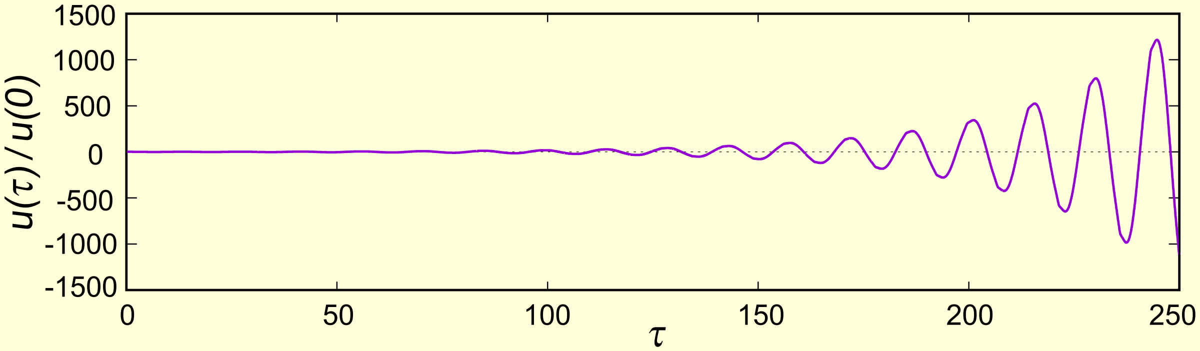


Fig.1. Resonant solution for $\varkappa$=0.02, a_{max}=0.985; $\mu \approx$0.029.

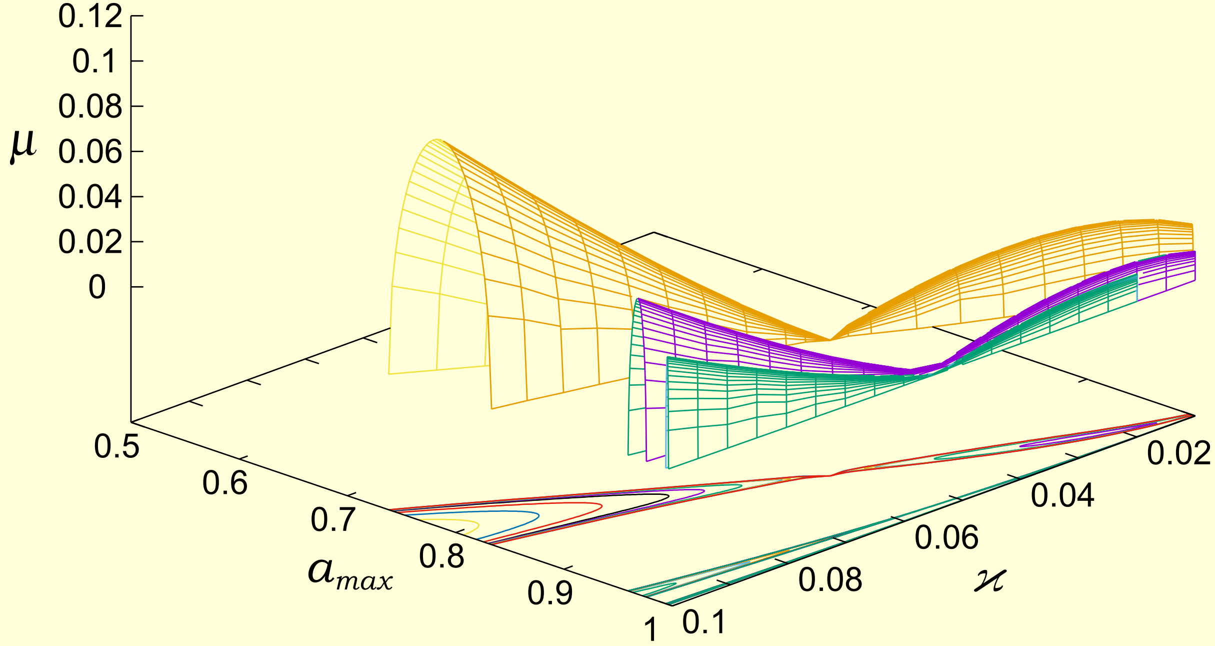


Fig.2. Zone structure and Floquet exponent for radial trajectories

3.1.2 Circular motion

$$\frac{d^2u}{dt^2} + \left(\frac{3\gamma(r_0)}{r_0^2} + \ddot{\psi}(t,r_0) + \frac{1}{2}\ddot{\chi}(t,r_0) + \chi''(t,r_0)\right) u = -\frac{1}{r_0}\tilde{\chi}'(t,r_0).$$

$$\Downarrow$$

$$\frac{d^2u}{d\tau^2} + \left\{ \frac{\gamma(\rho_0)}{\rho_0^2} + \frac{\varkappa}{2} e^{3-\rho_0^2} \left[2V_{\max}\left(1 - \ln a^2\right) + \left(3 - 2\rho_0^2\right) a^2 \right.\right.$$

$$\left.\left. - a^2 \ln a^2 \left(1 + 2\ln a^2\right) + 4\overline{a^2 \ln a^2}\right] \right\} u = -\varkappa \widetilde{a^2 \ln a^2} e^{3-\rho_0^2},$$

where ρ_0 is determined from the equation

$$\gamma(\rho_0) = \varkappa \rho_0^2 e^{3-\rho_0^2} \left\{ V_{\max}\left[1 - \frac{1}{2\rho_0^2}\left(1 - \frac{\sqrt{\pi}\,\mathrm{erf}\,\rho_0}{2\rho_0} e^{\rho_0^2}\right)\right] + \overline{a^2 \ln a^2} \right\}.$$

This is **inhomogeneous singular** Hill equation !

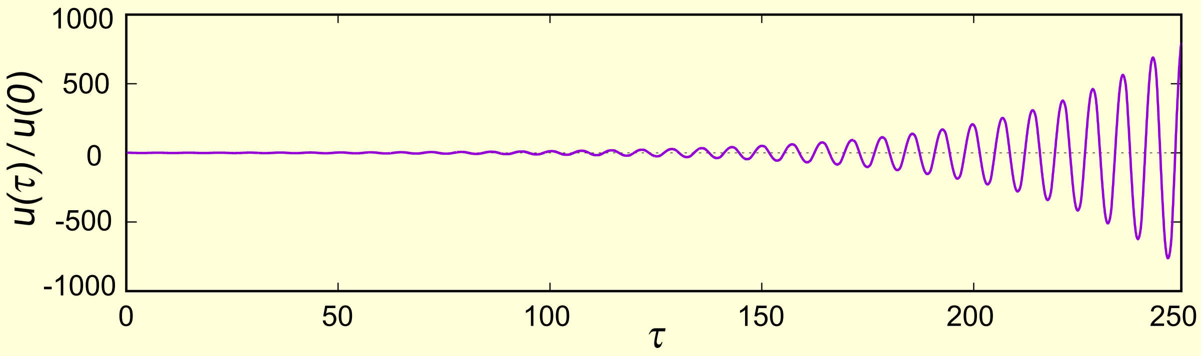


Fig.3. Resonant solution for ϰ=0.03, a_{max}=0.7847, ρ_0=0.1; μ ≈0.0283.

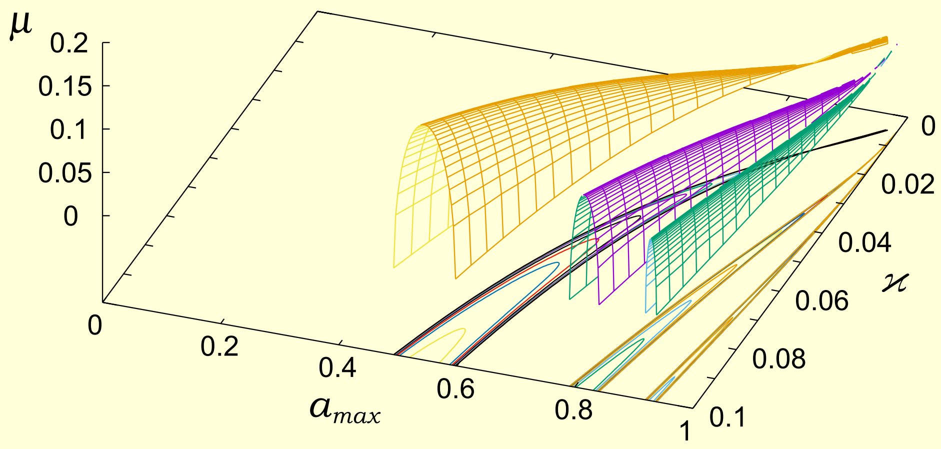


Fig.4. Floquet exponent for circular orbits, ρ_0=0.1

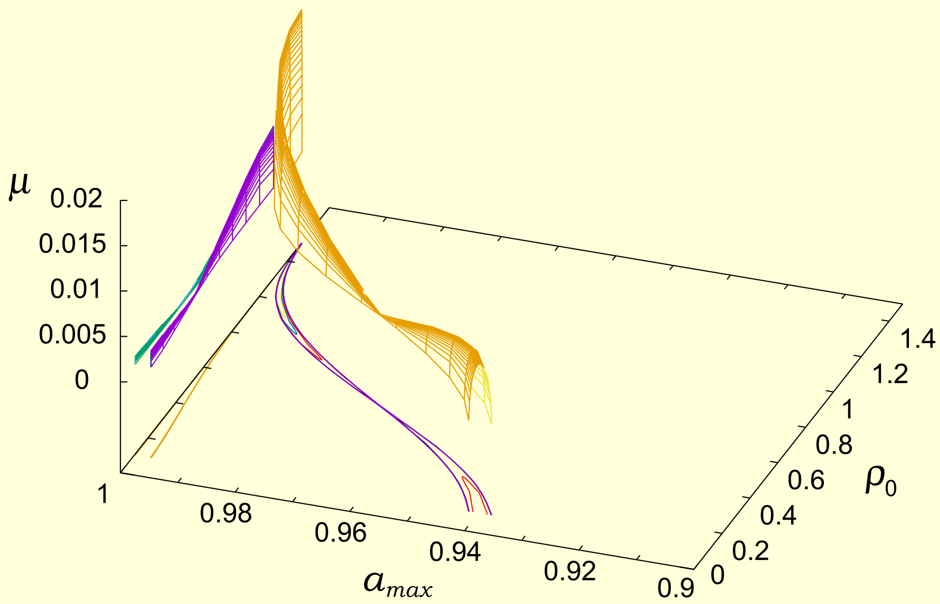


Fig.5. Floquet exponent for circular orbits, ϰ=0.01

4 Cosmological implications

The energy density of the lump oscillates in time and decays as

$$T_0^0 \sim m^2 \sigma^2 a^2(\tau) \rho^2 e^{3-\rho^2},$$

where $\tau = mt, \quad \rho = mr$.

Therefore, the characteristic size of the lump is $\sim m^{-1}$, and the oscillation period $T_g \approx (2m)^{-1} T$, where $T \sim 10$ is the oscillation period of $a(\tau)$.

For example, for $m \sim 10^{-22}$eV we have the lump of the size ~ 0.06 pc, oscillating with the period ~ 1 year.

To obtain the Sun-sized lump we need to assume $m \approx 2.7 \times 10^{-16}$eV, that gives $T_g \approx 12$ seconds.


Thank you
for your attention!