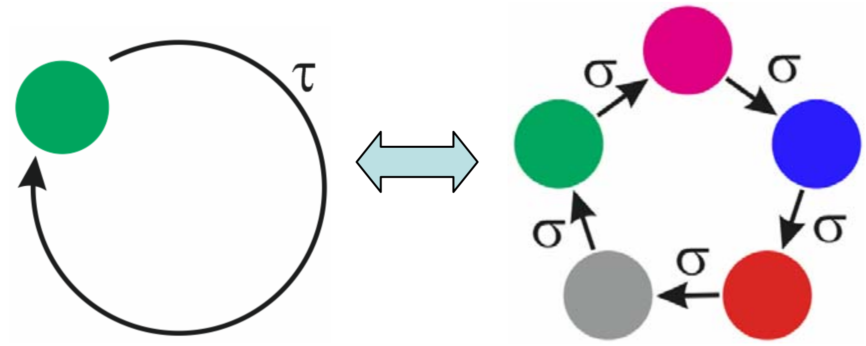

О соотношении динамики автогенератора с запаздывающей обратной связью и кольца связанных автогенераторов

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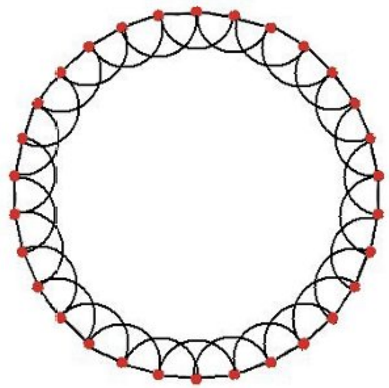
XXVI Научная сессия Совета РАН по нелинейной динамике 18.12.2017 Москва

Chimera states in networks

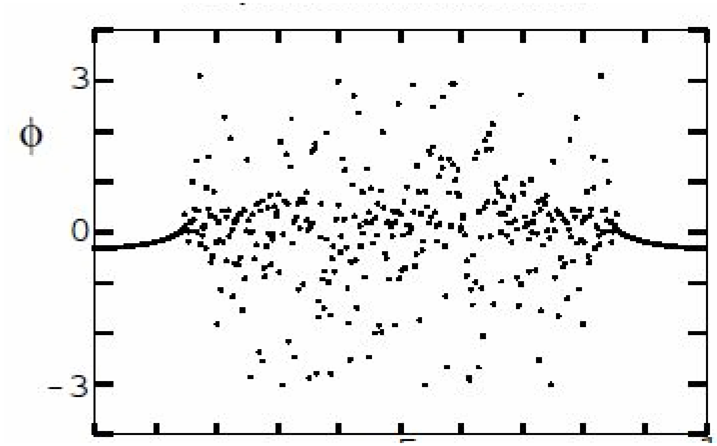
Kuramoto and Battogtohk, 2002
Abrams and Strogatz, Phys. Rev. Lett 2004

$$\frac{\partial \phi}{\partial t} = \omega - \int_{-\pi}^{\pi} G(x - x') \sin[\phi(x, t) - \phi(x', t) + \alpha]dx'$$

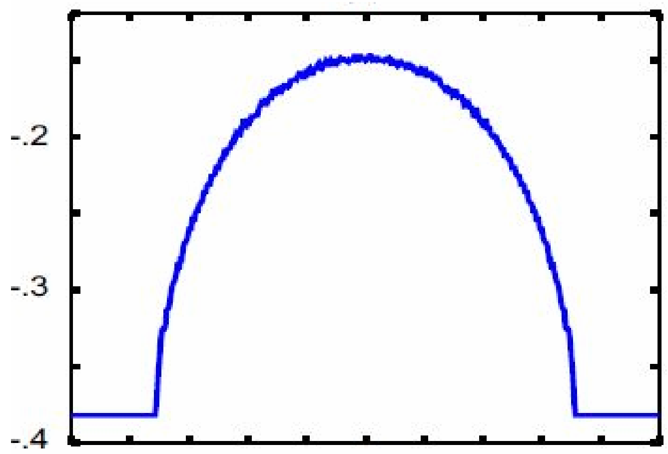


Network $\Rightarrow$ coherent and incoherent domains

Instant phases



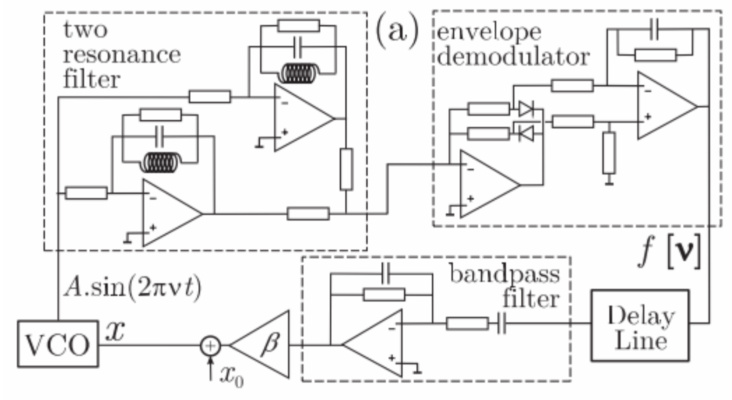
Average frequencies



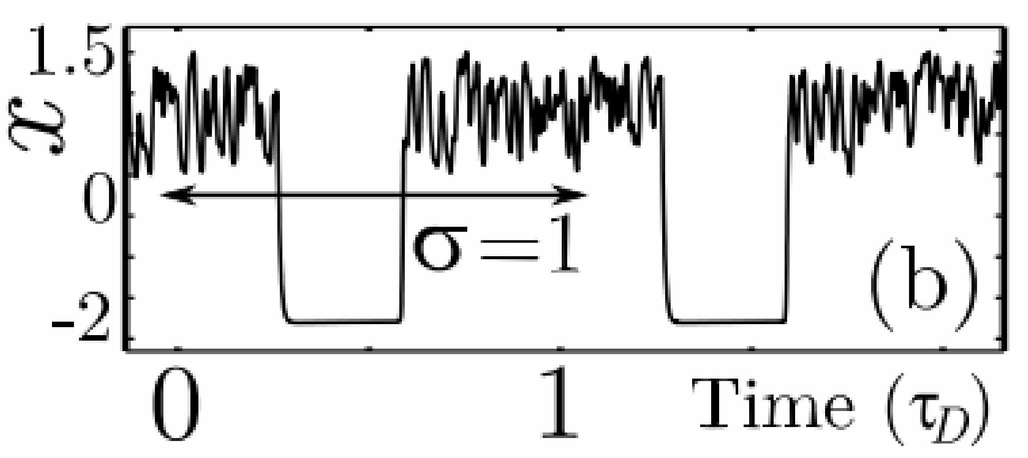
(Virtual) chimera states

Larger et al, Phys. Rev. Lett. 2013

$$\varepsilon x' = -\delta y - x + \beta f[x(s-1)]$$
$$y' = x$$



Coherent and incoherent phases



Space-time representation of a delayed dynamical system

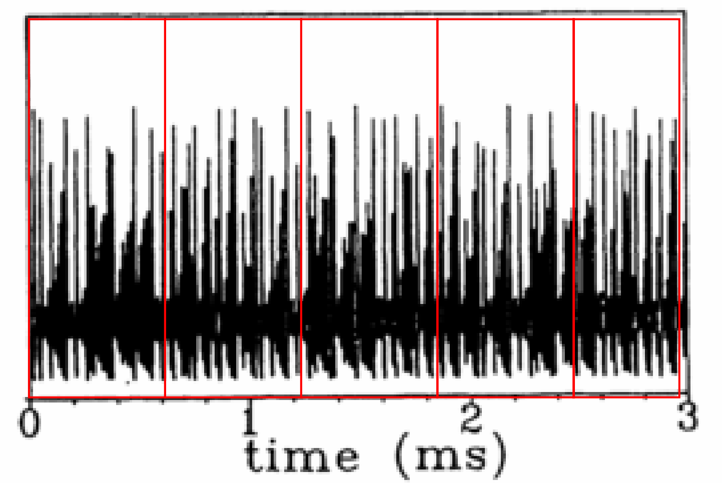
Arecchi et al, Phys. Rev. A 1992
Giacomelli et al, Phys. Rev. Lett. 1994
Giacomelli and Politi, Phys. Rev. Lett. 1996

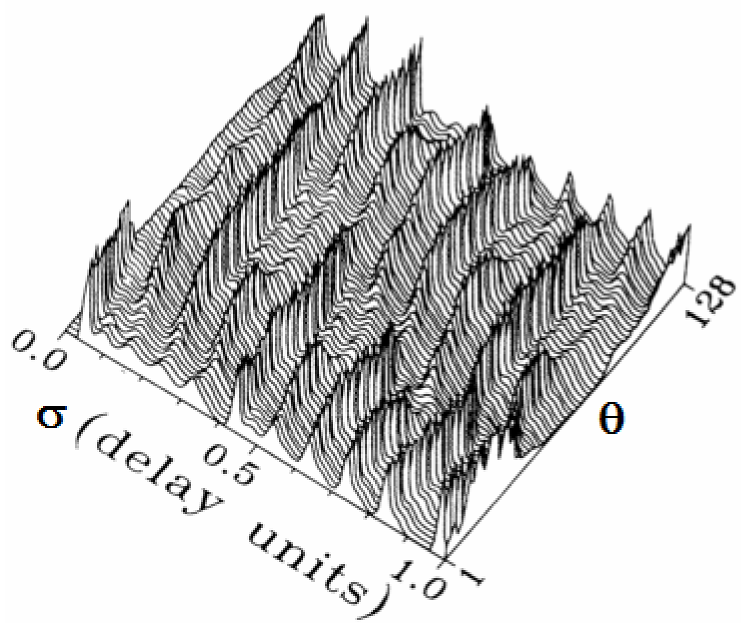
$$\dot{x}(t) = F(x(t), x(t-\tau))$$

$$t = \sigma + \theta T$$

$\sigma \in [0, T]$ - (pseudo) space

$\theta \in \mathcal{N}$ - (slow) time



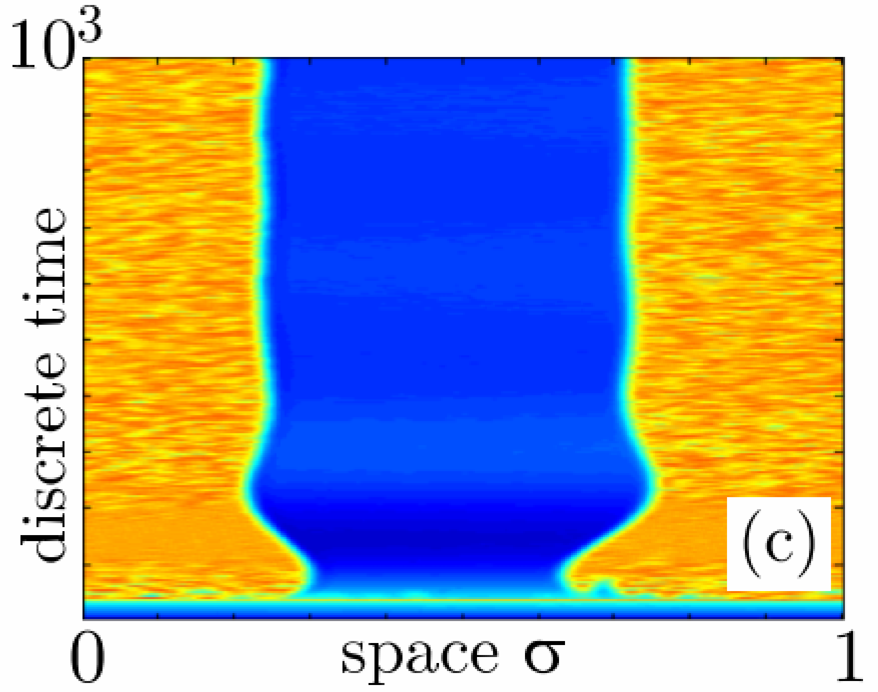


(Virtual) chimera states

Larger et al, Phys. Rev. Lett. 2013

$$\varepsilon x' = -\delta y - x + \beta f[x(s-1)], \qquad y' = x$$

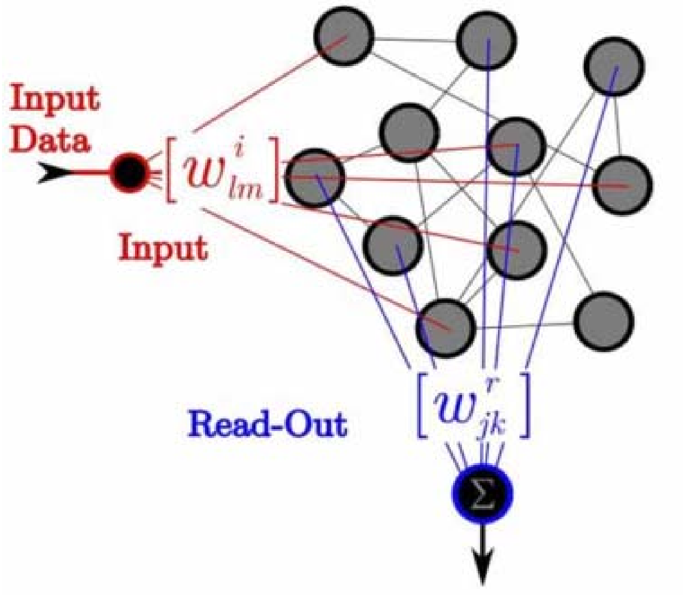
Coexistence of “coherent” and “incoherent” domains



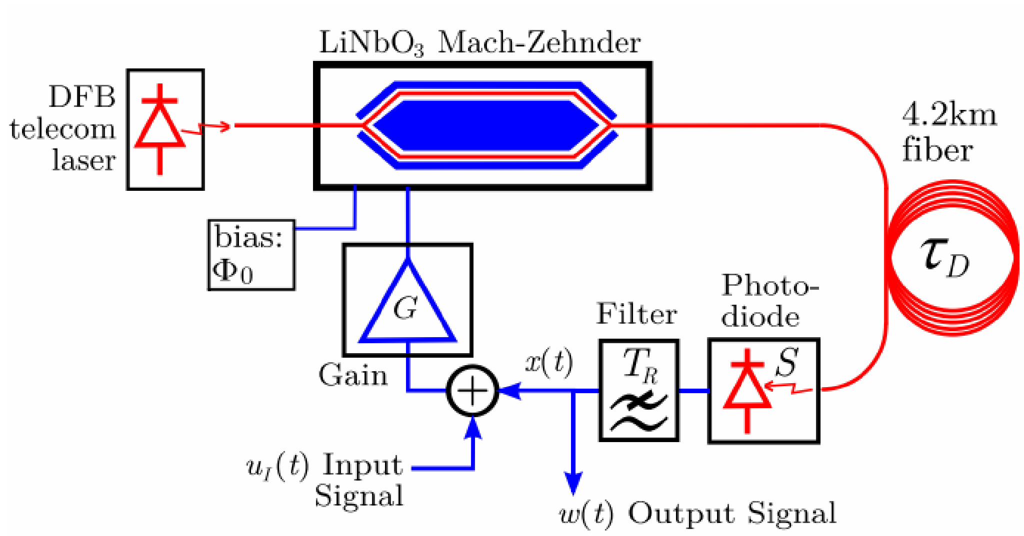
Reservoir computing

Larger et al, Opt. Express 2012

Complex network of nonlinear nodes

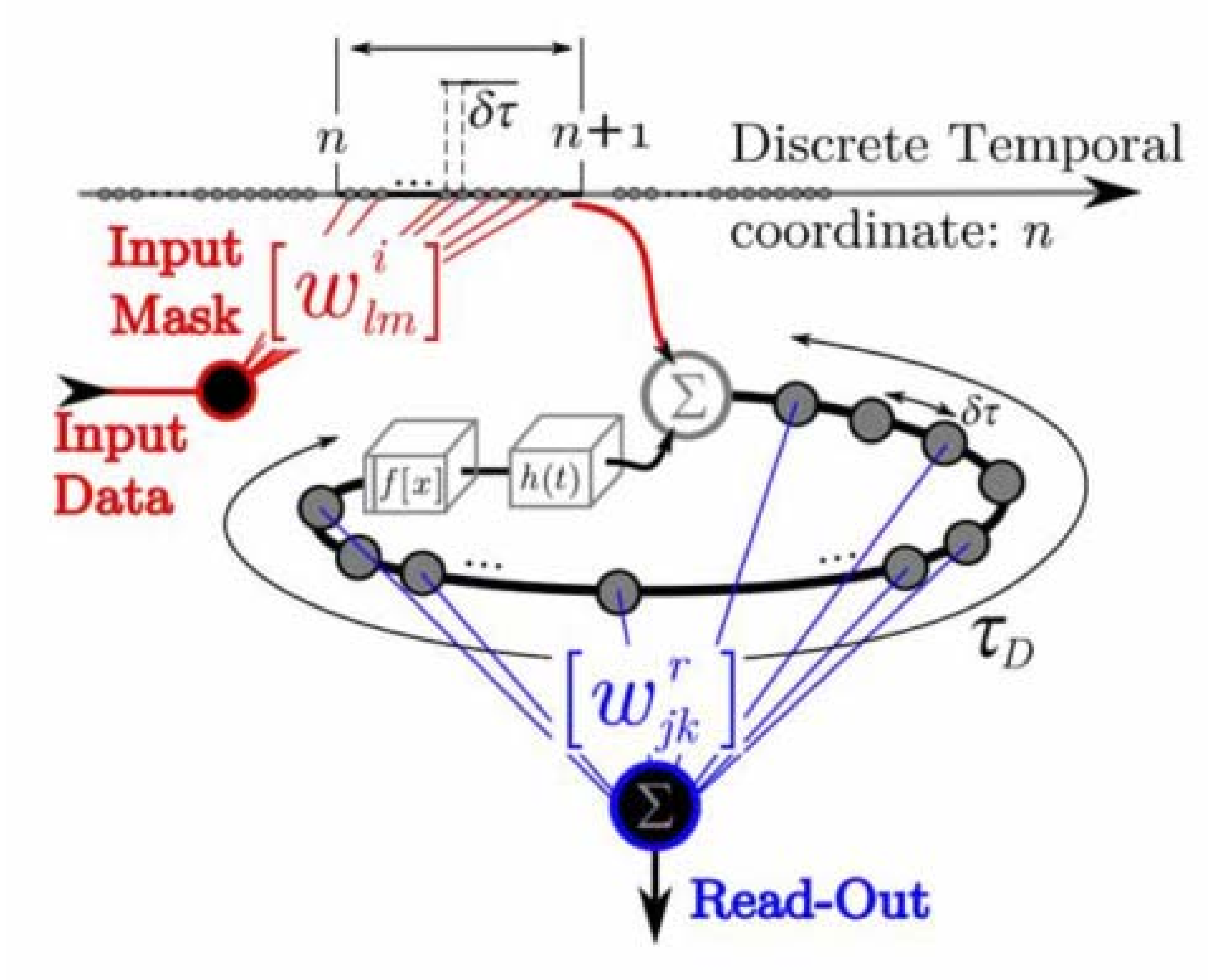


Optoelectronic element + delayed feedback



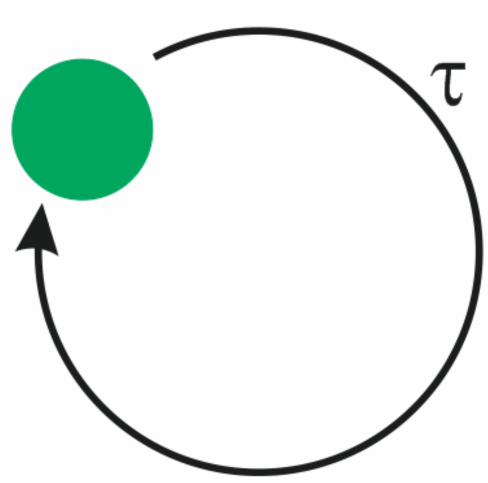
Reservoir computing

Larger et al, Opt. Express 2012

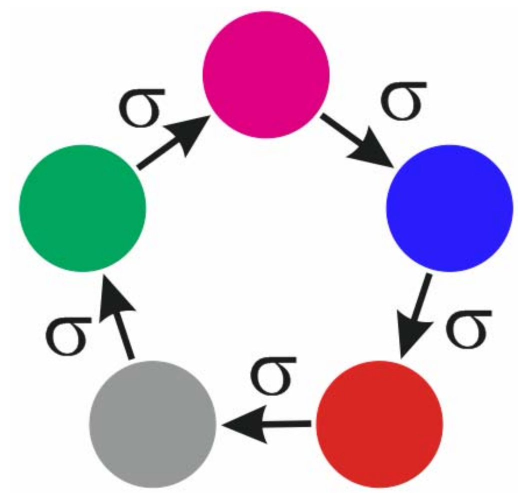


Single oscillator vs. ring of oscillators

Single oscillator with
delayed feedback



Ring of oscillators with
feed-forward delayed coupling



$$\frac{dx(t)}{dt} = f\left(x(t), x(t-\tau)\right)$$

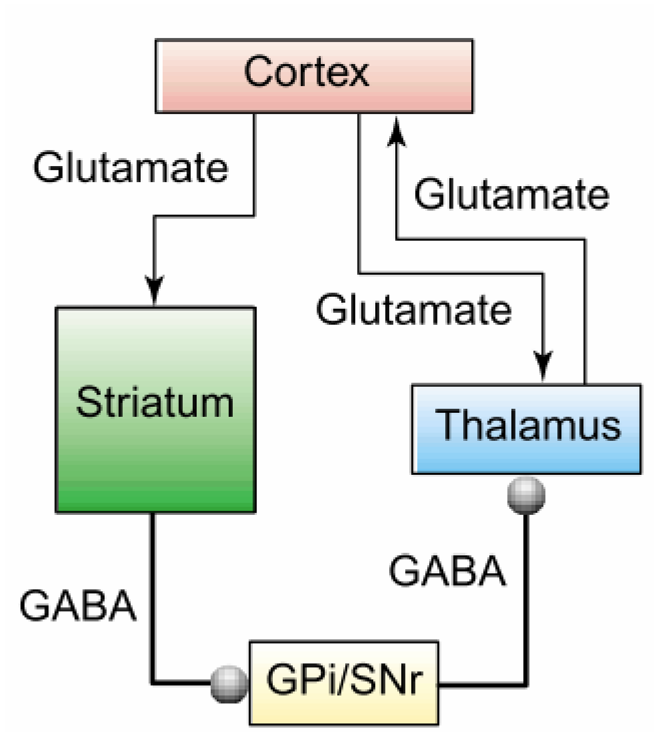
$$\frac{dx_j(t)}{dt} = f\left(x_j(t), x_{j-1}(t-\sigma)\right)$$

Rings (loops) in neuroscience

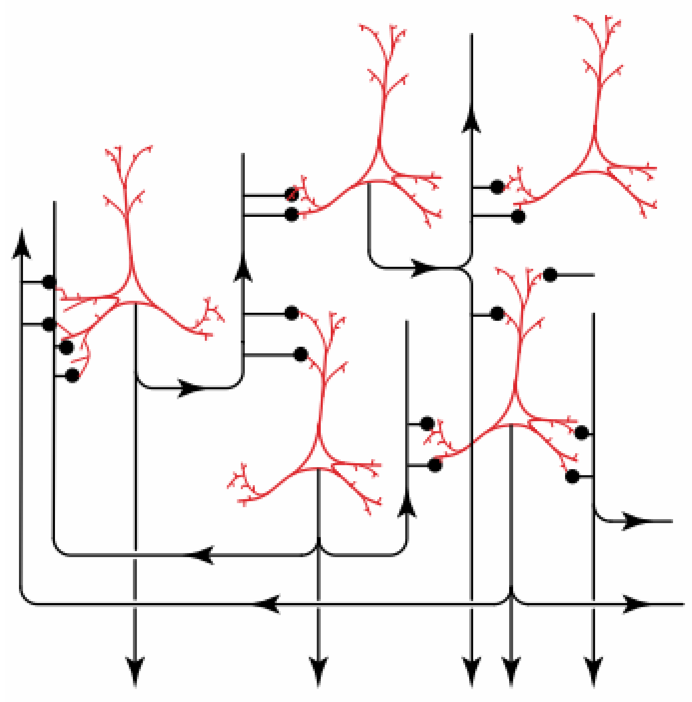
Wang, TRENDS in Neurosciences 2001

Reverberation as potential mechanism for working memory

Thalamocortical loop (and others)



Local cortical recurrent loops

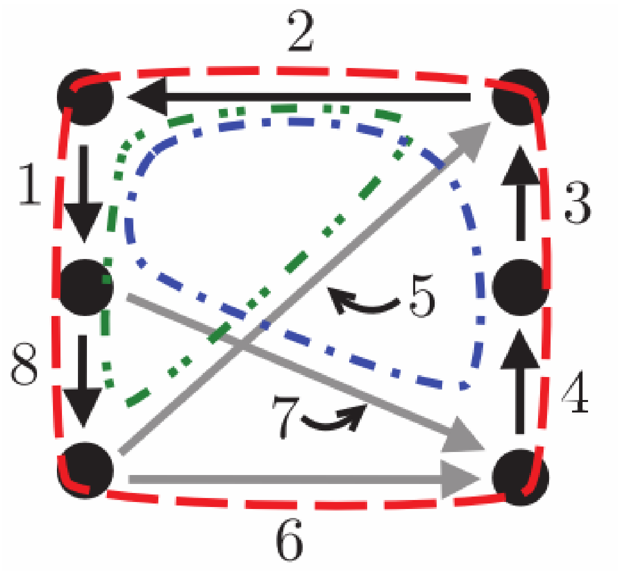


Ring motifs in complex networks

Sums of delays along fundamental semicycles matter:

Reduction of delays

Lücken et al, EPL 2013



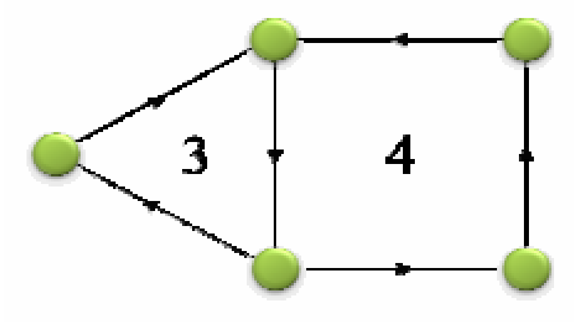
$T1 = \tau1+\tau8+\tau6+\tau4+\tau3+\tau2$

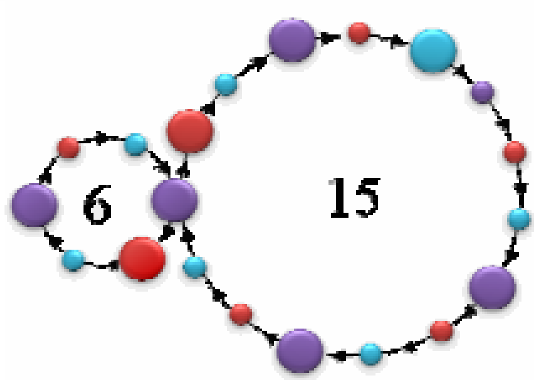
$T2 = \tau1+\tau7+\tau4+\tau3+\tau2$

$T3 = \tau1+\tau8+\tau5+\tau2$

Chaotic synchronization

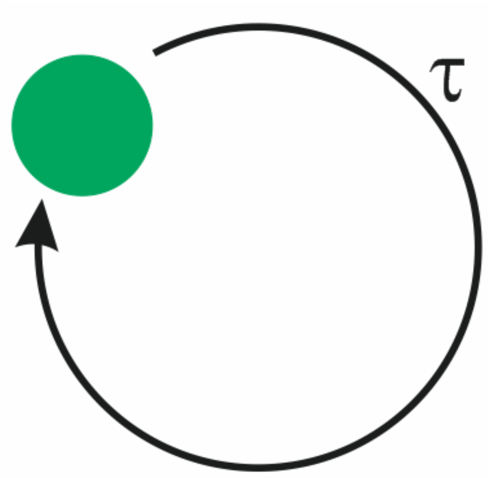
Kanter et al, EPL 2011





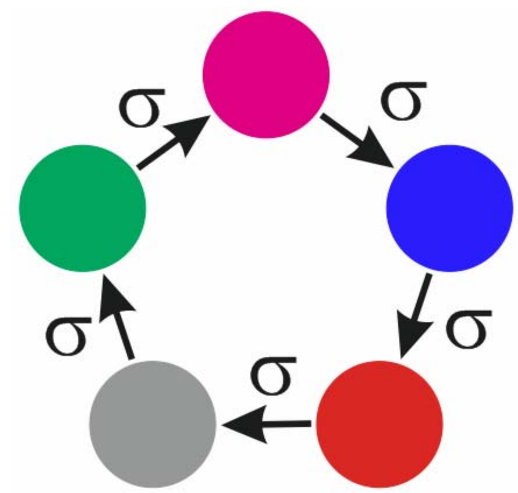
Single oscillator vs. ring of oscillators

Single oscillator with
delayed feedback



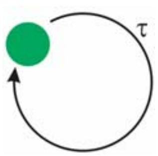
$$\frac{dx(t)}{dt} = f\left(x(t), x(t-\tau)\right)$$

Ring of oscillators with
feed-forward delayed coupling

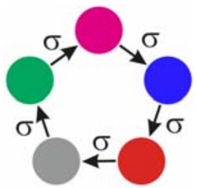


$$\frac{dx_n(t)}{dt} = f\left(x_n(t), x_{n-1}(t-\sigma)\right)$$

Single oscillator vs. ring of oscillators



$$\frac{dx(t)}{dt} = f\left(x(t), x(t-\tau)\right)$$ **(1) = SINGLE**



$$\frac{dx_n(t)}{dt} = f\left(x_n(t), x_{n-1}(t-\sigma)\right)$$ **(2) = RING**

$x(t) = h(t)$ is a T-periodic solution of **(1)** for $\tau = \tau_0$

then $x(t) = h(t)$ also solves **(1)** for $\tau = \tau_0 + kT$

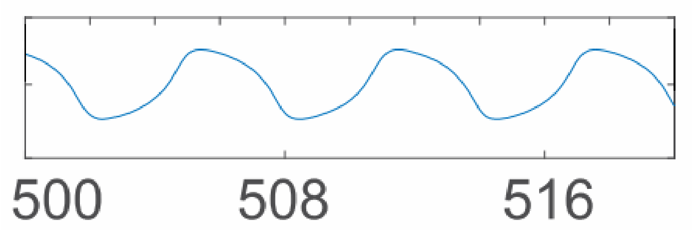
moreover, $x_n(t) = h(t + n\theta)$ is a solution of **(2)** for

$\theta = MT/N$ and $\sigma = \tau_0 + kT - \theta$, where $M = 0, 1, \ldots N\text{-}1$ is wave-number

Example: Van-der-Pol oscillator

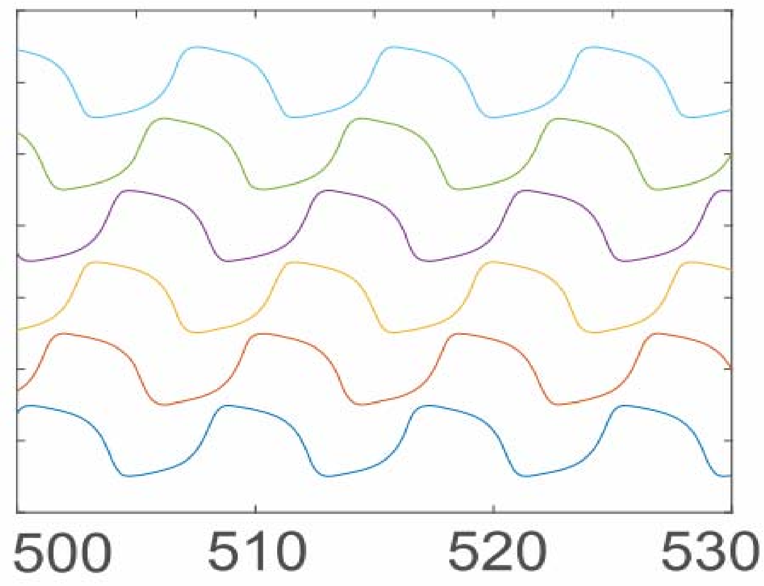
$$\frac{d^2x(t)}{dt^2} = \alpha[1 - x(t)^2]\frac{dx(t)}{dt} - x(t) + \kappa x(t-\tau)$$

Single oscillator



Periodic solution

Ring of oscillators



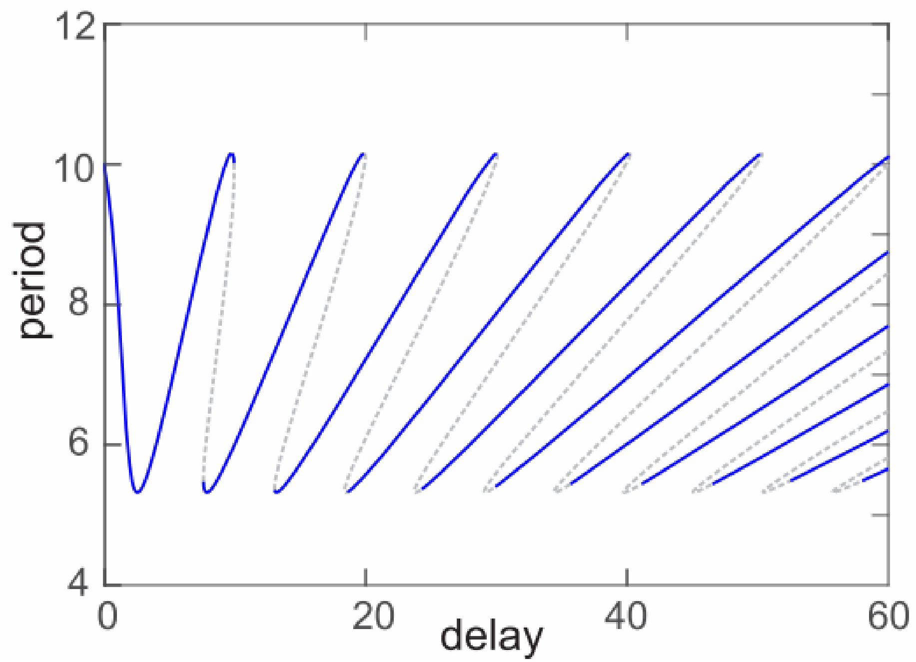
* Periodic solutions of single oscillator $\Rightarrow$ Rotating waves in rings

Example: Van-der-Pol oscillator

$$\frac{d^2x(t)}{dt^2} = \alpha[1 - x(t)^2]\frac{dx(t)}{dt} - x(t) + \kappa x(t-\tau)$$

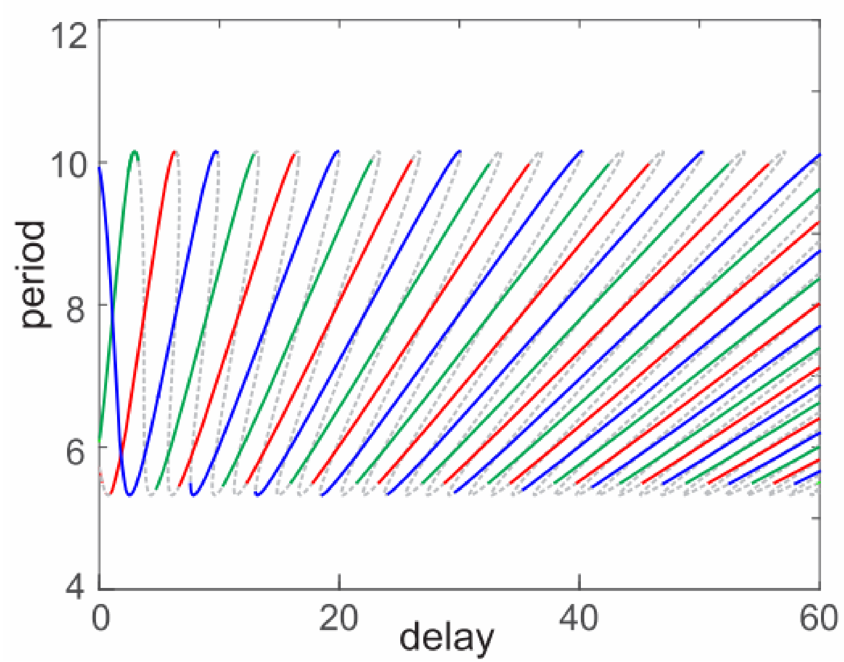
Single oscillator

$\tau_0 \Rightarrow \tau = \tau_0 + kT$



Ring of oscillators

$\tau_0 \Rightarrow \sigma = \tau_0 + (k - M/N)\,T$



Stability analysis

Single oscillator

$$\frac{d\delta(t)}{dt} = A(t)\delta(t) + B(t)\delta(t-\tau)$$

$$A(t) = \partial_1 f[h(t), h(t-\tau)]$$

$$B(t) = \partial_2 f[h(t), h(t-\tau)]$$

$$\delta(t) = p(t)e^{\lambda t}$$ - Floquet ansatz

$$\frac{dp(t)}{dt} = [A(t) - \lambda \mathrm{Id}]p(t) + e^{-\lambda\tau}B(t)p(t-\tau)$$

Stability analysis

Ring of oscillators

$$\frac{d\delta_n(t)}{dt} = \partial_1 f(h(t+n\theta), h(t+(n-1)\theta-\sigma))\delta_n(t) + \partial_2 f(h(t+n\theta), h(t+(n-1)\theta-\sigma))\delta_{n-1}(t-\sigma)$$

$\delta_n(t) = r_n(t)e^{\lambda t}$ - Floquet ansatz

$$\frac{dr_n(t)}{dt} = [A(t+n\theta) - \lambda \mathrm{Id}]r_n(t) + e^{-\lambda\sigma}B(t+n\theta)r_{n-1}(t-\sigma)$$

$p_n(t) = r_n(t-n\theta)$ - time shift

$p_n(t) = \sum_{m=1}^{N} \hat{p}_m(t)e^{2\pi imn/N}$ - Fourier transformation

$$\frac{d\hat{p}_m(t)}{dt} = [A(t) - \lambda_m \mathrm{Id}]\hat{p}_m(t) + e^{-\lambda_m\sigma - i\psi_m}B(t)\hat{p}_m(t-\tau)$$

Stability analysis

Ring of oscillators

$$\frac{d\hat{p}_m(t)}{dt} = [A(t) - \lambda_m \mathrm{Id}]\hat{p}_m(t) + e^{-\lambda_m \sigma - i\psi_m} B(t)\hat{p}_m(t-\tau)$$

$$\tau = \sigma + \theta$$

Single oscillator

$$\frac{dp(t)}{dt} = [A(t) - \lambda \mathrm{Id}]p(t) + e^{-\lambda\tau} B(t)p(t-\tau)$$

Characteristic eq.

$$F(\lambda, e^{-\lambda\tau}) = 0$$

Yanchuk and Perlikowski, Phys. Rev. E 2009

Spectrum for large delays

Characteristic eq.

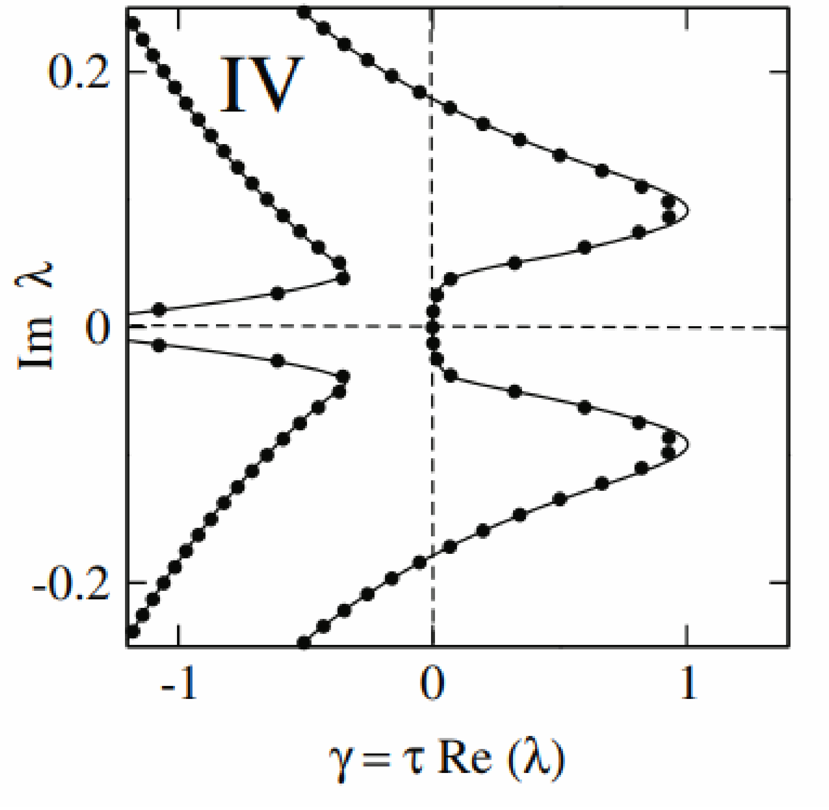
Single oscillator

$$F(\lambda, e^{-\lambda\tau}) = 0$$

"Weak" spectrum

$$\lambda = i\omega + \frac{\gamma}{\tau}$$

$$\omega = \omega(\gamma)$$



Spectrum for large delays

Yanchuk and Perlikowski, Phys. Rev. E 2009

Characteristic eq.

Single oscillator

$$F(\lambda, e^{-\lambda\tau}) = 0$$

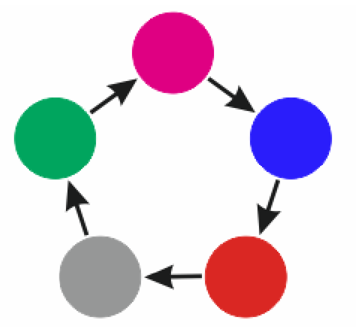
Ring of oscillators

$$F(\lambda, e^{-\lambda\sigma - i\psi_m}) = 0$$

The same "weak" spectrum

* **Stability of periodic solutions of single oscillator and rotating waves in ring (with all wave-numbers) is the same for large delays**

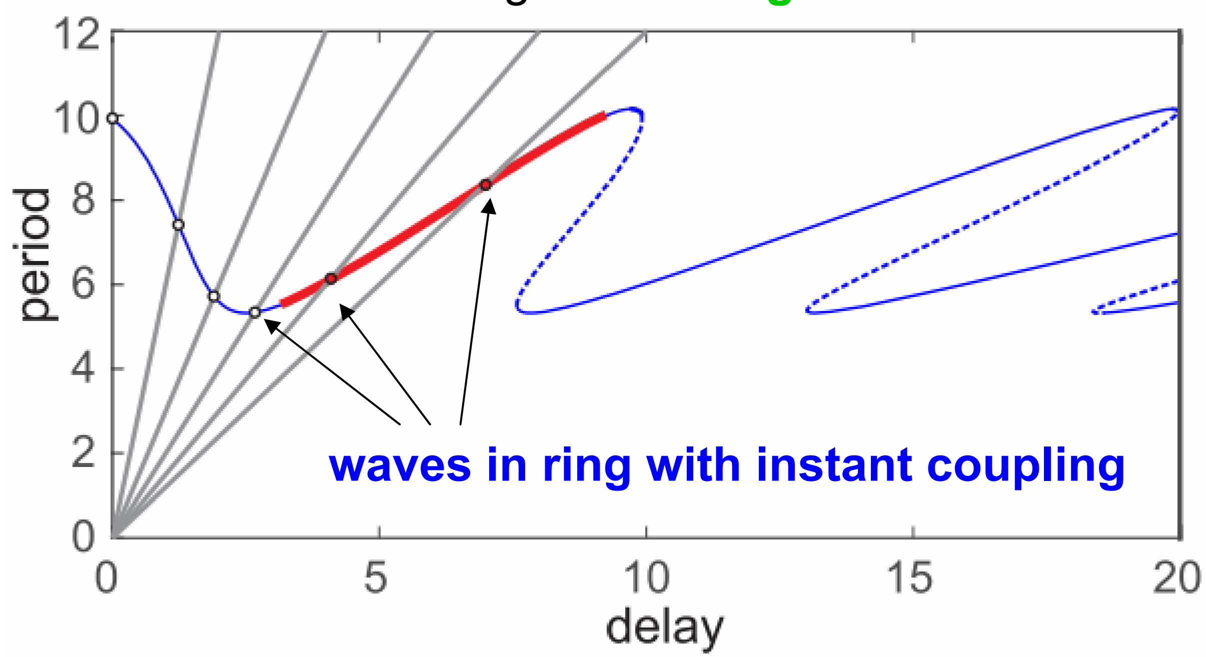
Ring with instant coupling



$\tau \Rightarrow \sigma = \tau - M/N\,T$

$\sigma = 0 : \tau = M/N\,T$ - delay and period are resonant

Bifurcation diagram for **single oscillator**



Stability analysis

Characteristic eq.

Single oscillator

$$F(\lambda, e^{-\lambda\tau}) = 0$$

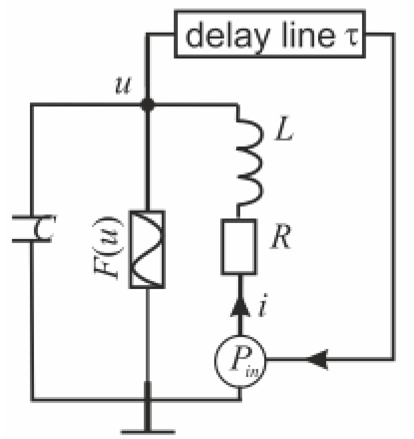
Ring of oscillators with instant coupling

$$F(\lambda, e^{-i\psi_m}) = 0$$

* **Stability of periodic solutions of single oscillator for large delays is sufficient for stability of rotating waves in ring with instant coupling**

* **For large number of oscillators in ring N >> 1 it is also a necessary condition**

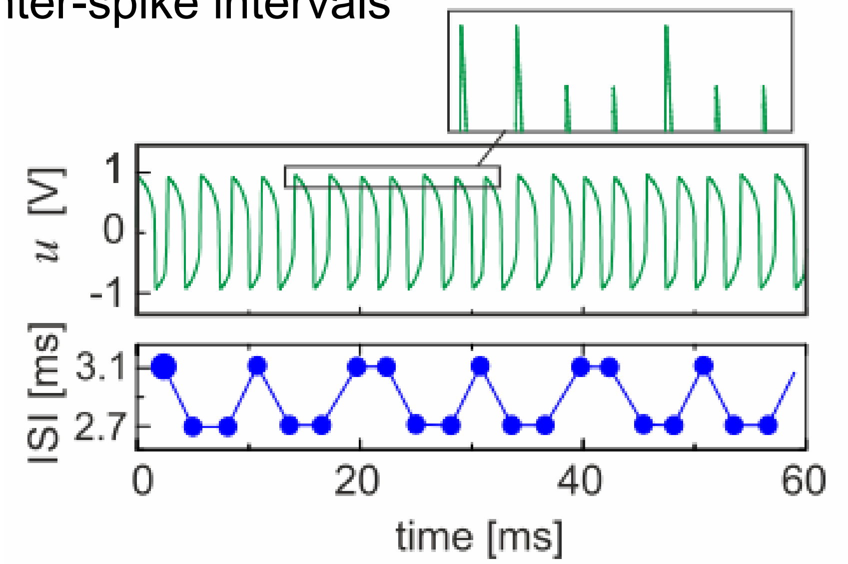
Example: multi-jittering

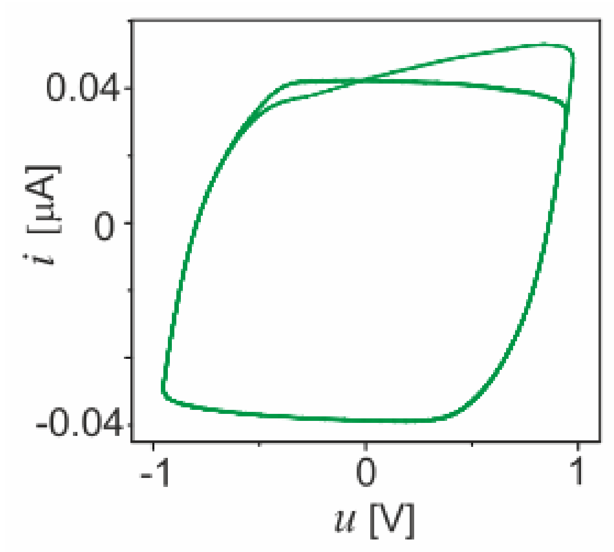


Single oscillator
with pulse delayed feedback

$$\frac{d\varphi}{dt} = 1 + Z(\varphi)\sum_{t_p} \delta(t - t_p - \tau)$$

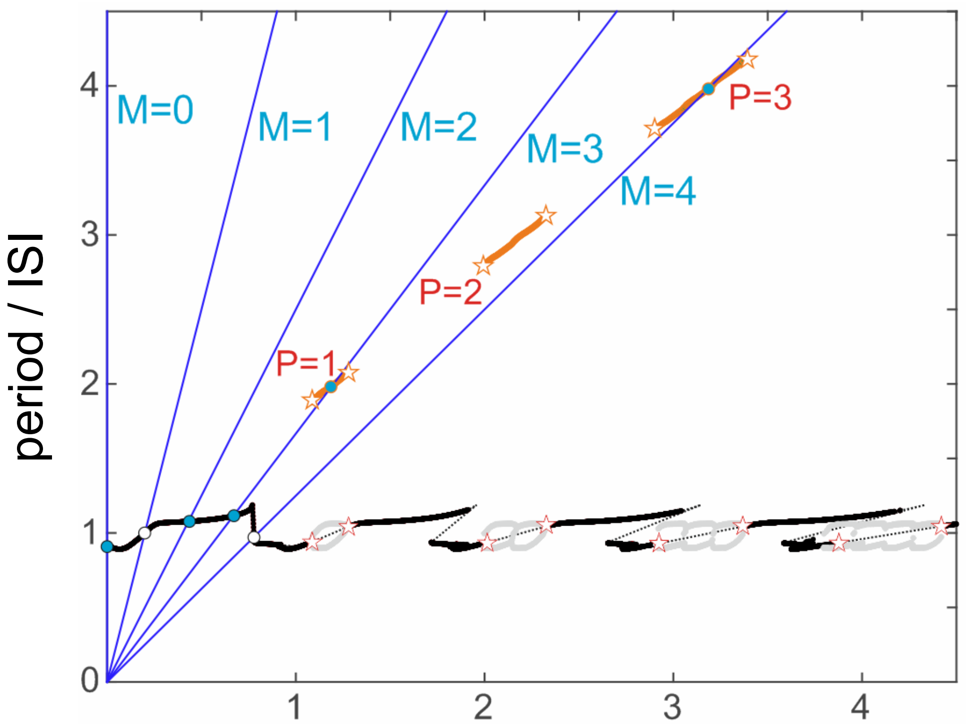
"Jittering" regimes with distinct inter-spike intervals





Multi-jittering waves

Bifurcation diagram for **single oscillator**

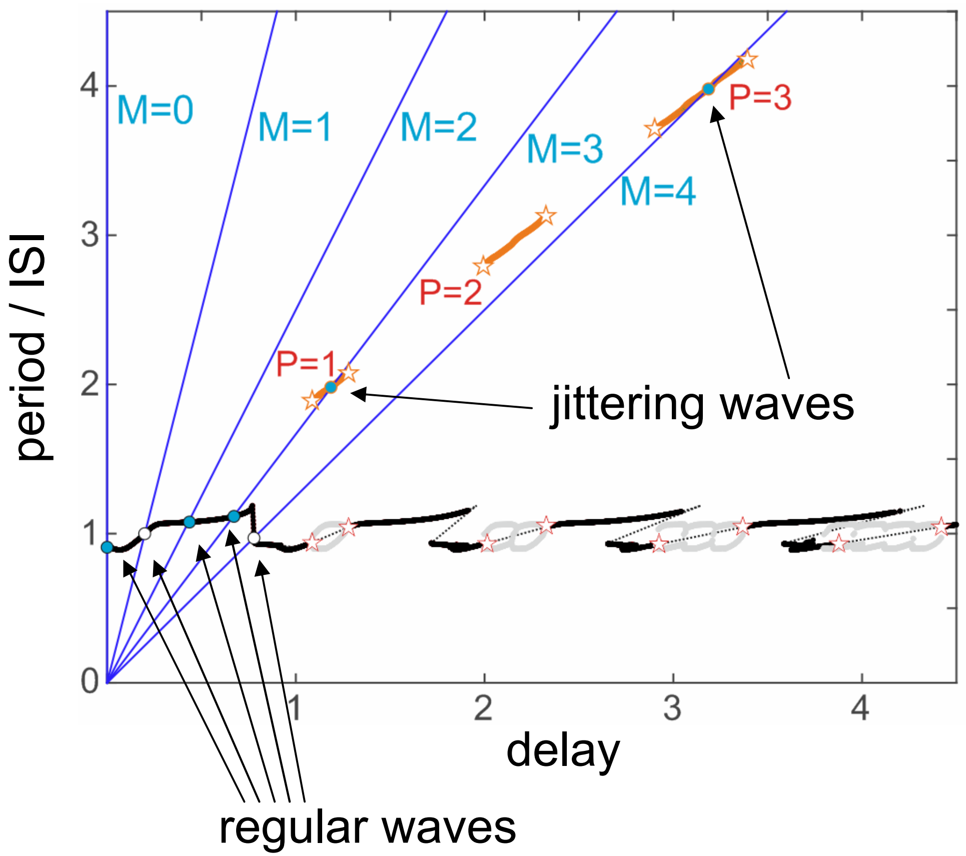


regular regimes

jittering regimes

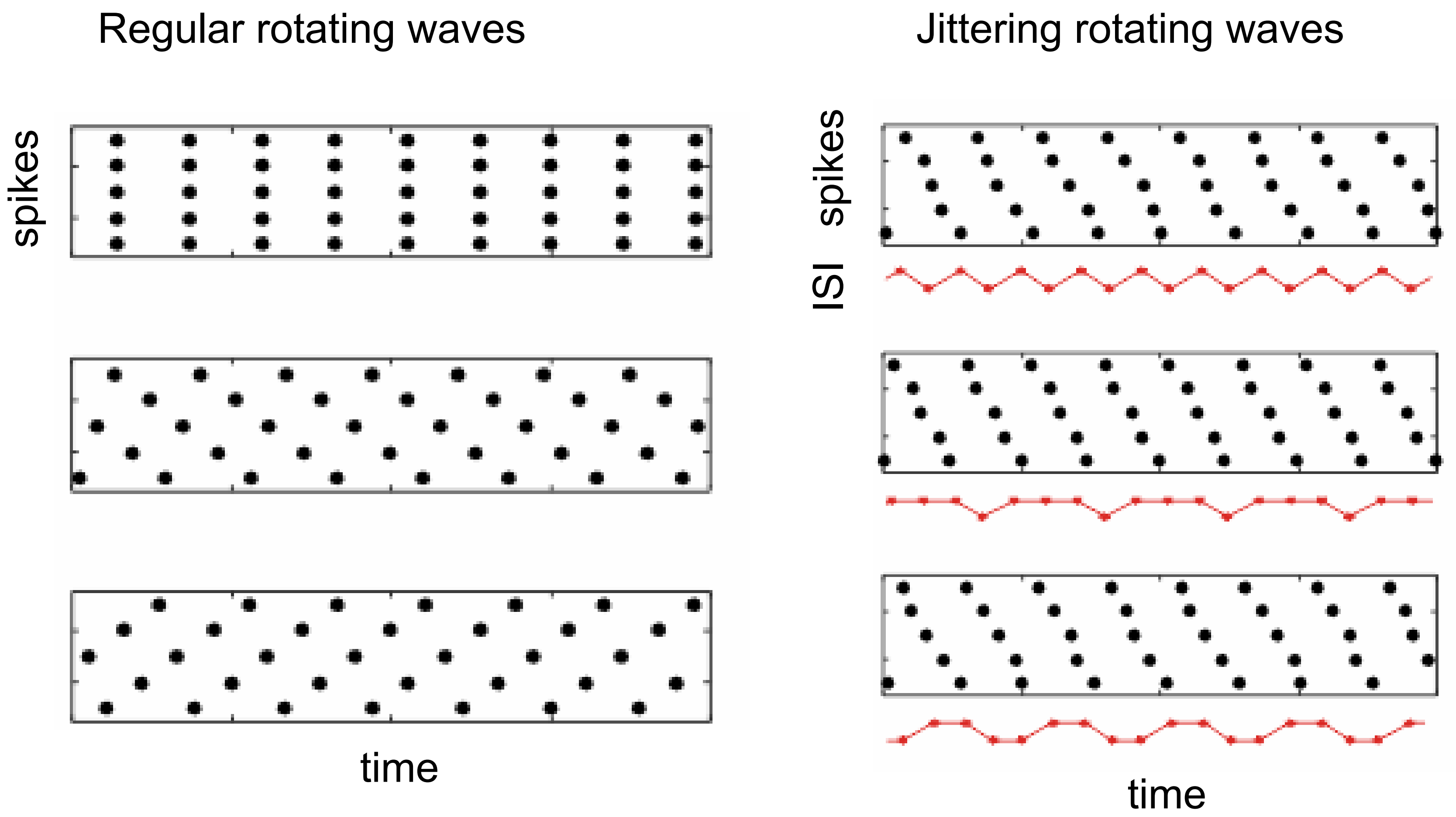
Jittering rotating waves

Bifurcation diagram for **single oscillator**



regular regimes
jittering regimes

Jittering rotating waves



Conclusions

* **Periodic solutions of single oscillator** $\Rightarrow$ **Rotating waves in rings**

$$\tau_0 \Rightarrow \sigma = \tau_0 + (k - M/N)\, T$$

* **Stability of periodic solutions of single oscillator and rotating waves in ring (with all wave-numbers) is the same for large delays**

* **Stability of periodic solutions of single oscillator for large delays is sufficient for stability of rotating waves in ring with instant coupling**

* **For large number of oscillators in ring N >> 1 it is also a necessary condition**

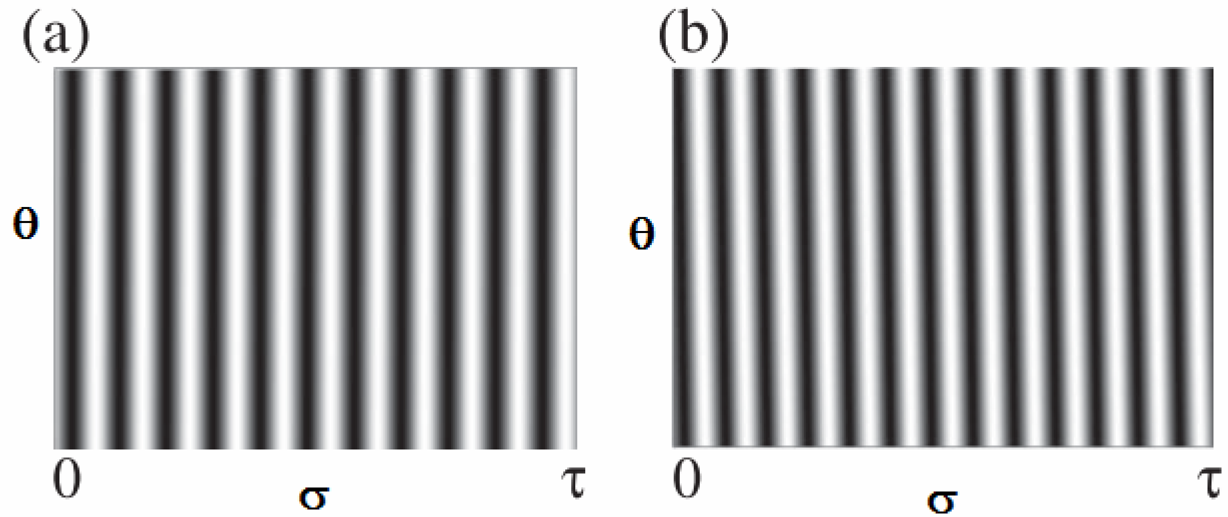
Klinshov, V., Shchapin, D., Yanchuk, S., Wolfrum, M., D'Huys, O., Nekorkin, V. Embedding the dynamics of a single delay system into a feed-forward ring. *Physical Review E*, *96*, 42217 (2017)

Eckhaus instability

Wolfrum and Yanchuk, Phys. Rev. Lett 2006

$$z' = (\alpha + i\beta)z - z|z|^2 + z_\tau$$

Multiplicity of coexisting periodic attractors



Complex patterns

Giacomelli and Politi, Phys. Rev. Lett. 1996

$$\dot{y} = \mu y - (1 + i\beta)|y|^2 y + \eta y_d$$