О РАСПРЕДЕЛЕНИИ КОНЦЕНТРАЦИИ КАПЕЛЬ БРЫЗГ В ВОЗДУШНОМ ПОТОКЕ НАД ВЗВОЛНОВАННОЙ ВОДНОЙ ПОВЕРХНОСТЬЮ

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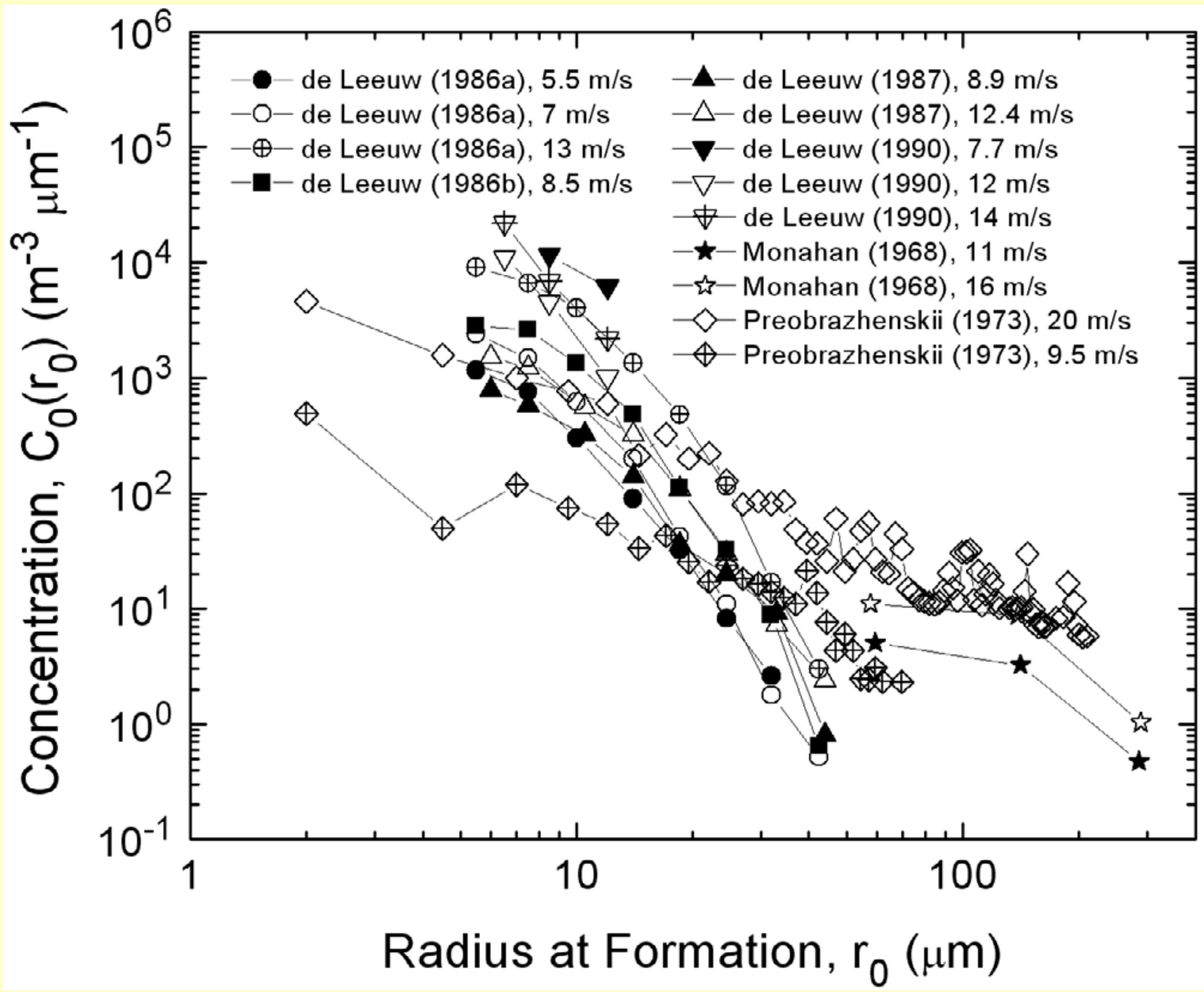
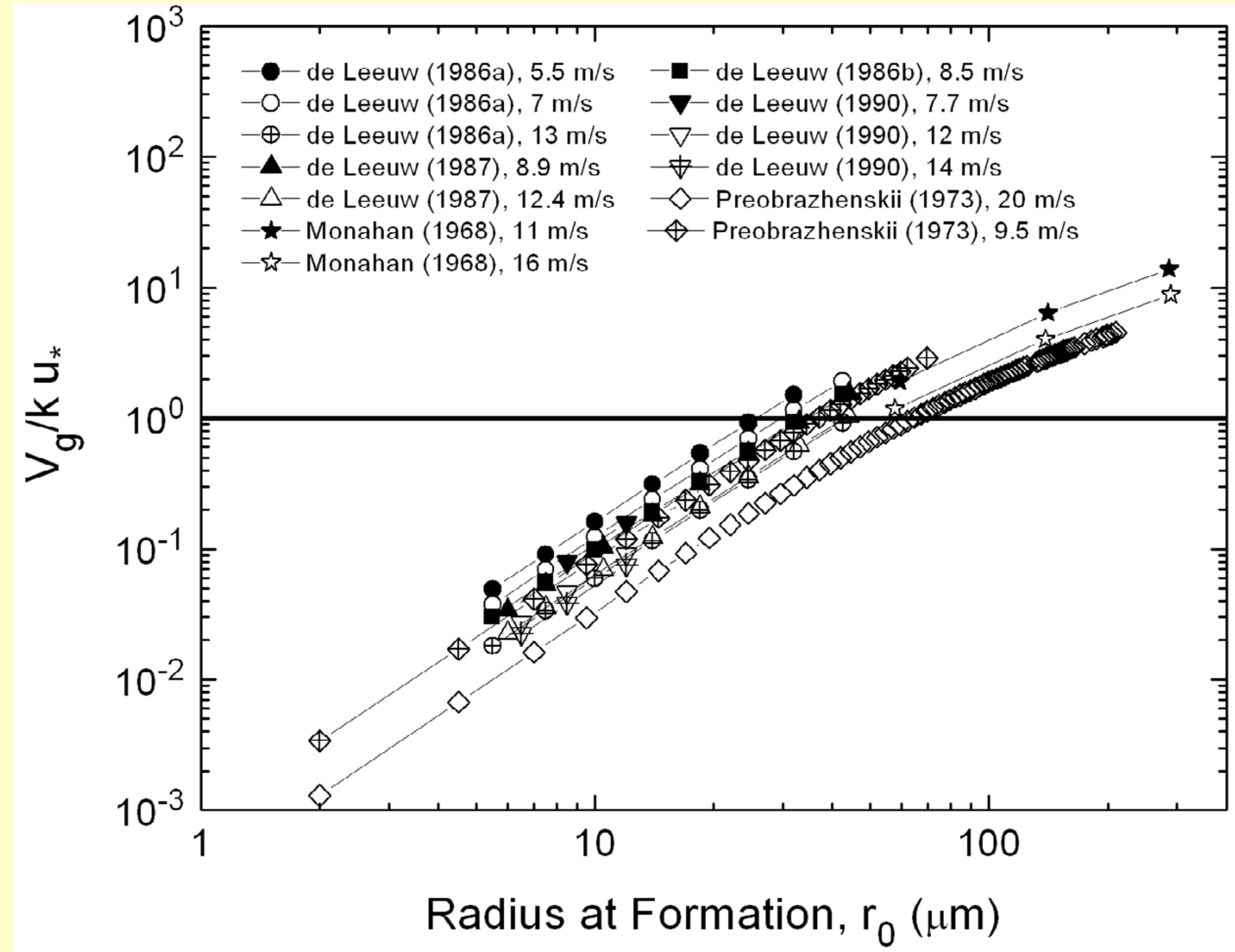
Data on droplets generation by the wind (Andreas et al. 2010, JGR)



Droplet number density at different wind speeds at heights from 1 to 2m.



Droplet terminal velocity normalized by friction velocity and Karman constant (k=0.4)

$$\left(V_g = g \frac{d^2}{18\nu} \frac{\rho_w}{\rho_a} \right)$$

Droplet concentration in lab experiments

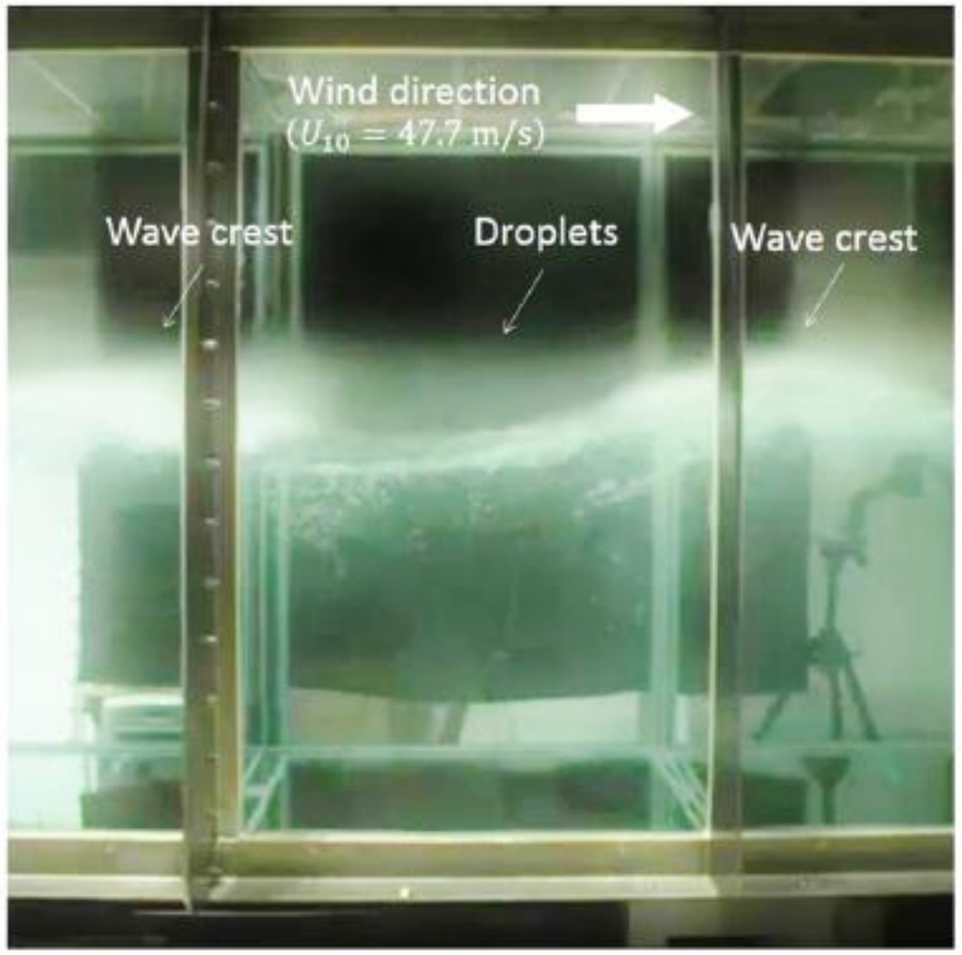
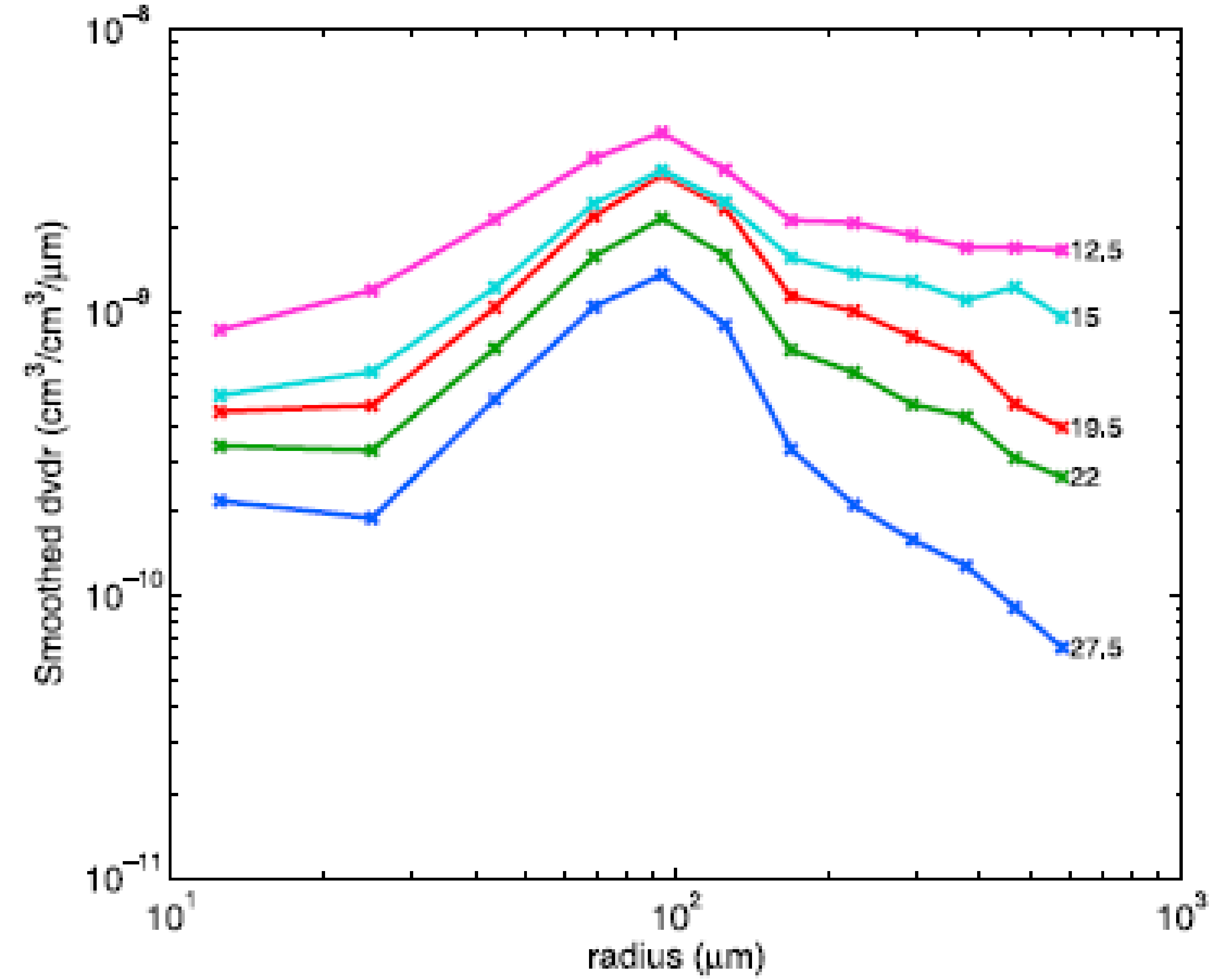


FIG. 6. Snapshot of droplets dispersing over breaking waves at $U_{10} = 47.7\,\mathrm{m\,s^{-1}}$.

Komori at al. 2018)



Droplet volume concentration for wind speed 16m/s at different heights above the water surface (in cm)

(Fairall et al.,2009)

Solution for concentration of particles passively settling over flat surface

Assumption:

$V_d = U_a - V_s$

where U_a – air velocity, V_s - particle settling velocity

Then using gradient closure for concentration turbulent flux

$$\left[CU_z\right]_t = -K_C \frac{dC}{d\eta} = -\frac{1}{Sc_t} K_U \frac{dC}{d\eta} \quad \text{where} \quad Sc_t = \frac{K_U}{K_C}$$

and for turbulent air momentum flux in the log-layer

$$\left[U_x U_z\right]_t = -K_U \frac{dU}{d\eta} = -u_*^2$$

the solution for stationary concentration profile is found:

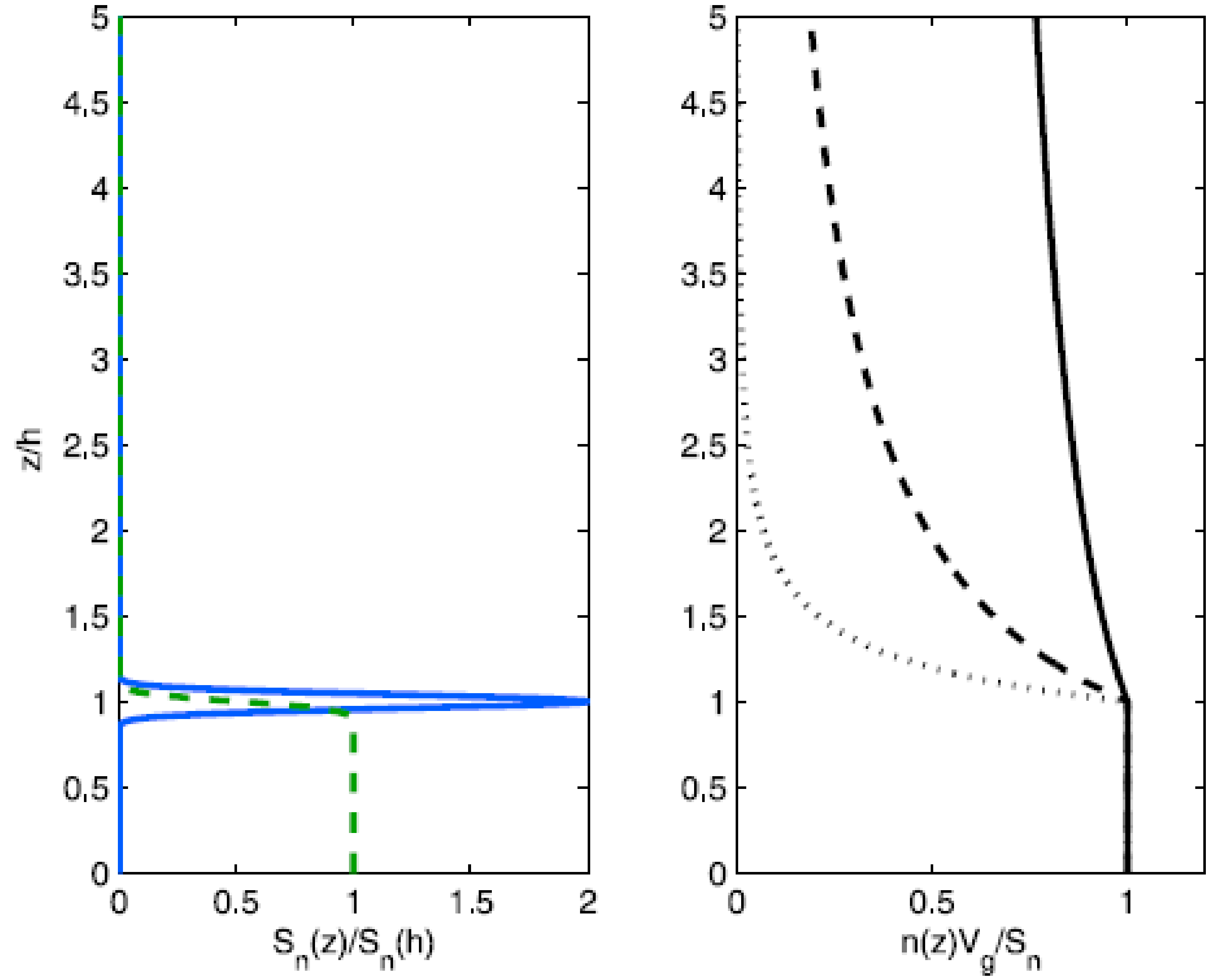
$$C = C_m \exp\left(-V_s U(\eta) Sc_t / u_*^2\right) \quad \text{or} \quad C = C_m \eta^{-\omega}, \quad \omega = V_s / \kappa u_*$$

(cf. e.g. Kudryavtsev 2006)

(for $Sc_t \approx 1$ Prandtl 1949, Barenblat 1953, Toba 1970, Barenblat & Golitsyn 1974)

Model profiles of droplet concentration

Droplets source profile



Droplets concentration profile

Figure 1. Sample normalized droplet profiles to illustrate relationships from section 2. (left) An example of a narrow Gaussian specification of Q_n (solid line) at source height h. The source flux term S_n (dashed line) is obtained from (3). (right) Expected normalized nonevaporating sea spray profiles computed using (13) for $z > h$ for droplets of 31 (solid line), 100 (dashed line), and 310 (dotted line) μm radius. In this case we have used $u_* = 1.6$ ms^{-1}. For $z < h$ we assume $n(z,r)$ is constant with the value $n(h,r)$.

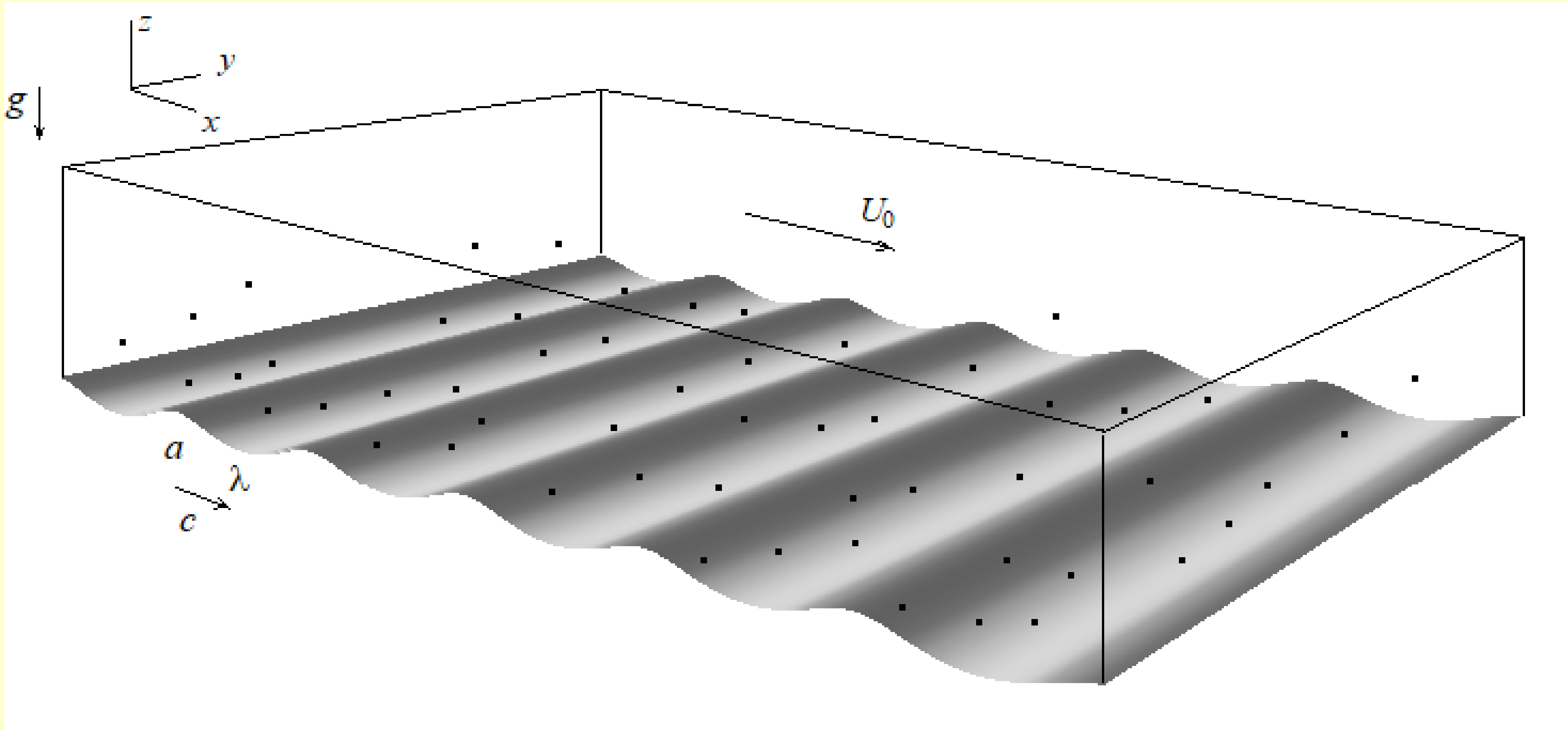
(From Fairall et al. 2009)

Цель работы

Возникают вопросы:
- Насколько точно приближение пассивно оседающей примеси «работает» для капель в воздушном потоке над взволнованной водной поверхностью?
- Каковы характеристики дисперсии капель (турбулентное число Шмитдта) в воздушном погранслое при приближении к водной поверхности, каково влияние шероховатости?

Попробуем ответить на эти вопросы с помощью прямого численного моделирования.

Schematic of numerical experiment



c=0.05 – wave celerity
ka =0.2 – maximum wave slope

Domain sizes: $L_x = 6\lambda \quad L_y = 4\lambda \quad L_z = \lambda$

$$\mathrm{Re} = \frac{U_0 \lambda}{\nu} = 15000$$ - bulk Reynolds number

Governing equations

Air momentum and continuity

$$\frac{\partial U_i}{\partial t}+\frac{\partial (U_i U_j)}{\partial x_j}=-\frac{1}{\rho_a}\frac{\partial P}{\partial x_j}+\frac{1}{\mathrm{Re}}\frac{\partial^2 U_i}{\partial x_j \partial x_j} \qquad \frac{\partial U_j}{\partial x_j}=0$$

Droplet coordinate and velocity

$$\frac{dr_i^n}{dt}=V_i^n \qquad \frac{dV_i^n}{dt}=\frac{1}{\tau_d}\left(U_i(r^n)-V_i^n\right)\left(1+0.15\,\mathrm{Re}_n^{0.687}\right)-\delta_{iz}g$$

$$\tau_d=\frac{d^2}{18\nu}\frac{\rho_w}{\rho_a}$$ - droplet response time

$$\mathrm{Re}_n=\frac{|U(r^n)-V^n|\,d}{\nu}$$ - droplet Reynolds number

Curvilinear coordinates

$$x = \xi - a\exp(-k\eta)\sin k\xi$$

$$z = \eta + a\exp(-k\eta)\cos k\xi$$

Shape of the water surface: $z_b(x) = a\cos kx + \frac{1}{2}a^2k(\cos 2kx - 1)$

Mapping over η: $\eta = 0.5\left(1 + \frac{\tanh\tilde{\eta}}{\tanh 1.5}\right)$ $\quad -1.5 < \tilde{\eta} < 1.5$

Grid of 360 x 240 x 180 nodes is employed with mesh sizes:

$\Delta x^+ \approx 6$ in the horizontal direction

$\Delta z_1^+ \approx 0.3$ near water surface in the vertical direction

$\Delta z_2^+ \approx 3$ in the middle of the domain

Boundary conditions

Air velocity = water velocity in the surface wave:

$$U(\xi, y, 0) = c\left(ka \cos kx(\xi,\eta) - 1\right)$$

$$V(\xi, y, 0) = 0$$

$$W(\xi, y, 0) = cka \sin kx(\xi,\eta)$$

No-slip condition at the upper moving plane :

$$U(\xi, y, 1) = 1 - c$$

$$V(\xi, y, 1) = 0$$

$$W(\xi, y, 1) = 0$$

All fields are x and y periodic

Droplets injection:

Drops falling on the water or reaching the upper plane are re-injected in the vicinity of the wave crests in the range:

$$0.01 < \eta / \lambda < 0.05 \quad (5 < \eta u_* \mathrm{Re} < 25)$$

$$n\lambda - 0.2 < \xi < n\lambda, \quad n = 1, \ldots, 6$$

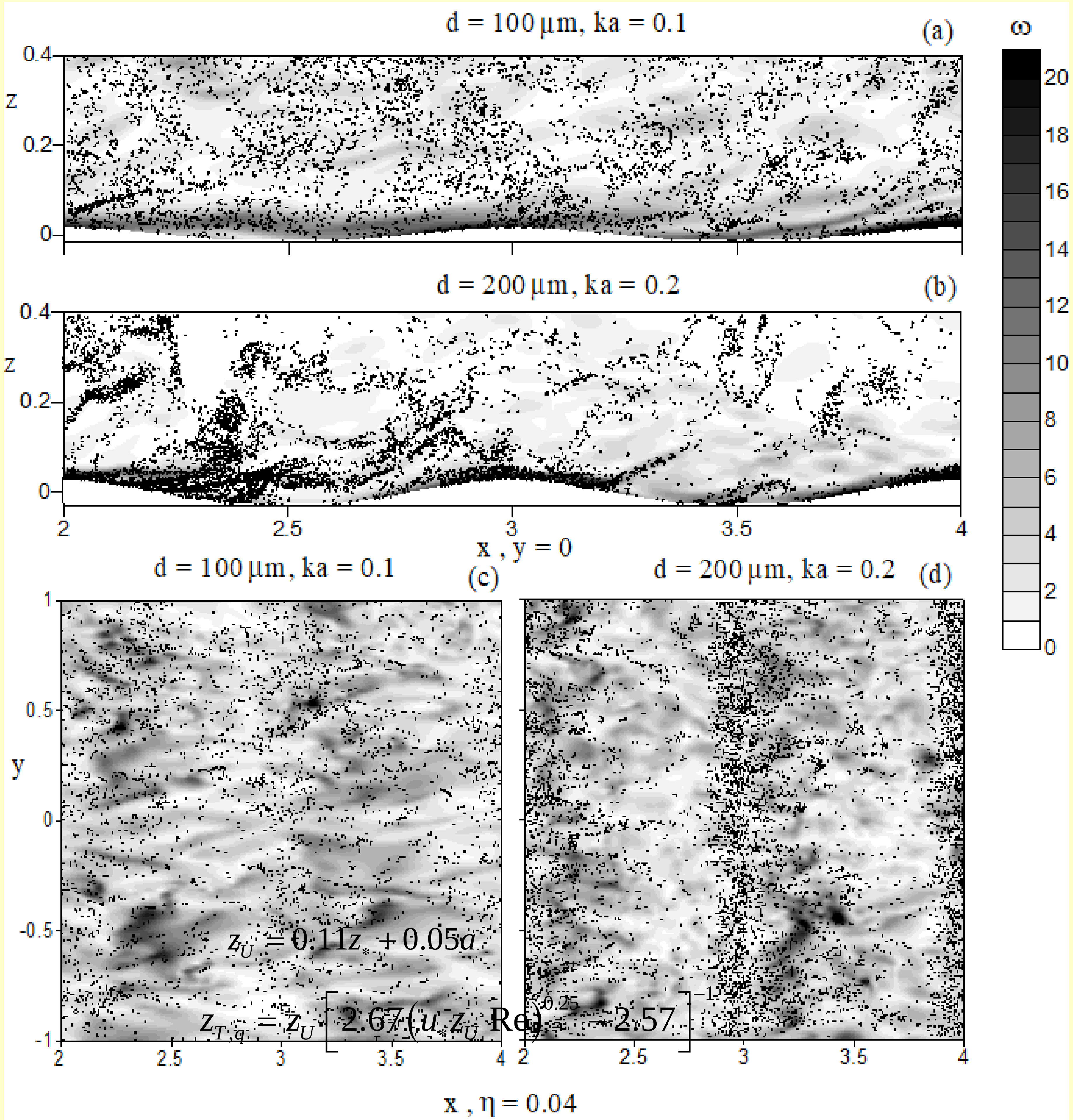
Drop velocity at injection is prescribed according to Andreas (2004), Troitskaya et al. (2016); for drops diameters :

$$d = 100,\, 200,\, 300\,\mu m$$

Instantaneous fields

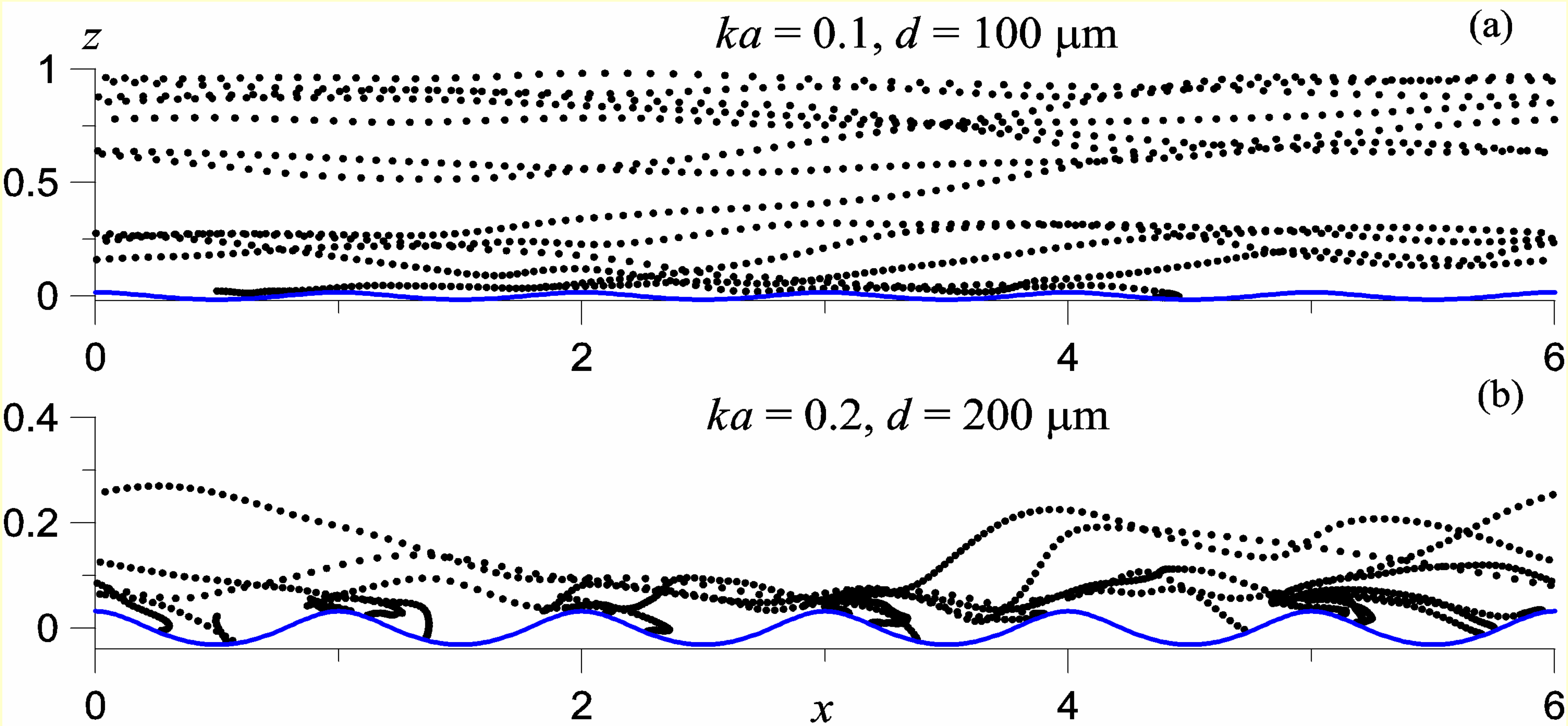
Side view

View from above



Vorticity modulus

Droplet trajectories



Side view

Mean profiles

Air velocity

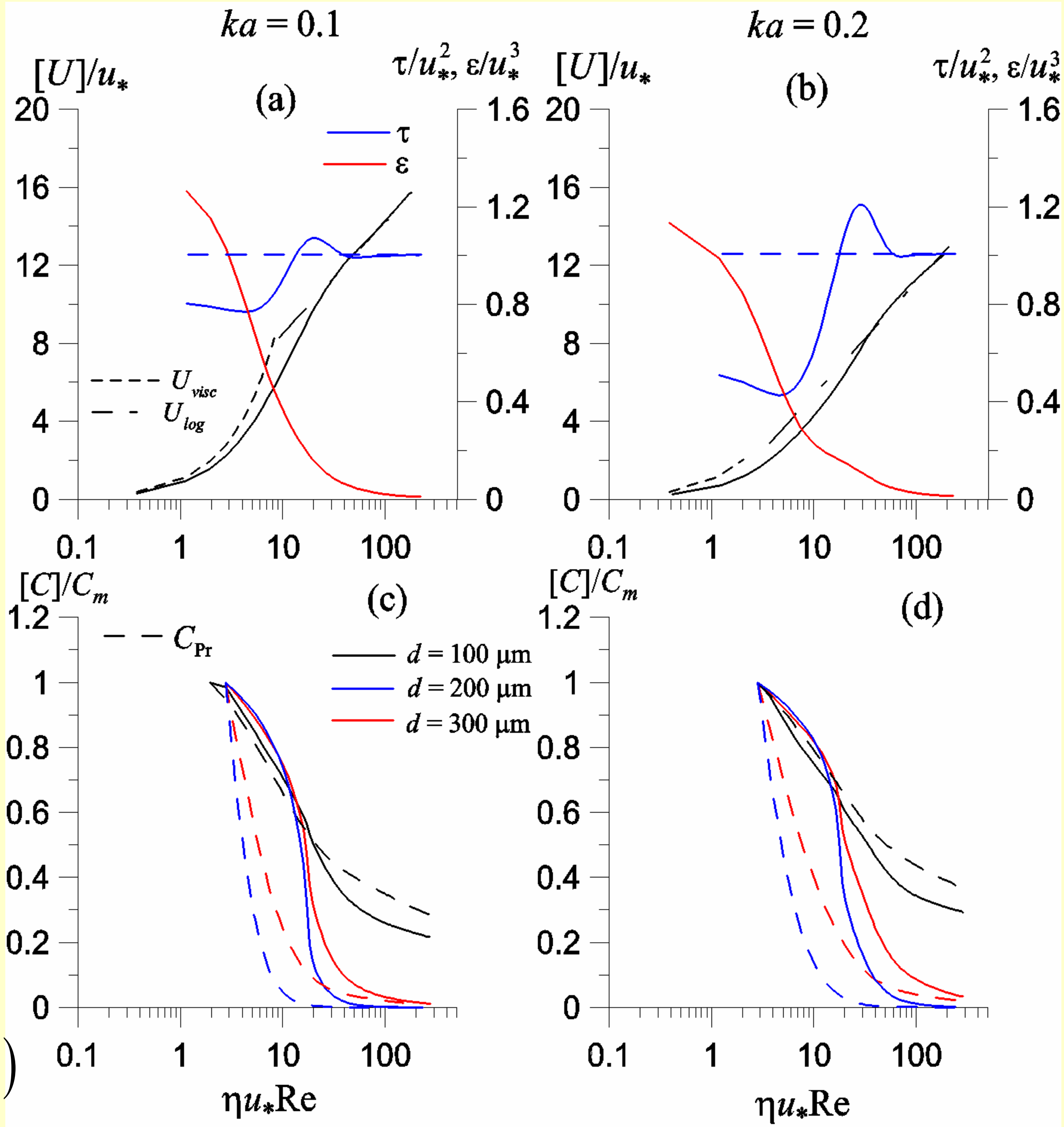
$$U_{visc} = u_* \eta \,\mathrm{Re}$$

$$U_{\log} = \frac{1}{\kappa} \ln \frac{\eta}{\eta_0}$$

$$\eta_0 = \frac{1}{u_* \mathrm{Re}} \mathrm{e}^{-2} + 0.3a$$

Droplet concentration

$$C_{\mathrm{Pr}} = C_m \exp\left(-V_s [U] / u_*^2\right)$$



Momentum flux

$$\tau_{full} = \tau + \tau_{wave} = u_*^2$$

$$\tau = \frac{1}{\mathrm{Re}} \frac{d[U]}{d\eta} + \tau_t$$

$$u_* \approx 0.03$$

$$\tau_t = -[U_x' U_z']$$

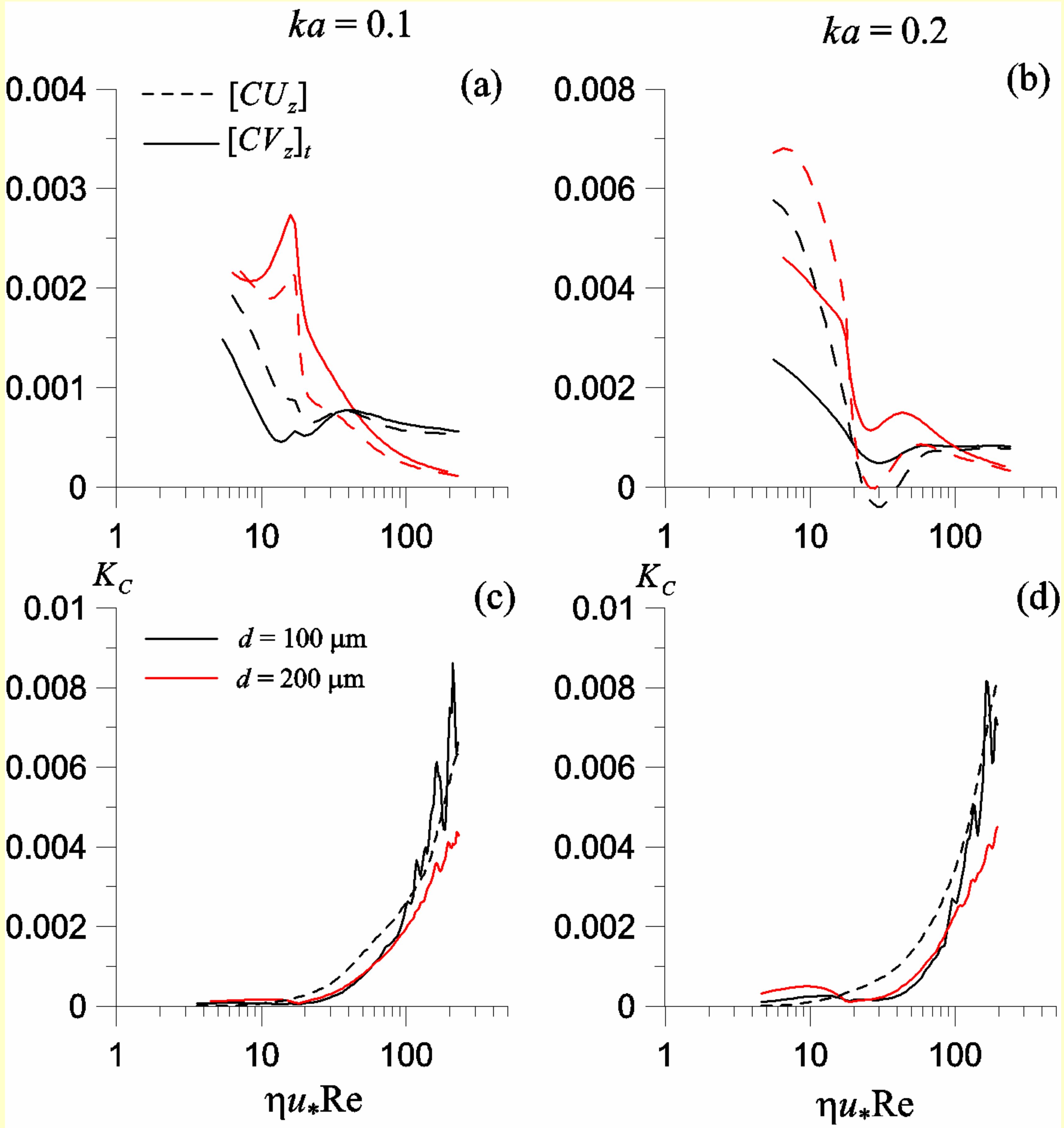
dissipation

$$\varepsilon = \left[\left(\frac{\partial U_i'}{\partial x_j} \right)^2 \right]$$

Mean profiles

Concentration flux

$$[CV_z]_t = [CV_z] - [C]V_s$$



Turbulent viscosity/diffusion coeffcients

$$K_U(\eta) = \frac{\tau_t}{d[U]/d\eta},$$

$$K_C(\eta) = -\frac{[CV_z]_t}{d[C]/d\eta},$$

Mean profiles

Turbulent Schmidt number

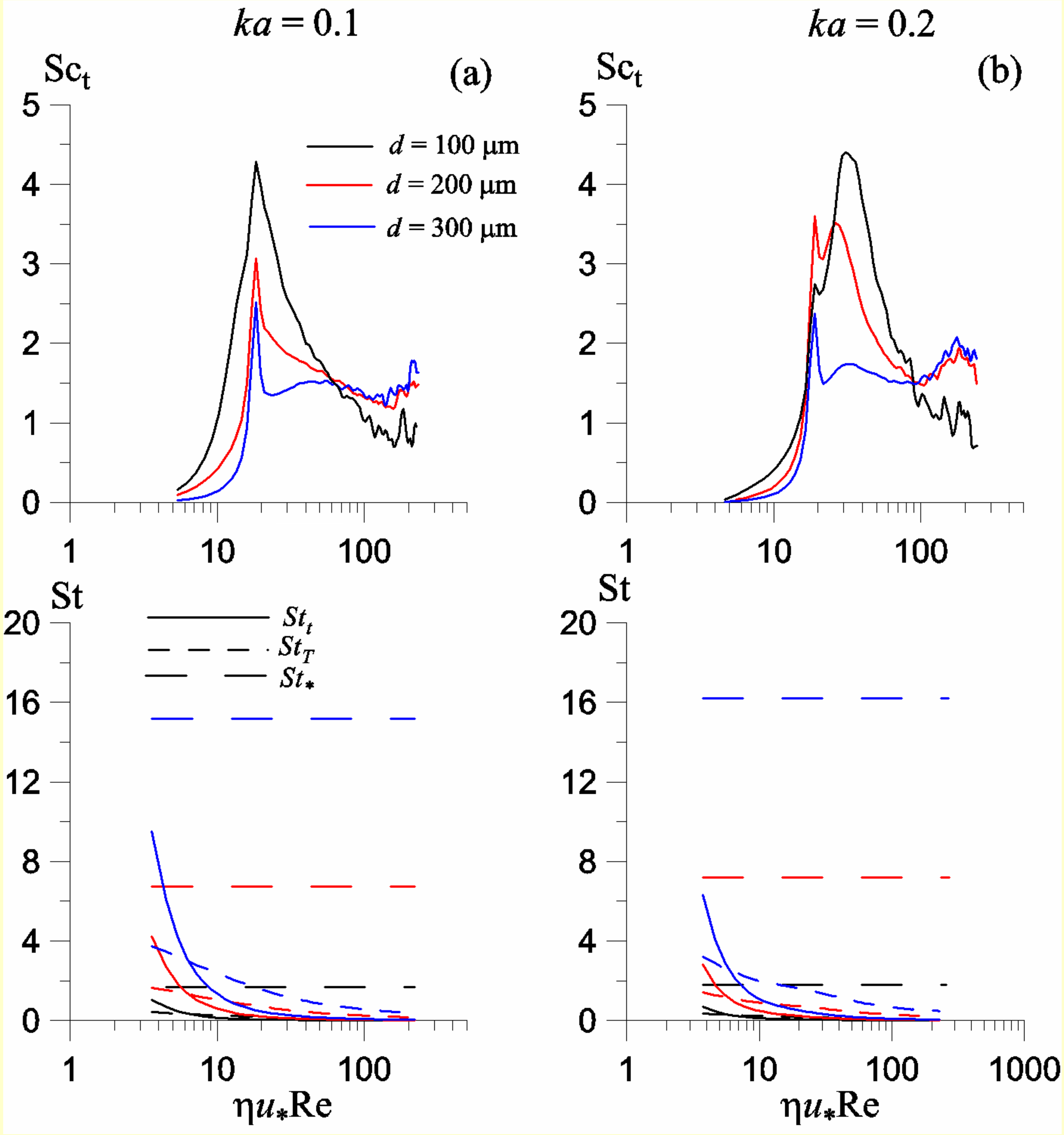
$$Sc_t = \frac{K_U}{K_C}$$

Stokes number

$$St_t = \frac{\tau_d \varepsilon}{E_t} \quad \text{integral scale}$$

$$St_T = \tau_d \left(\frac{\varepsilon}{15\nu} \right)^{1/2} \quad \text{Taylor scale}$$

$$St_* = \frac{\tau_d \mathrm{Re}}{u_*^2} \quad \text{wall scale}$$



Заключение

Потоки и профили концентрации капель удовлетврительно описываются решениями в приближении пассивно оседающей примеси лишь случае относительно малых размеров капель и пологих волн. Отличие возрастает с увеличением крутизны поверхностных волн. Турбулентное число Шмитдта капель существенно превышает 1 вблизи водной поверхности, где концентрация капель в DNS существенно превосходит предсказание теории (решение Прандтля).