XXXI session of the scientific council of RAS on “Nonlinear dynamics”

Nonlinear generation of sound waves by electromagnetic waves in space plasma

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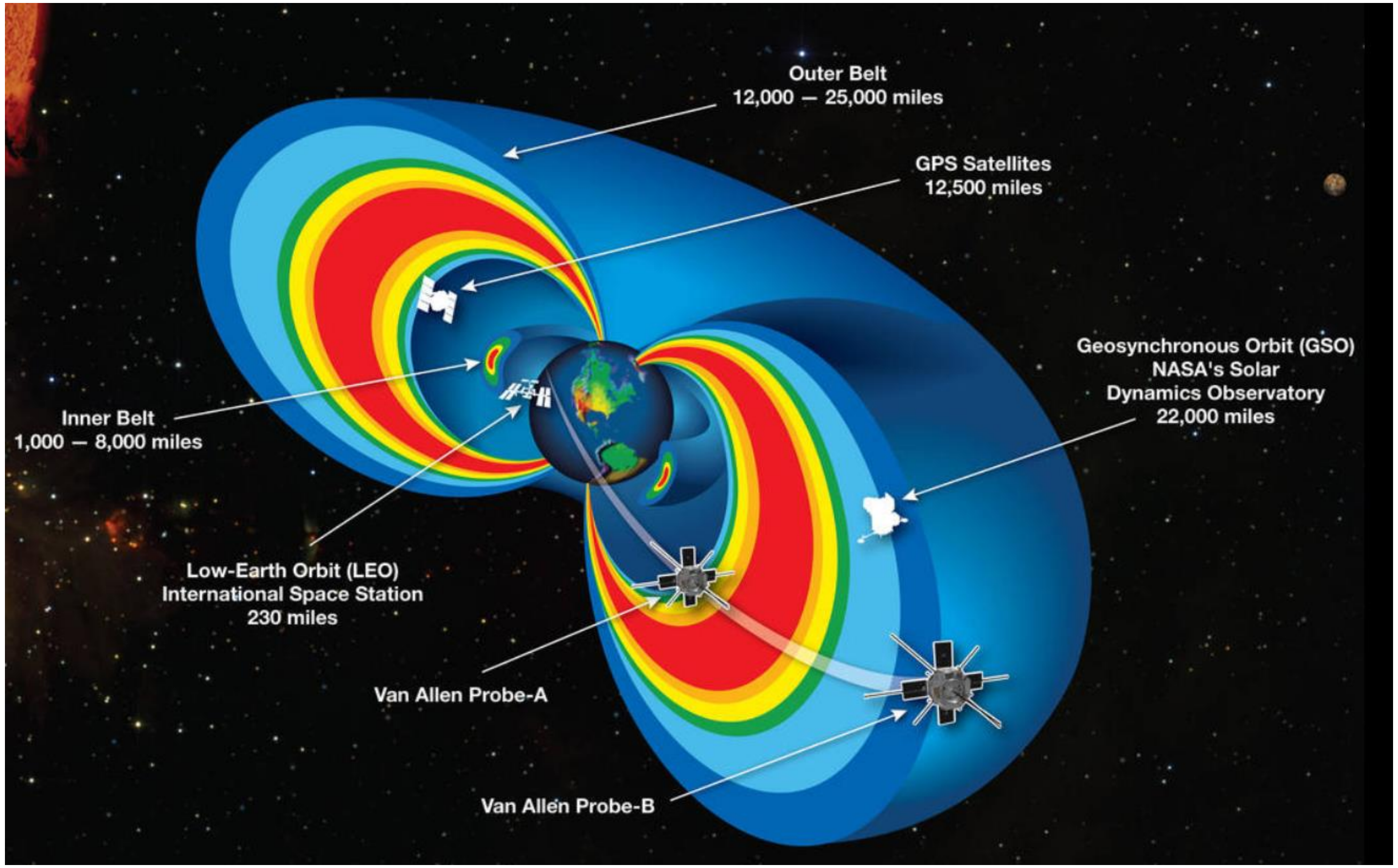
The Earth's Radiation Belts / Van Allen Probes Spacecraft /



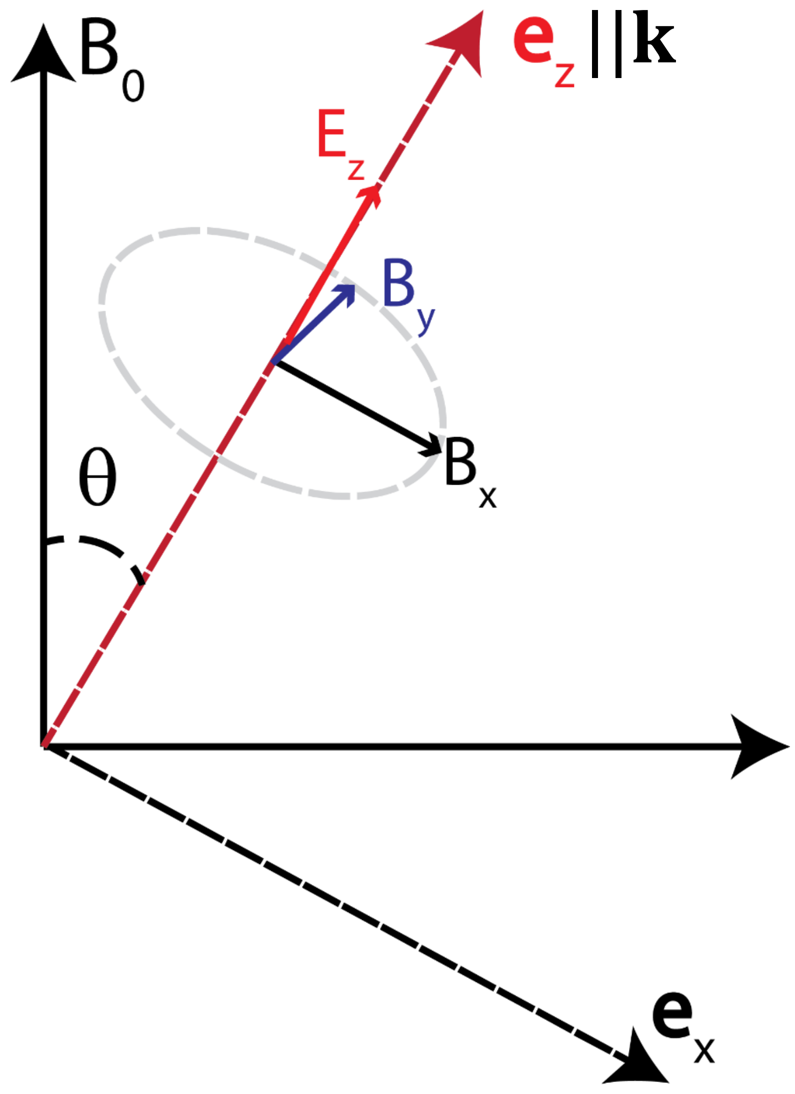
~10 keV electrons with $T_{\perp} > T_{||}$ provide free energy for generation of whistler waves in the radiation belts

[Andronov&Trakhtengerts, 1964]

sub-relativistic electron fluxes in the Earth's radiation belts are strongly affected by whistler waves

plenty of nonlinear physics involved and observed !

Whistler Waves / Electron Plasma Mode /



whistler wave magnetic and electric fields can be decomposed into electromagnetic and electrostatic

$$\mathbf{B} = \mathrm{rot}\,\mathbf{A}, \qquad \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \frac{\partial \Phi}{\partial z}\,e_z$$

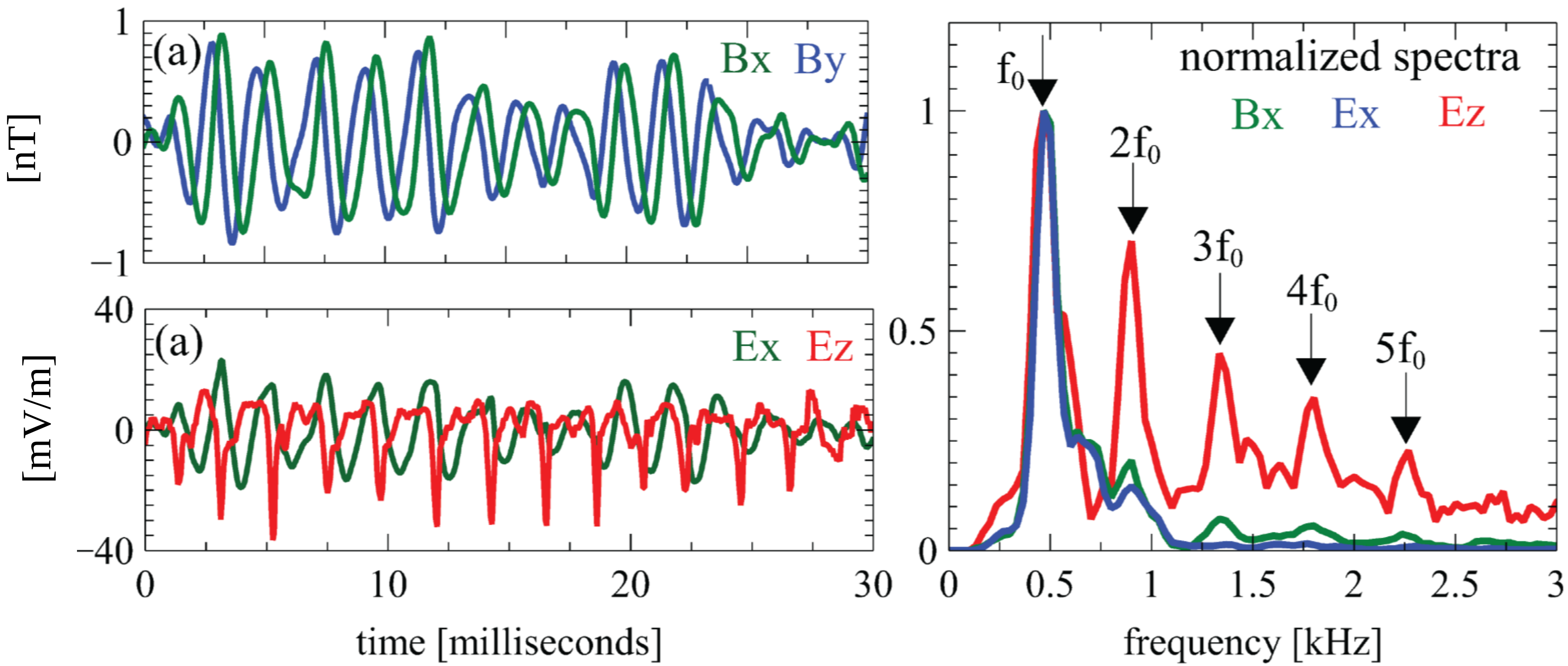
strictly parallel propagation

whistler waves are purely electromagnetic & circularly polarized

oblique propagation

whistler waves have non-zero electrostatic field parallel to propagation direction $\mathbf{k}$

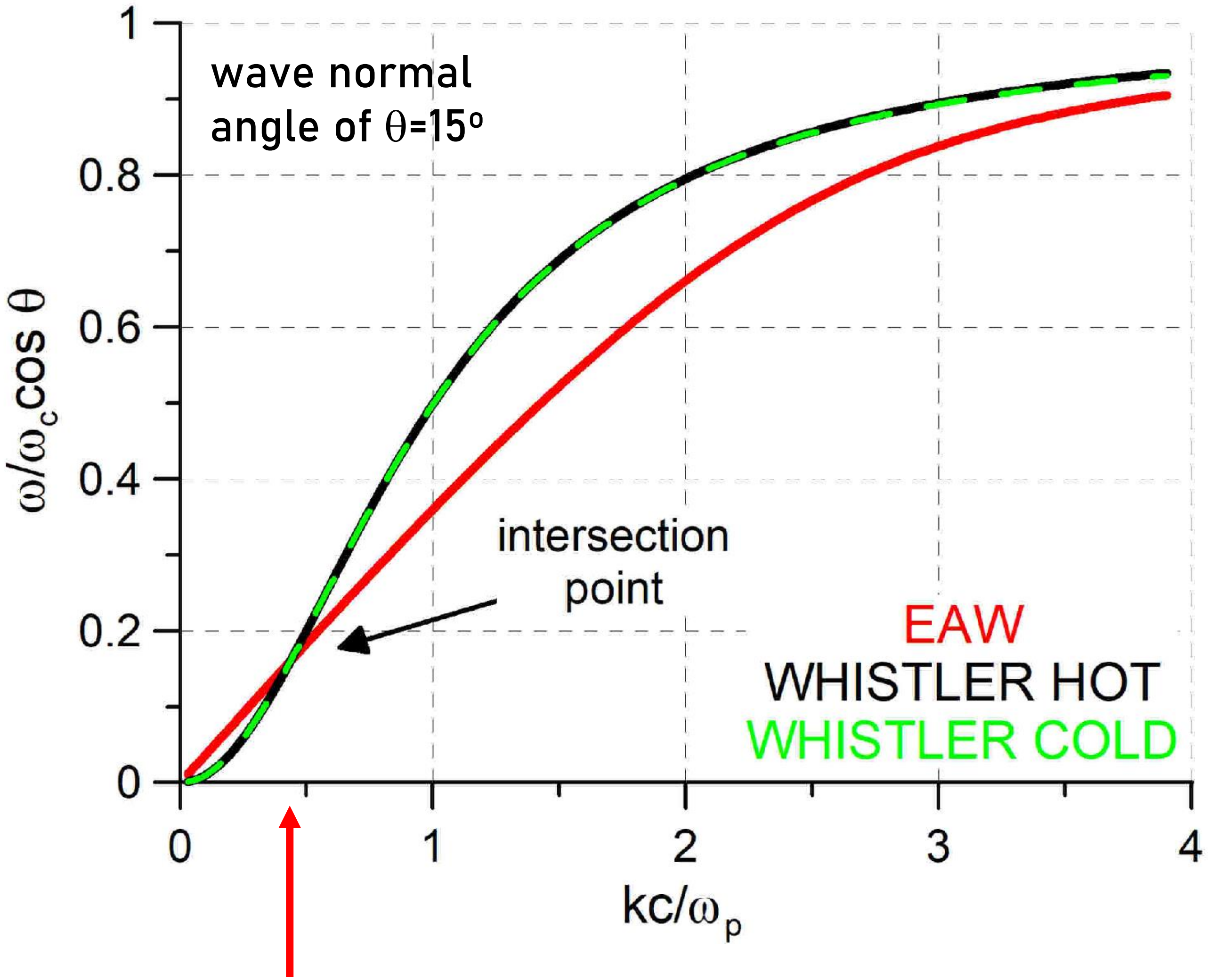
A Novel Nonlinear Phenomenon / Van Allen Probes Observations /



electromagnetic components (Bx & Ey, By & Ex) look rather ordinarily:
whistler wave propagating at ~15º to background magnetic field

electrostatic field Ez consists of nonlinear spikes phase-locked with electromagnetic components

Two-Fluid Linear Dispersion Relation



crossover wavenumber

k_c

computed using two-fluid MHD:
cold & hot electron fluids,
ions are immobile.

cold & hot electron parameters are
adopted from Van Allen Probes observations

whistler wave mode

$$\omega \approx \omega_c \cos\theta \frac{k^2c^2}{k^2c^2+\omega_p^2}, \quad \omega_p \gg \omega_c$$

electron-acoustic mode

$$\omega \approx k\, v_{EA} \cos\theta$$

$$v_{EA} = (T_h/m_e)^{1/2}(n_l/n_0)^{1/2}$$

Nonlinear Dynamics of Whistler Waves in a Two-Temperature Electron Plasma

$$\frac{d}{dt}\left[u_j - \frac{eA_x}{mc}\right] = -2\pi f_c\, v_j \cos\theta,$$

$$\frac{d}{dt}\left[v_j - \frac{eA_y}{mc}\right] = 2\pi f_c \left(u_j \cos\theta + w_j \sin\theta\right),$$

$$\frac{dw_j}{dt} = \frac{e}{m}\frac{\partial \Phi}{\partial z} - \frac{1}{m\, n_j}\frac{\partial\left(T_j\, n_j\right)}{\partial z} - 2\pi f_c\, v_j \sin\theta - \frac{e}{mc}\left[u_j \frac{\partial A_x}{\partial z} + v_j \frac{\partial A_y}{\partial z}\right],$$

momentum conservation

$$\frac{dn_j}{dt} = -n_j \frac{\partial w_j}{\partial z}, \quad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + w_j \frac{\partial}{\partial z},$$

density

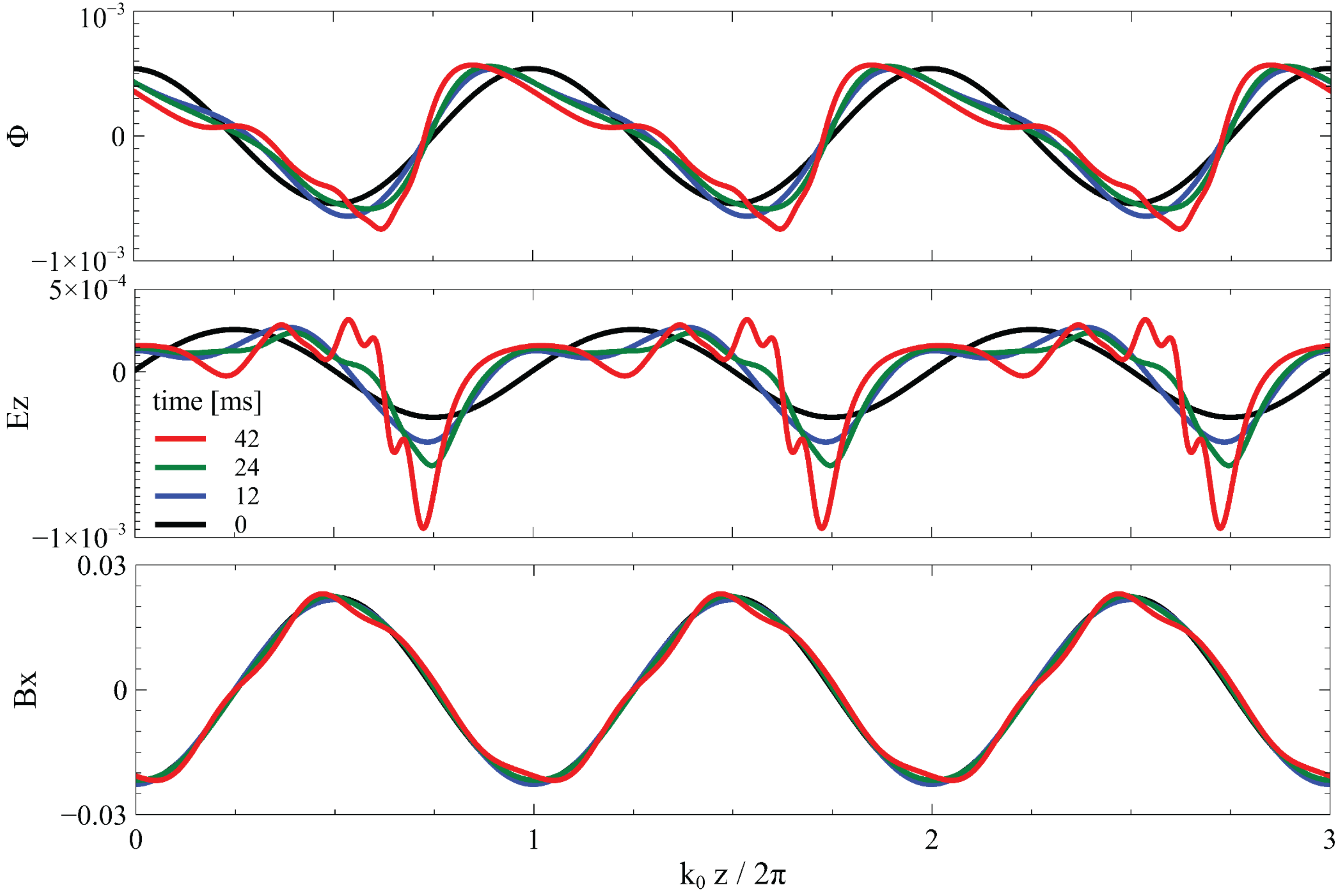
where $j = l, h$ corresponds to the low- and high-energy populations, n_j , (u_j, v_j, w_j) and T_j are electron densities, bulk velocities and temperatures, $-e$ and m are electron charge and mass. The Maxwell equations for the electrostatic and vector potentials are $\partial^2\Phi/\partial z^2 = 4\pi e \left(\sum_j n_j - n_0\right)$ and $\partial^2 \mathbf{A}/\partial z^2 = (4\pi e/c) \sum_j \left(n_j u_j\, \hat{x} + n_j v_j\, \hat{y}\right)$, where n_0 is the unperturbed electron density

we study evolution of initially monochromatic whistler wave by numerically solving 11 two-fluid MHD equations

initial condition: a whistler wave with a particular wavenumber, realistic finite initial amplitude and wave normal angle of 15^0

whistler waves with initial wavenumbers far from the crossover point, $k = k_c$, exhibit no distortions

Nonlinear Evolution of Whistler Wave / Initial Wave Number $k \sim k_C$ /

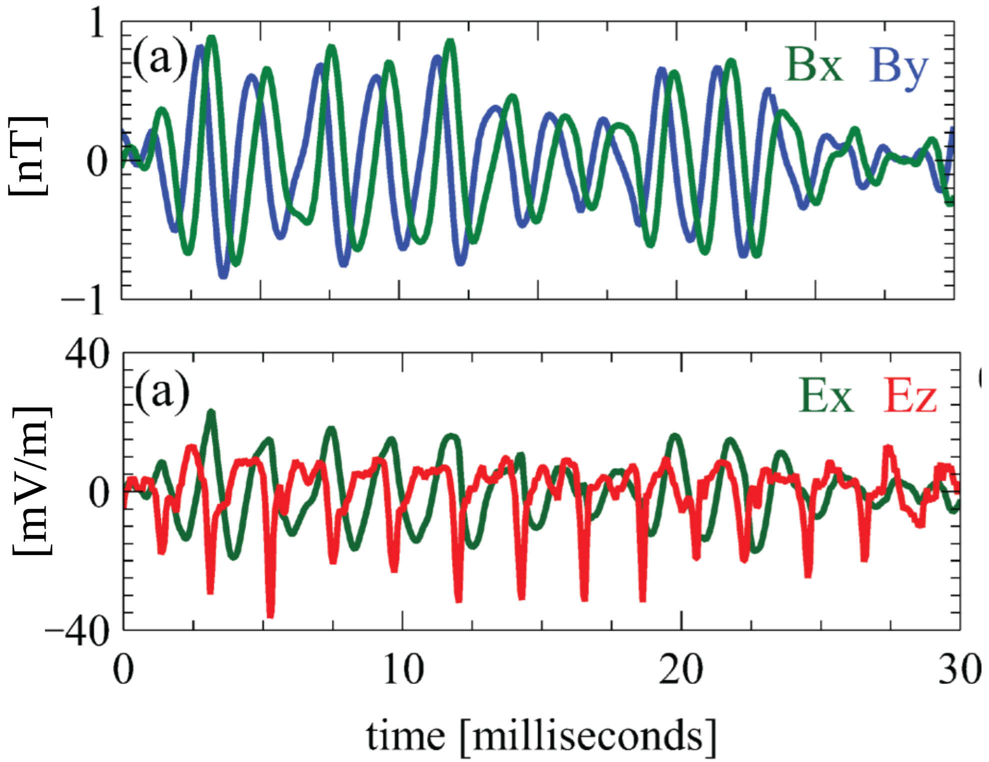


electrostatic potential exhibits signatures of the classical steepening

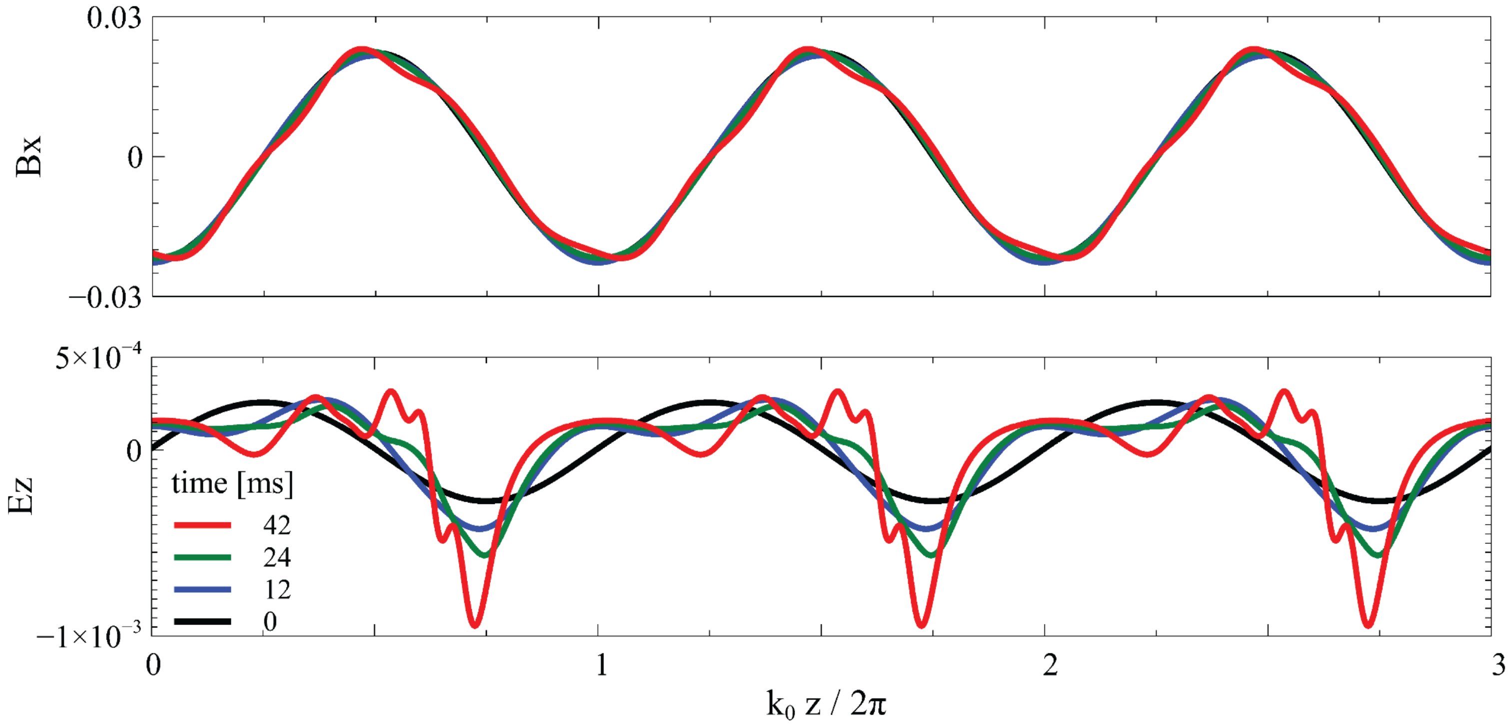
after ~40 ms electrostatic field Ez consists of spikes

electromagnetic components are essentially not distorted

Observations



Numerical Simulations



Theoretical Interpretation

$$\frac{d}{dt}\left[u_j - \frac{eA_x}{mc}\right] = -2\pi f_c\, v_j \cos\theta,$$

$$\frac{d}{dt}\left[v_j - \frac{eA_y}{mc}\right] = 2\pi f_c\, (u_j \cos\theta + w_j \sin\theta)\,,$$

$$\frac{dw_j}{dt} = \frac{e}{m}\frac{\partial \Phi}{\partial z} - \frac{1}{m\, n_j}\frac{\partial\,(T_j\, n_j)}{\partial z} - 2\pi f_c\, v_j \sin\theta - - \frac{e}{mc}\left[u_j \frac{\partial A_x}{\partial z} + v_j \frac{\partial A_y}{\partial z}\right],$$

$$\frac{dn_j}{dt} = -n_j \frac{\partial w_j}{\partial z}, \quad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + w_j \frac{\partial}{\partial z},$$

momentum conservation & density

where $j = l, h$ corresponds to the low- and high-energy populations, n_j , (u_j, v_j, w_j) and T_j are electron densities, bulk velocities and temperatures, $-e$ and m are electron charge and mass. The Maxwell equations for the electrostatic and vector potentials are $\partial^2\Phi/\partial z^2 = 4\pi e\left(\sum_j n_j - n_0\right)$ and $\partial^2 \mathbf{A}/\partial z^2 = (4\pi e/c)\sum_j (n_j u_j\, \hat{x} + n_j v_j\, \hat{y})$, where n_0 is the unperturbed electron density

near the crossover frequency compressional velocities w_j become non-negligible

nonlinear terms in hydrodynamic equations results in generation of electron-acoustic waves at the harmonics

$$(\omega_0, k_0) \rightarrow (2\omega_0, 2k_0), (3\omega_0, 3k_0)\ldots..$$

the process is equivalent to steepening of sound waves

<u>steepening time scale</u>

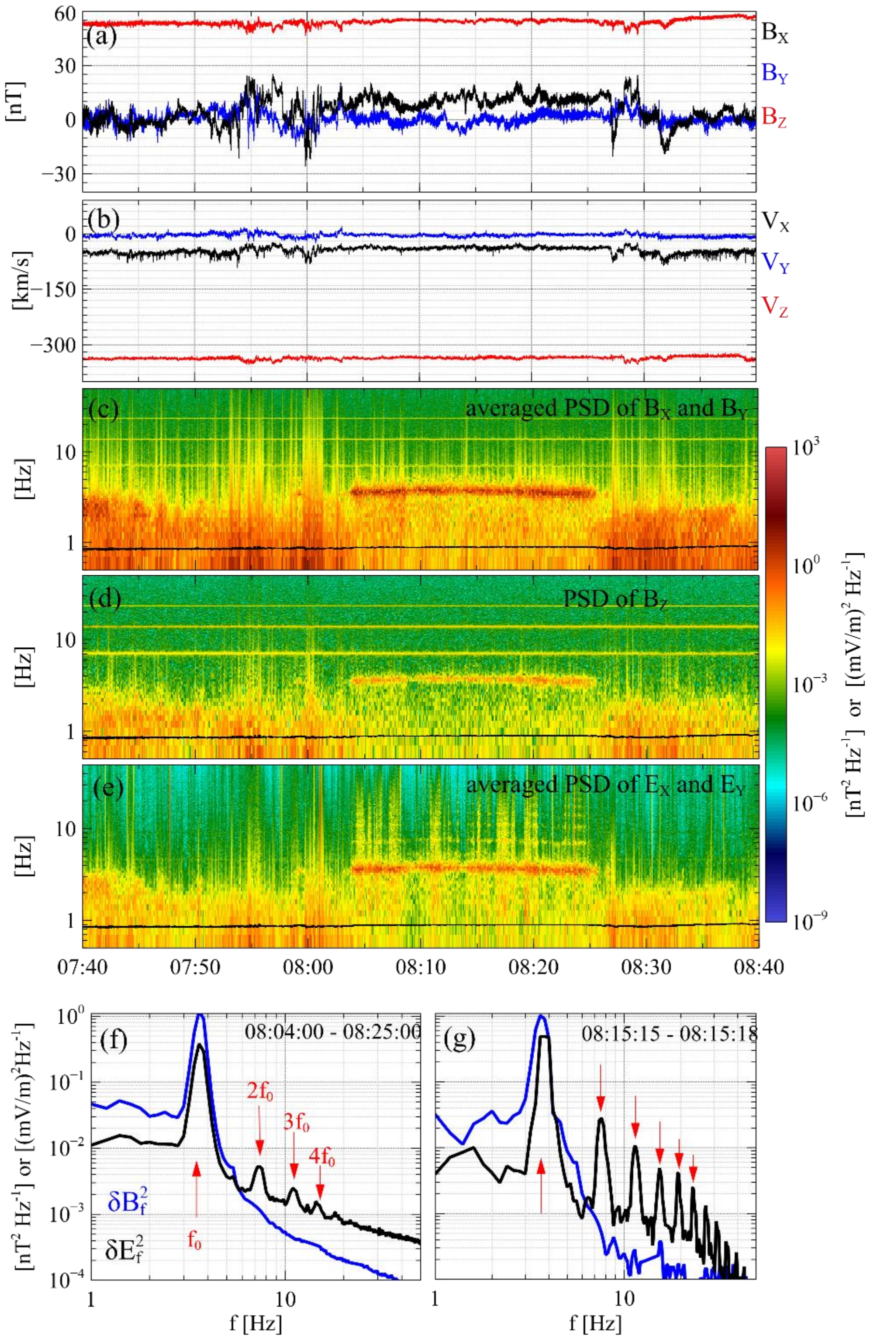
$$\tau_s \sim 2A(mT_h)^{1/2}\,(eE_0)^{-1} \sim 40 \text{ ms}$$

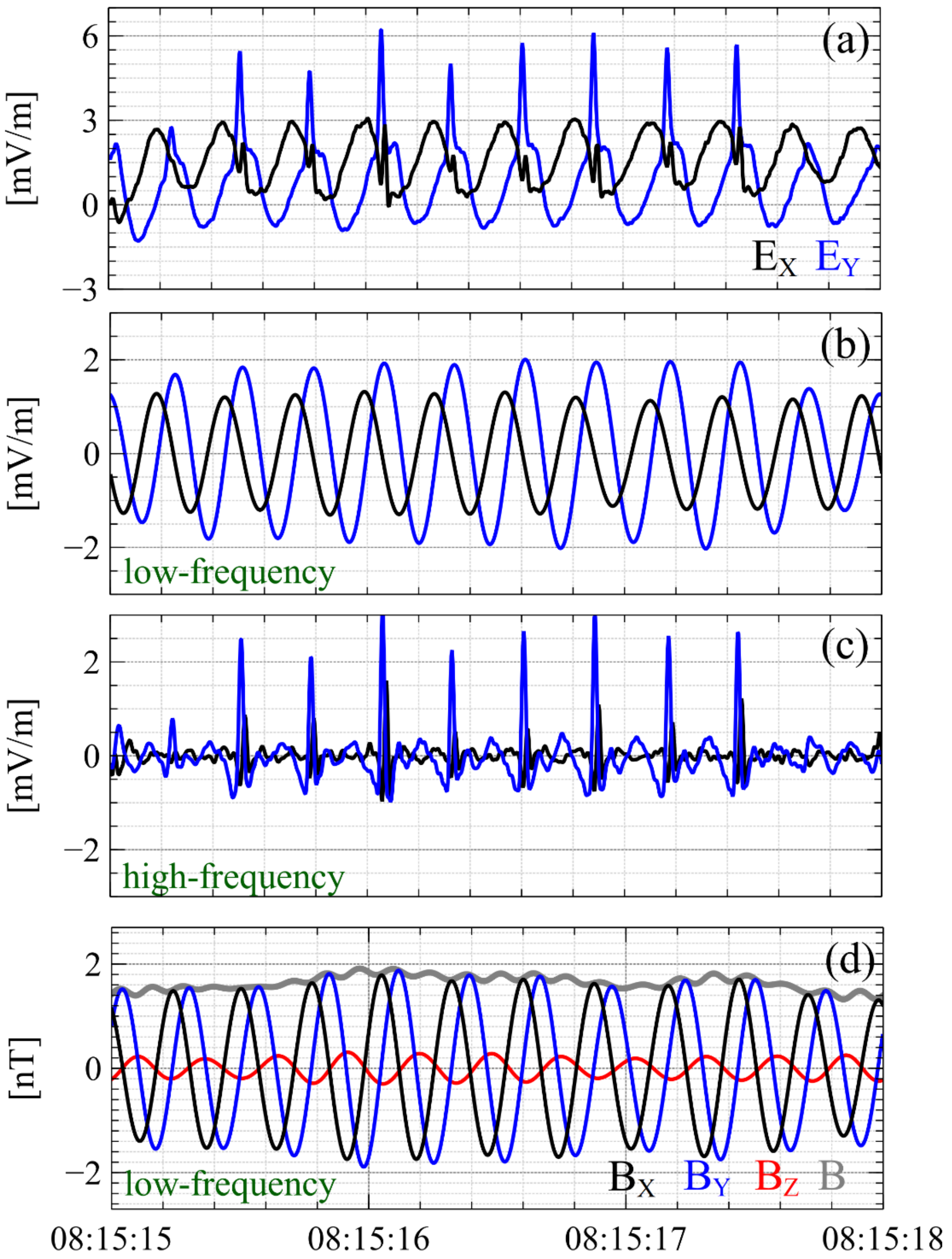
$$A^{-1} = (n_h/n_l)^{1/2}(3 + n_l/n_h)$$

Thank You!

Vasko I. Y. , Agapitov O., Mozer F., Bonnell J., Artemyev A. V., Krasnoselskikh V., and Tong Y. (2018),
Electrostatic Steepening of Whistler Waves
Physical Review Letters, https://doi.org/10.1103/PhysRevLett.120.195101

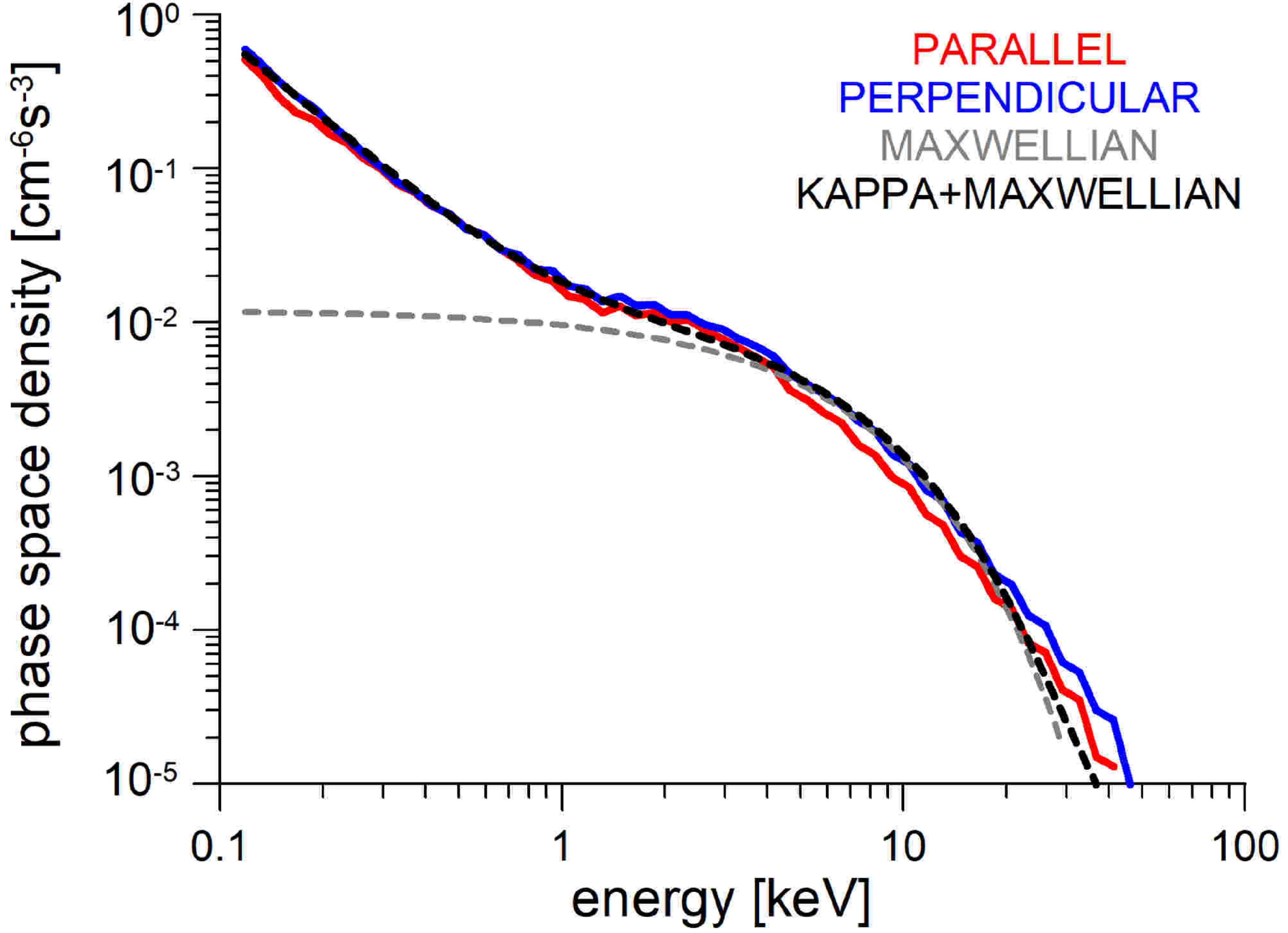
Similar Parker Solar Probe Observations in the Solar Wind





Back Up

Electron Energy Spectrum / Van Allen Probes Observations /



there are at least two electron populations

cold electrons with power law distribution

$$f_\kappa \sim n_\kappa \left[1 + \frac{E}{\kappa T_\kappa}\right]^{-(\kappa+1)}$$

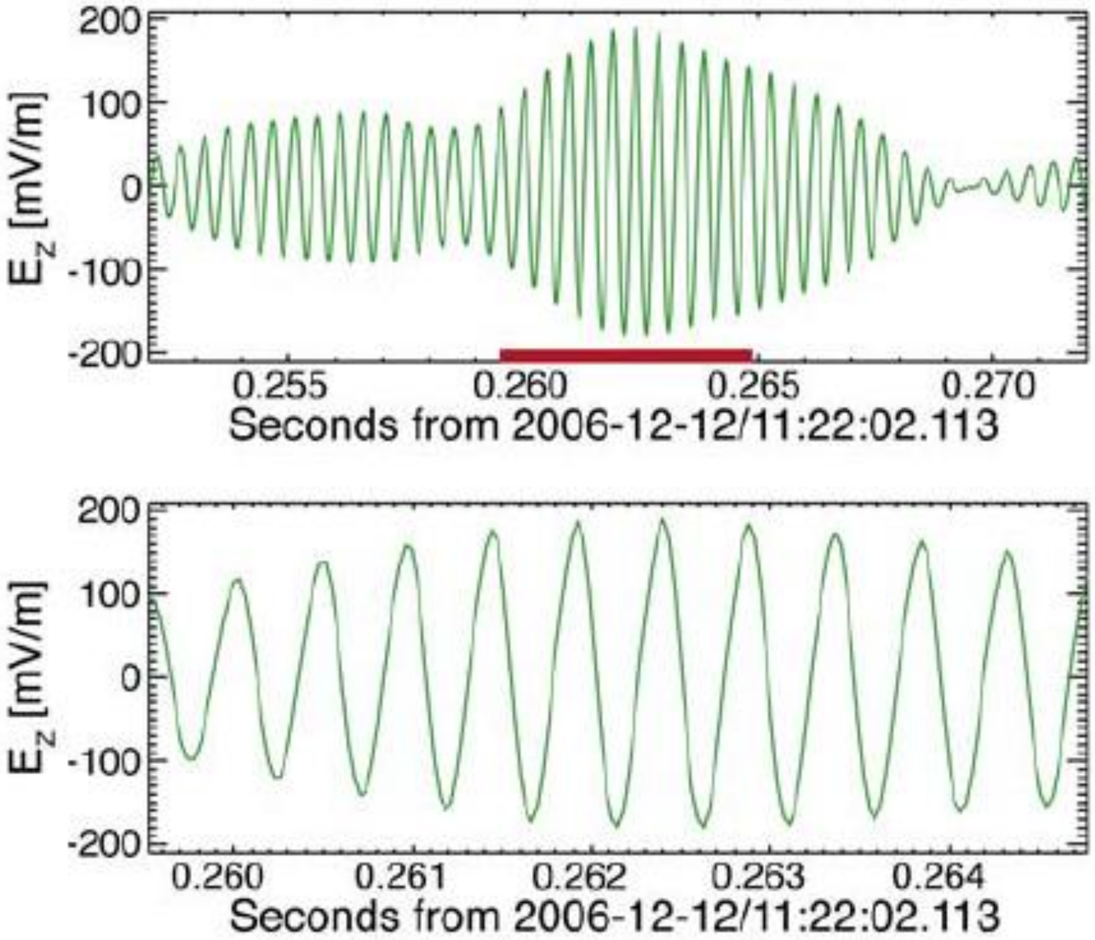
$$n_\kappa \sim 2.5\ \mathrm{cm}^{-3}\ \ T_\kappa \sim 1\ \mathrm{eV},\ \kappa \sim 1$$

hot electrons with Maxwell distribution

$$f_M \sim n_M \exp[-E/T_M]$$

$$n_M \sim 1.8\ \mathrm{cm}^{-3},\ \ T_M \sim 4.5\ \mathrm{keV}$$

Large-Amplitude Whistler Waves



even the largest-amplitude whistlers
(up to a few hundred mV/m)
exhibit quasi-sinusoidal waveforms
indicating thereby absence of
any nonlinear effects related to E^2

thus, the effect we are looking at is of
resonant nature

Cattell et al., GRL, 2008