Dynamics of particles and plankton under the action of nonlinear internal waves

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Background

The name ***plankton*** is derived from the Greek adjective πλαγκτός (*planktos*), meaning *errant*, and by extension, *wanderer* or *drifter*, and was coined by Victor Hensen in 1887. While some forms are capable of independent movement and can swim hundreds of meters vertically in a single day (a behavior called diel vertical migration), their horizontal position is primarily determined by the surrounding water movement, and plankton typically flow with ocean currents. This is in contrast to ***nekton*** organisms, such as fish, squid and marine mammals, which can swim against the ambient flow and control their position in the environment.

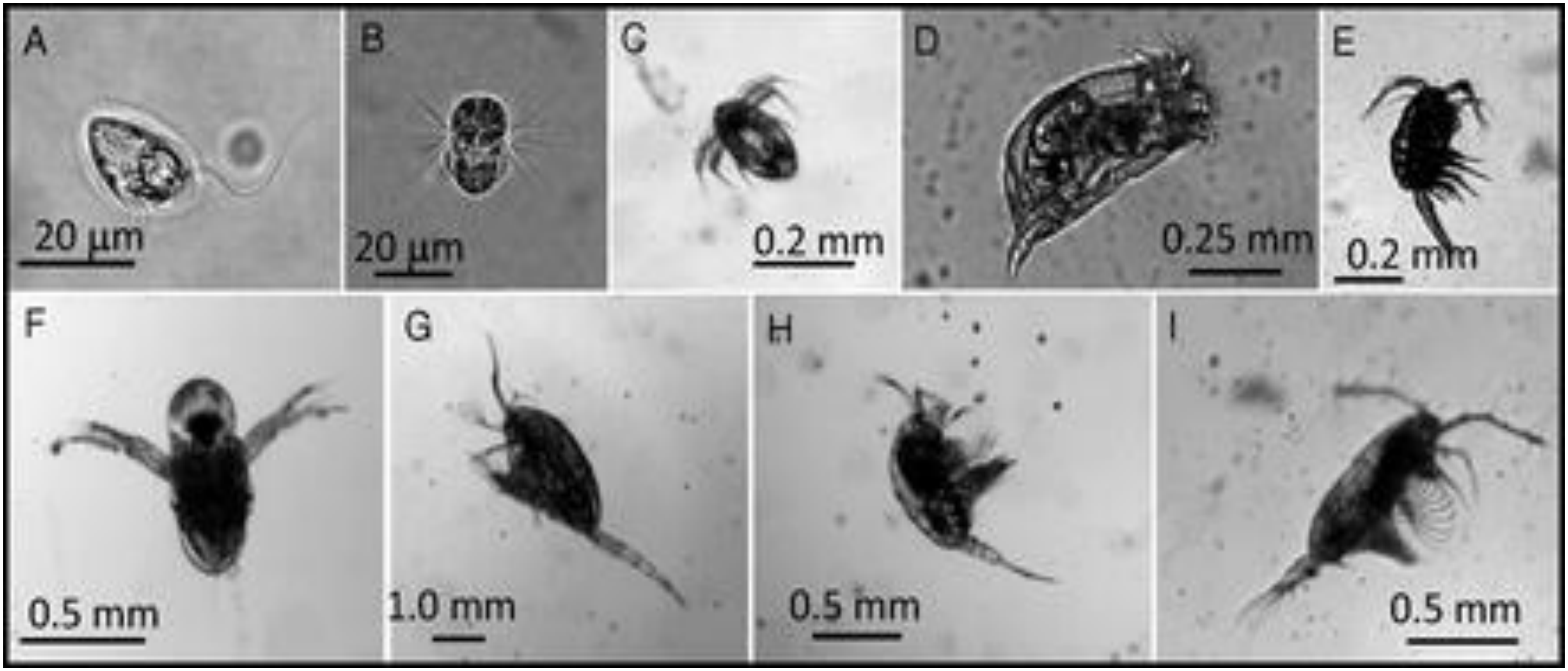
Phytoplankton (from Greek *phyton*, or plant) live near the water surface where there is sufficient light to support photosynthesis.

Zooplankton (from Greek *zoon*, or animal), are small crustaceans and other animals that feed on other plankton. Some of the eggs and larvae of larger nektonic animals,… are included here.

(From Wikipedia)

Some plankton species

Kiørboe et al. PNAS, 111 (32), 11738–11743, 2014

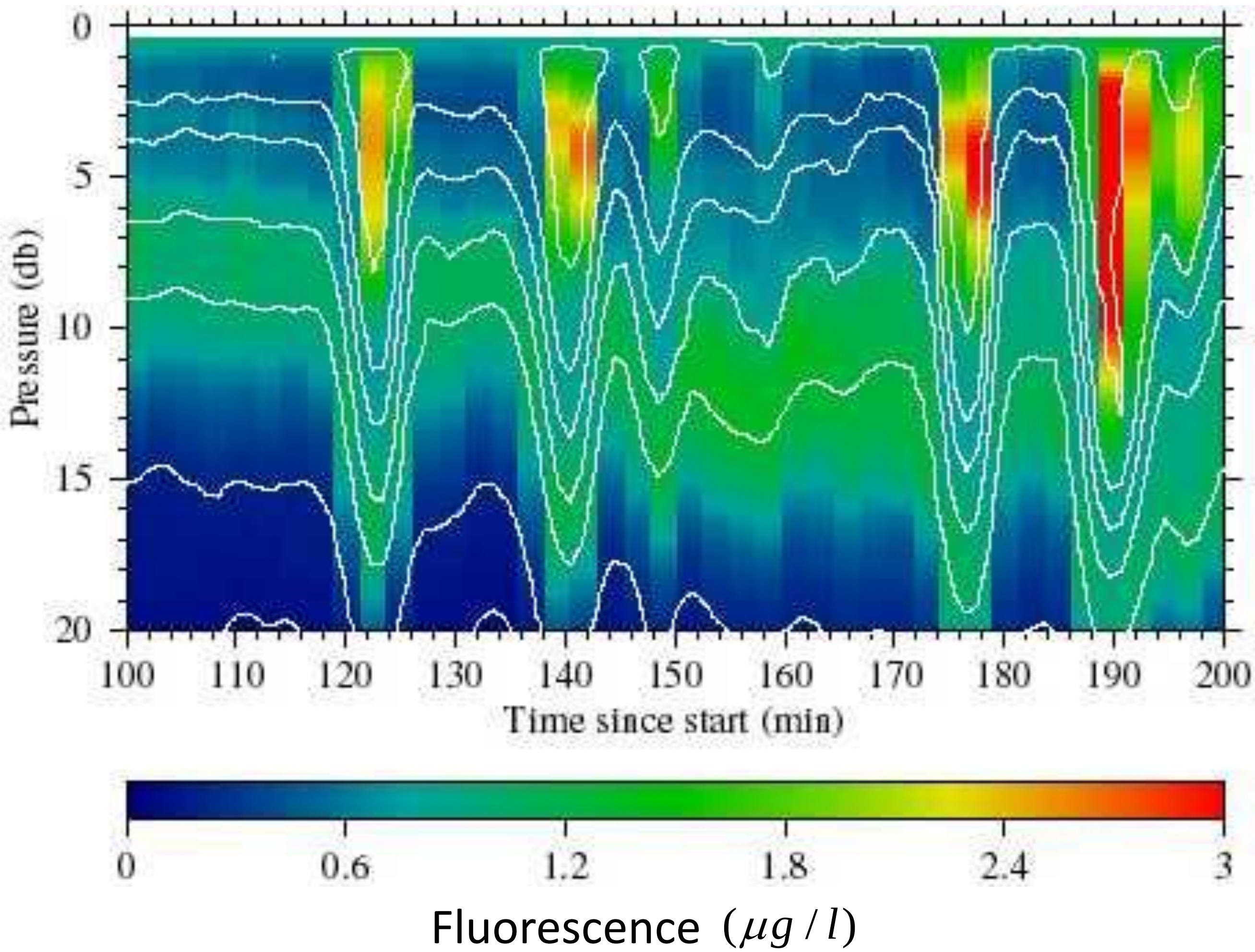


The organisms with their diverse propulsion equipment. (A–I) The dinoflagellate Oxyrrhis marina (A), the ciliate Mesodinium rubrum (B), nauplius of the copepod Acartia tonsa (C), the rotifer Brachionus plicatilis (D), the copepod Oithona davisae (E), the cladoceran Podon intermedius (F), the copepod Metridia longa (G), the copepod T. longicornis (H), and the copepod A. tonsa (I).

Plankton patchiness in high-frequency internal waves

Lennert-Cody and Franks, Marine Ecology Progress Series, 186, 59-66, 1999

Chlorophyll fluorescence and temperature contours (1 C from 11 to 17 C)
during a coastal bloom of phytoplankton *Lingulodinium polyedrum.*
Mission Beach, CA, Apr. 17, 1997. General depth is 30 m.



Modeling of plankton dynamics

Motion of a passive particle (incl. phytoplankton)

In general (T. R. Auton et al., J. Fluid Mech., 197, 241-257, 1988):

$$\boxed{\rho_p \frac{d\mathbf{u}_p}{dt} = \rho \frac{D\mathbf{u}}{Dt} - \frac{\rho}{2}\frac{d(\mathbf{u}_p - \mathbf{u})}{dt} - \hat{S}(\mathbf{u}_p - \mathbf{u}) + \Delta\rho\mathbf{g}.}$$

$\Delta\rho = \rho_p - \rho$ (p – particle)

$\mathbf{u}_p = d\mathbf{X}_p(t)/dt,\ \mathbf{u}(\mathbf{r}(t),t)$

$D/Dt = \partial/\partial t + (\mathbf{u}\nabla),$

$d/dt = \partial/\partial t + (\mathbf{u}_p\nabla)$

Gravity will be neglected for a while

For small Reynolds numbers, $\mathrm{Re} = R|\mathbf{u}_p - \mathbf{u}|/\nu << 1$:

$$\mathbf{u}_p = \mathbf{u} + n\mathbf{g},\ n = 2R^2\Delta\rho/9\nu\rho_f.$$

R is effective particle radius

For large Reynolds numbers, Re >>1, we have:

$$\mathbf{u}_p = q\mathbf{u},\ \ q = \sqrt{3\rho_f/(\rho_f + 2\rho_p)}.$$

q defines inertia here (and buoyance with gravity)
q = 1 is for a neutrally buoyant particle

First, *q* = 1 or neglect stratification, then

For the particle position vector $\mathbf{X}_\mathrm{p}$ we have

$$\frac{d\mathbf{X}_p}{dt} = q\mathbf{u}_p[\mathbf{X}_p(t)].$$

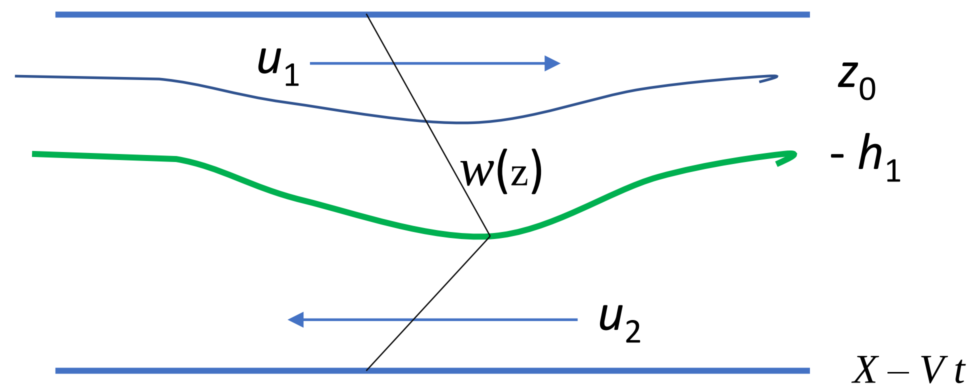
Passive particle motion in a long travelling wave

$$\mathbf{u} = \mathbf{u}(z, \xi = x - Vt).$$

Using the continuity equation $w_z = -u_x = -u_\xi$, we have

$$\frac{dz_p}{dt} = qw$$
$$\frac{d\xi}{dt} = qu - V$$

$$\Rightarrow \quad \frac{dz_p}{d\xi} = \frac{qw}{-V + qu} = \frac{q\int u_\xi dz}{V - qu}.$$



For a long wave in a two-layer fluid:

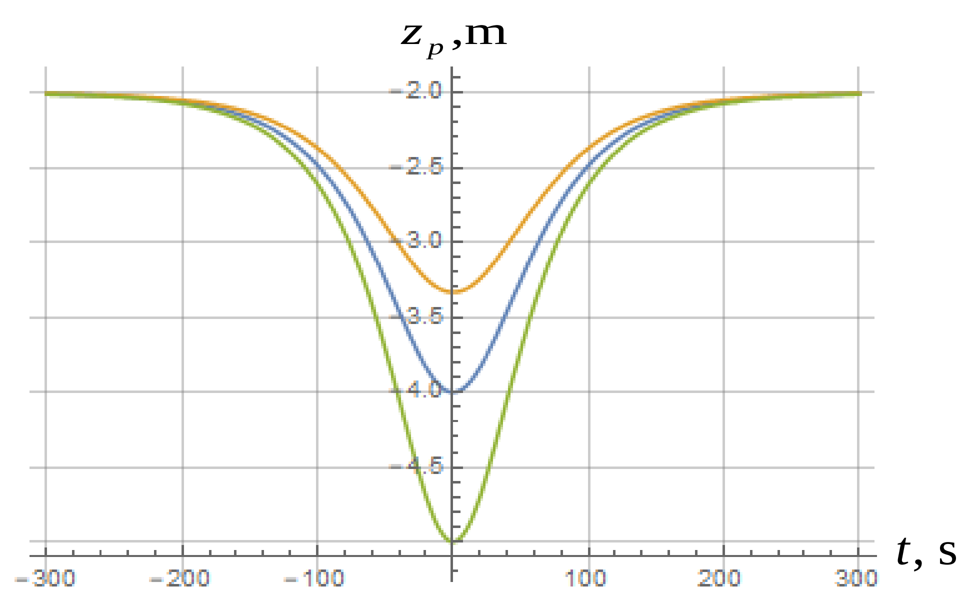
In the upper layer, $u_1 \approx u(\xi),\ w_1 \approx -zu_\xi$

For an initial condition $z_p = z_0$ at $u = 0$,

we have

$$\boxed{z_p = z_0 \frac{V}{V - qu(\xi)}}$$

Soliton: $u = u_0 \operatorname{sech}^2(k\xi)$ $\Rightarrow$



Particle displacement from the depth z_0 = − 2m. The wave parameters are $k = 2\pi / 600$ m^{-1}, $A/h_1 = -1$, V = 1 m/s, u_0 = 0.5 m/s (it corresponds to the displacement amplitude $A = -h_1$ = -10 m).
Blue: q = 1, Green: q = 1.2, yellow: q = 0.8.

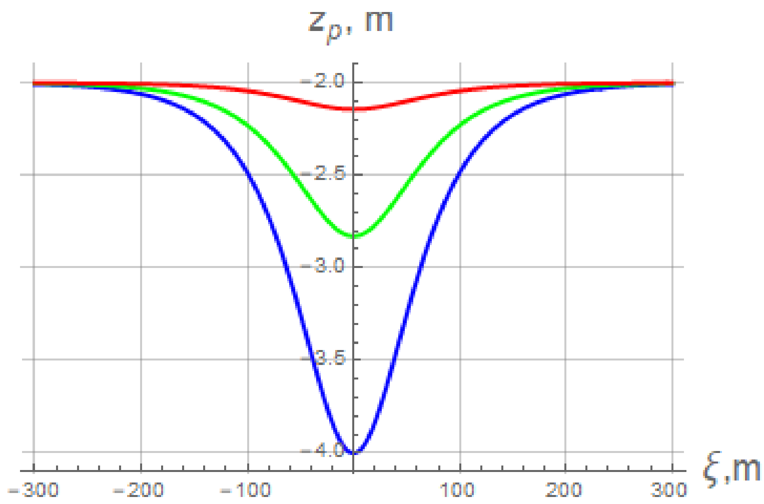
Active (swimming) particles

Lennert-Cody – Franks model:

$$w_p = qw - \alpha qw$$

$$\boxed{z_p = z_0 \left(\frac{V}{V - qu(\xi)} \right)^{(1-\alpha)}}$$

α = 0: Passive particle
α = 1: Depth-keeping



(or $-Vt$ at a fixed x)

Particle displacement with q = 1 from the depth z_0 = − 2m,

The wave parameters are $k = 2\pi/600$ m^{-1}, $u_0 = 0.5V$ so that $A/h_1 = -1$. The motility parameter a is 0 (blue), 0.5 (green), and 0.9 (red).

Concentration dynamics of plankton

$$\frac{\partial C}{\partial t} + qu\frac{\partial C}{\partial x} + qw\frac{\partial C}{\partial z} = -\nabla\left(\mathbf{u}_{ps}C\right)$$

$$C = C(z, \xi = x - Vt)$$

$$(qu - V)\frac{\partial C}{\partial \xi} - qz\frac{\partial u}{\partial \xi}\frac{\partial C}{\partial z} = -\left(\frac{\partial u_{ps}C}{\partial \xi} + \frac{\partial w_{ps}C}{\partial z}\right) = S$$

Characteristic system

$$\frac{d\xi}{qu - V} = -\frac{dz}{qzu_{\xi}} = \frac{dC}{S}$$

Passive particles ($u_p = w_p = 0$):

$$\boxed{C = F\left[z(1 - qu(\xi)/V)\right]}$$

Active particles ($u_p = 0,\ w_p = -a\,w$):

$$\boxed{C = (1 - qu(\xi)/V)^{-\alpha} F\left[z(1 - qu(\xi)/V)^{(1-\alpha)}\right]}$$

F is an arbitrary function

Case1: Homogeneous initial distribution

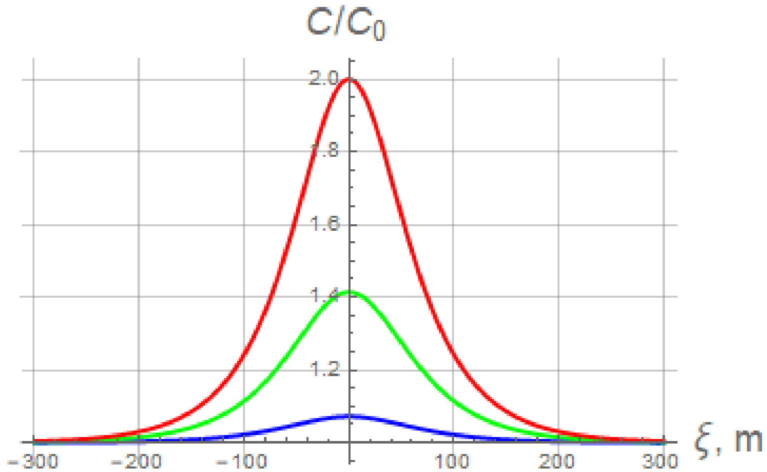
$$C(z, u = 0) = C_0 \text{ , then } F = C_0$$

Passive particles: $C = C_0 = \text{const}$

Active particles:

$$\boxed{C = \frac{C_0}{(1 - qu(\xi)/V)^{\alpha}}}$$

(For small u/V this corresponds to the Lennert-Cody-Franks' result).



The relative plankton concentration for $q = 1$, in the reference frame moving with the wave velocity V.

The wave parameters are $k = 2\pi/600$ m^{-1}, $u_0 = 0.5V$ so that $A/h_1 = -1$. The motility parameter a is 0.1 (blue), 0.5 (green), and 1 (red).

Case 2: A thin plankton layer

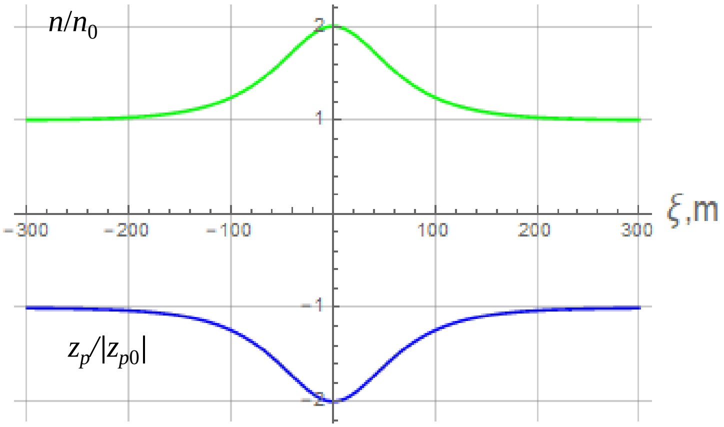
(often observed)

$$C(z, u=0) = n_0 \delta(z - z_0), \text{ then}$$

$$C = \frac{n_0}{(1 - qu(\xi)/V)^{\alpha}} \delta\left[z(1 - qu(\xi)/V)^{(1-\alpha)} - z_0 \right]$$

$$\boxed{n(\xi) = \int_{-\infty}^{\infty} C dz = \frac{n_0}{1 - qu(\xi)/V}}$$

-Both for passive and active particles
Independent of α



Top: the relative number of particles with q = 1 in a thin layer affected by a soliton. The wave parameters are the same as above. Upper curve: relative variation of n. Bottom: variation of a passive particle depth normalized by its initial modulus.

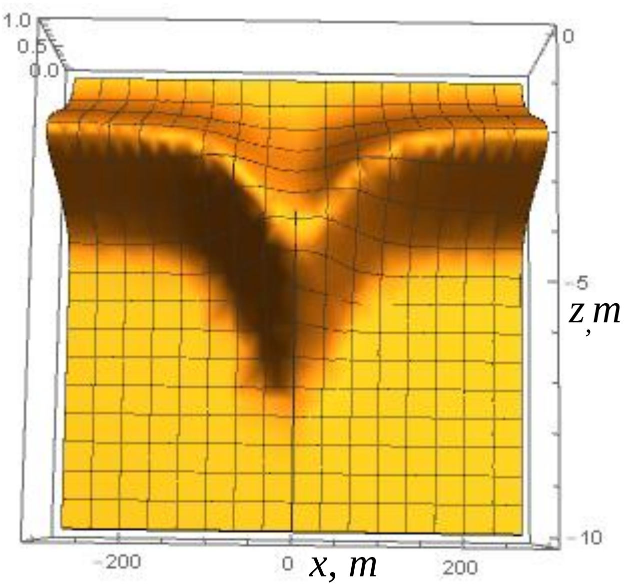
Case 3:Gaussian vertical distribution

$$C(z,u=0)=C_0\exp[-\beta^2(z-z_0)^2]$$

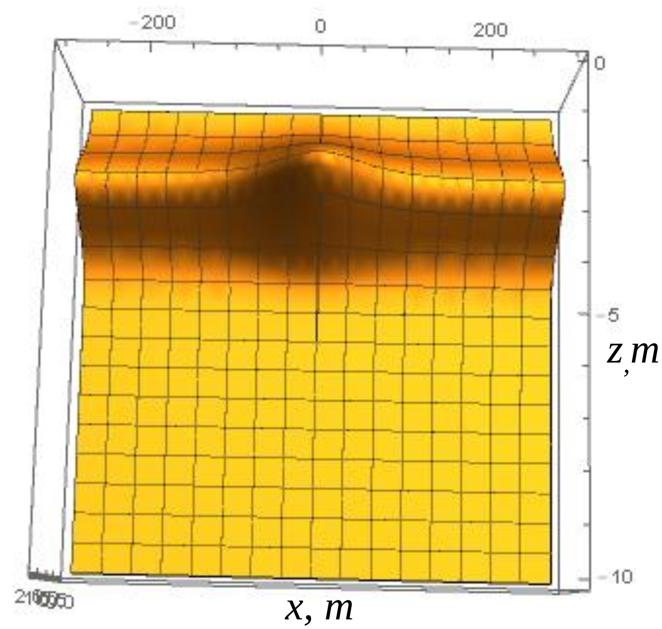
$$C=\frac{C_0}{(1-qu(\xi)/V)^{\alpha}}\exp\left[-\beta^2\left(z(1-qu(\xi)/V)^{(1-\alpha)}-z_0\right)^2\right]$$

$$\boxed{n(\xi)=\int_{\infty}^{\infty}Cdz=\frac{C_0\sqrt{\pi}}{\beta(1-qu(\xi)/V)}}$$

Again, independent of α



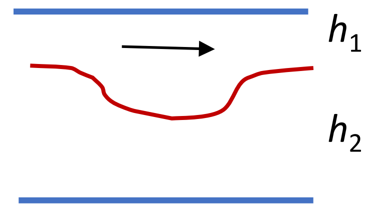
Passive particles
3D plot of C/C_0 for α = 0
(C_{max} = const but n varies)



Depth-keeping particles
3D plot of C/C_0 for α =1
(Both C_{max} and n vary)

Passive particles in a strong soliton, 2-layer fluid

W. Choi and R. Camassa, J. Fluid Mech.,396, 1, 1999
L. Ostrovsky and J. Grue, J. Fluid Mech., 15, 2934-48, 2003

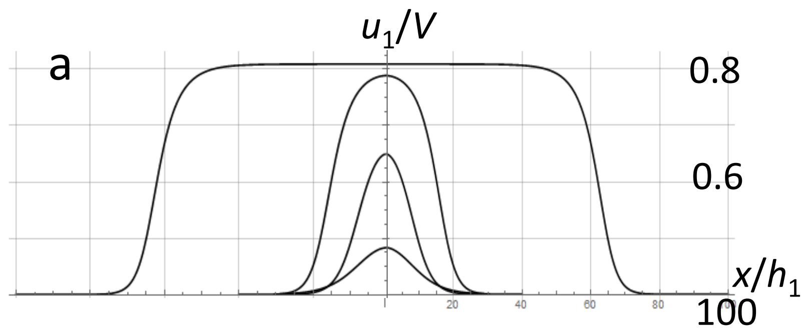


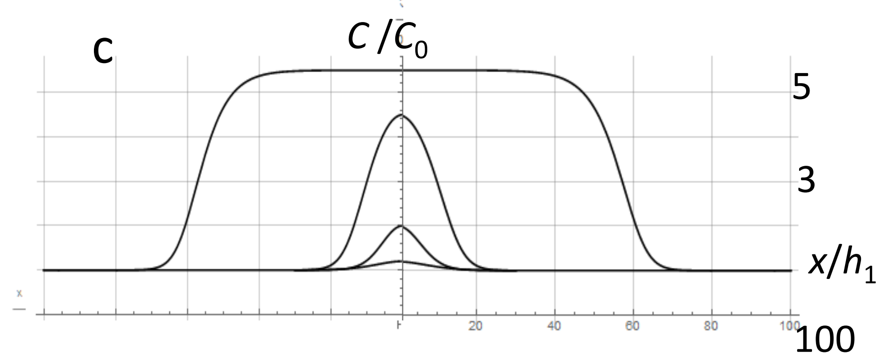
Profile of a strong soliton

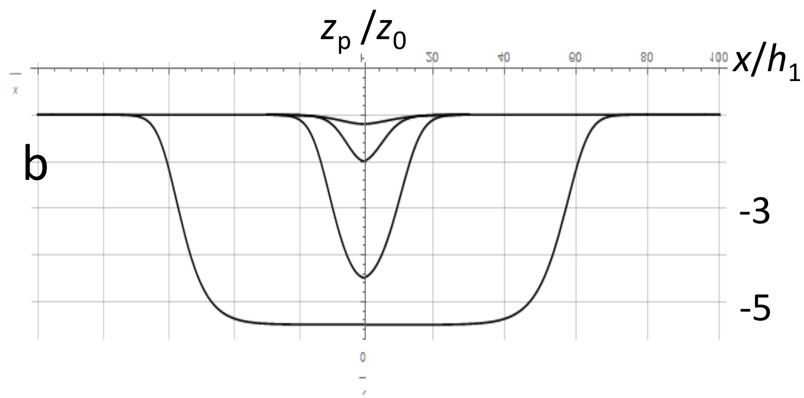
$$\frac{ds}{dX} = -s\sqrt{\frac{3(1+q)[1-(1+s)(1-qs)/U^2]}{(1-qs)+(1+s)/q}}$$

$$s = \eta / h_1, u_1 / V = s / /(1+s), q = h_1 / h_2, X = \xi / h_1, a = A / h_1,$$

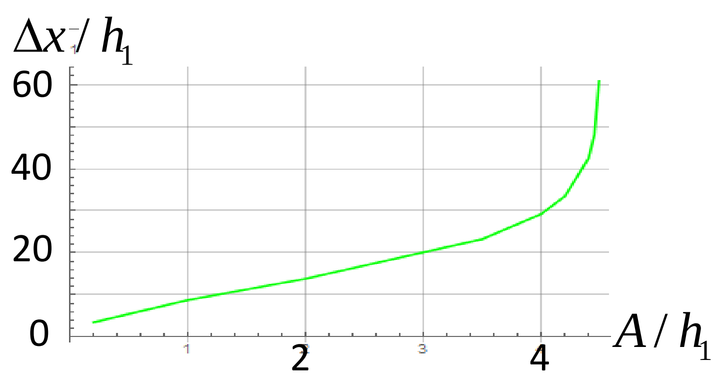
$$u_1 = V\eta / (h_1 + \eta), U = V / c_0, U^2 = (1+a)/(1-qa), A_{max} \approx (h_2 - h_1)/2$$







a) Horiz. velocity in the upper layer. b) Particle displacement. c) Relative number of thin-layer particles in a soliton. For three soliton amplitudes. $h_1/h_2 = 0.1$.



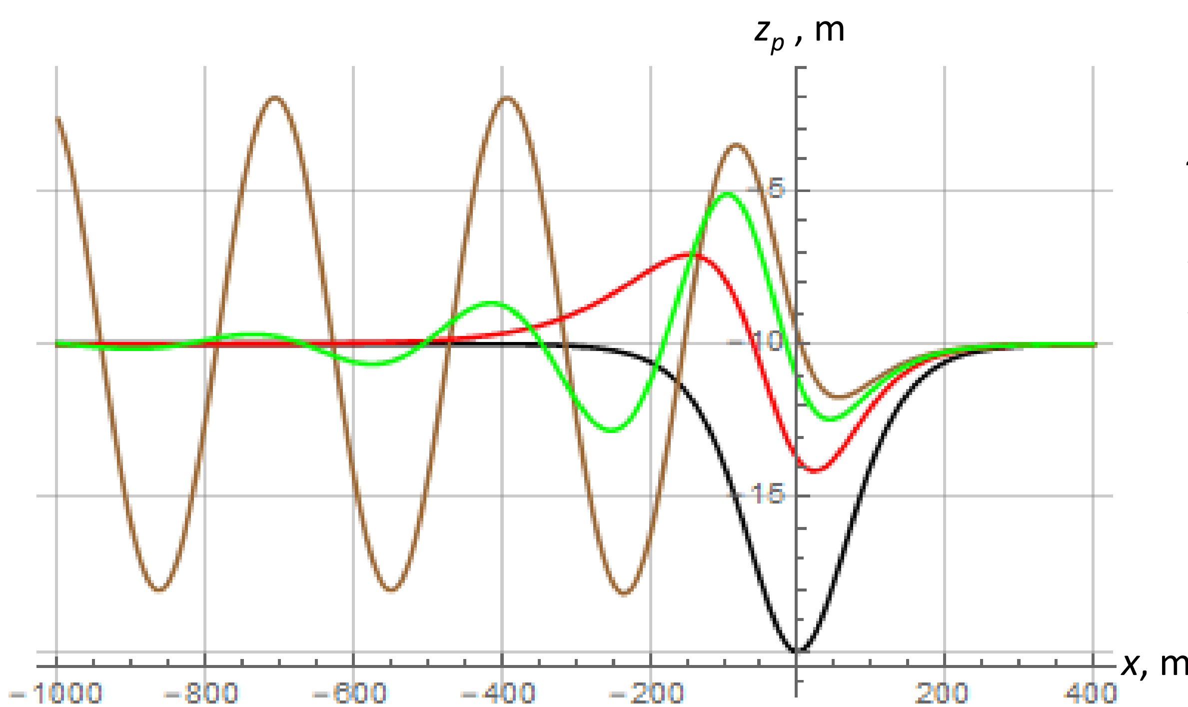
Horiz. drift vs. soliton amplitude

Effect of buoyancy

Weak nonlinearity: $\eta = Af(z)\,\mathrm{sech}^2(k\xi)$

$$\frac{d^2 z_p}{d\xi^2} = -\frac{(q-\alpha)w_\xi}{V} + \frac{9\nu}{2a^2V^2}\left(V\frac{dz_p}{d\xi} + w\right) + \frac{N^2}{V^2}(z_0 - z_p)$$

$$w = \eta_t = -V\eta_\xi = -2VkAf(z)\,\mathrm{sech}^2(k\xi)\tanh(k\xi)$$



Particle displacement from the depth z_0 = – 10m, The wave parameters are $k = 2\pi / 600\ \mathrm{m}^{-1}$, A = 7 m, V = 1 m/s, $F(z_0)$ = 1, $q = 1$, a = 0, $\nu = 10^{-6}$ m^2/s , N = 0.02 rad/s
Brown: ν = 0
Black: N = 0, R = 1 mm
Red: N = 0.02 rad/s, R = 1 cm
Green: N = 0.02 rad/s, R = 2.2 cm

Conclusions

- The motion of a particle in an internal wave depends on its size (Reynolds number) and for zooplankton, on its motility.
- In a stratified layer, larger particles oscillate after soliton passing.
- Concentration of particles varies differently for their initially homogeneous distribution and for a thin layer which is common in the upper ocean.
- For a thin layer, the number per unit length is the same for passive and active plankters.
- Turbulent diffusion can be significant for a thin plankton layer.
- **For details, see L.A. Ostrovsky, Wave Motion, 114 (2022) 103013**