Эффект несохранения топологического заряда в коаксиальном лазере

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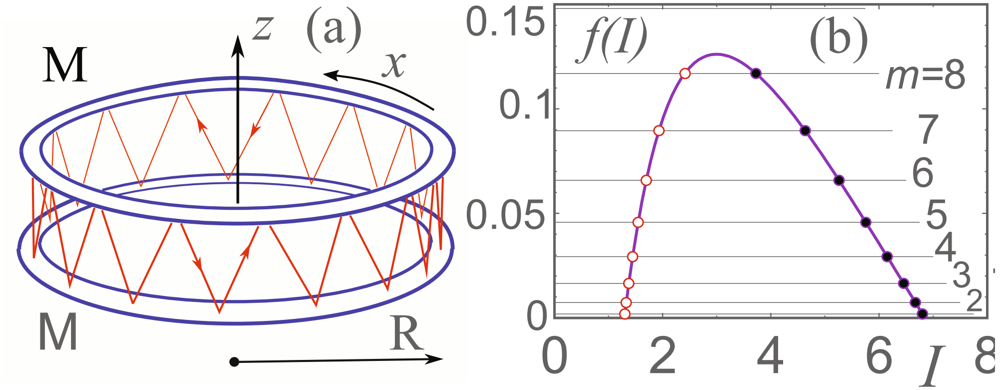
Показана ограниченность топологической защищенности характеристик структур лазерного излучения и выявлены механизмы изменения топологического заряда в коаксиальном лазере.

XXIX Научная сессия Совета РАН по нелинейной динамике. 14-15 декабря 2020 г.

The scheme and governing equation



- Not standard ring laser;
- Not a whispering gallery mode.

1D-coaxial laser. Not simply connected aperture.

$$\frac{\partial E}{\partial t} += (i \quad d)\frac{\partial^2 E}{\partial x^2} \quad f(|E|^2)E \qquad (1)$$

$$E(L,t) = E(0,t), \quad L \quad 2\pi R = \quad \lambda \qquad (2)$$

E – complex electric field envelope;
$I = |E|^2$ – intensity.

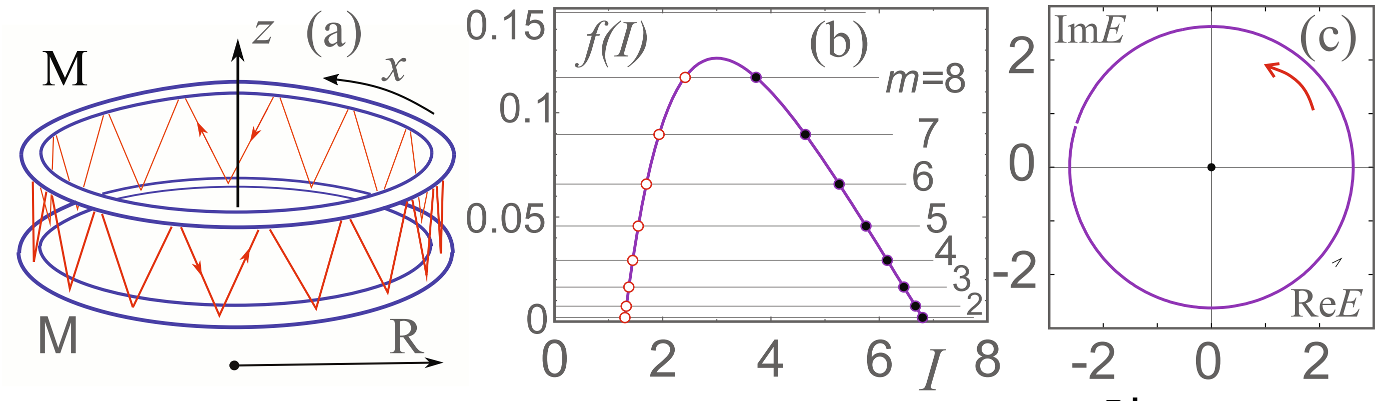
$$f(I) = \frac{g_0}{1+I} - 1 - \frac{a_0}{1+\beta I} \qquad (3)$$

saturable gain — nonresonant loss — saturable absorption

$\mathrm{Im}\, f = 0$ - small frequency detunings

In the limit $L \to \infty$ we get the known case with bright solitons with typical width w_0 under boundary condition $E(x \to \pm\infty, t) \to 0$.

Plane-wave solutions



$$\frac{\partial E}{\partial t} = (i \quad d)\frac{\partial^2 E}{\partial x^2} \quad f(|E|^2)E$$

$$E(L,t) = E(0,t), \quad L \quad 2\pi R$$

Phase curve

$$\{\mathrm{Re}\,E(x,t), \mathrm{Im}\,E(x,t)\}$$

$$0 < x < L, \quad t = \mathrm{const}$$

$$E = A_h \exp(iK_x x - iK_x^2 t) \quad (4) \qquad K_x = 2\pi m / L, \quad (5)$$

$$m = 0, \ \pm 1, \ \pm 2, \ldots, \ \pm m_{\max} \quad \text{–topological charge} = \delta\Phi/(2\pi) \qquad \delta\Phi = \int_0^L \frac{\partial \Phi}{\partial x} dx \quad (6)$$

$$m_{\max} = \text{Integer part}[K_{\max} L / (2\pi)] \qquad K_{\max}^2 = \max f(I) / d \quad (7)$$

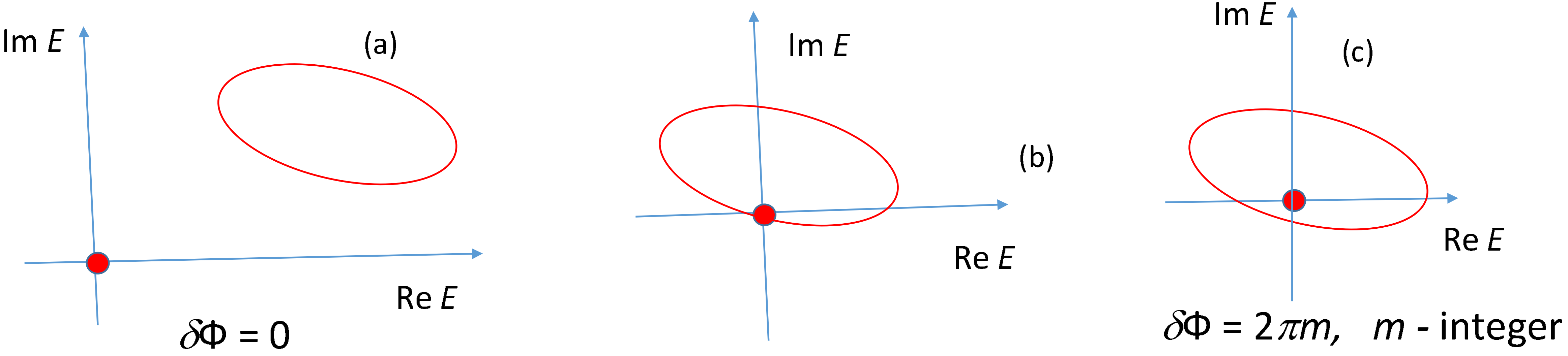
$$dK_x^2 = f(I_h), \quad I_h \quad |A_h|^2 \quad (8)$$

Energy flow (Poynting vector) $S = \Phi d \quad / dx \quad KI_h \quad 2\pi m I_h / L$

Linear stability condition: $df/dI < 0$. (9) For $L < L_{cr} \sim w_0$ only regimes with homogeneous intensity distribution are possible.

Phase curve $\{\mathrm{Re}\,E(x,t), \mathrm{Im}\,E(x,t)\}, \quad 0 < x < L, \quad t = \mathrm{const}$

In the nondegenerate case, when *x* changes from 0 to *L*, the closed phase curve is traversed once.



For nondegenerate phase curve without self-intersections, topological charge can change only by 1.

If the curve is close to the origin, sharp phase variation by about π occurs in a very narrow range of *x*

Conservation of topological charge?

Linear mode, intensity $I \ll I_{sat}$

$$\frac{\partial E}{\partial t} = (i \quad d)\frac{\partial^2 E}{\partial x^2} + f_0 E, \quad f_0 < f_m(0) \quad const \quad 0 \qquad (1)$$

$$E(x,t) = \sum_m A_{m0} \exp(\ \gamma_m t)\exp(iK_m x), \quad K_m \quad 2\pi m / L, \qquad (2)$$

$$\gamma_m = (i + d)K_m^2 - f_0, \quad m = 0, \ 1, \ 2, \ldots$$

Initial conditions:

nonzero A_{00} and A_{m0} only

$$E(x,t) = a(t) \quad b(t) \exp(iK_m x)$$

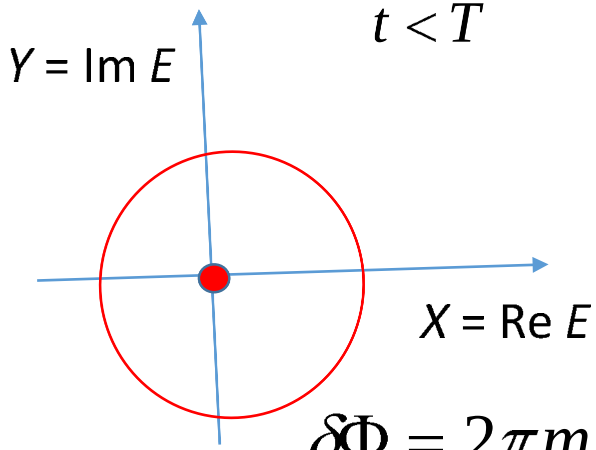
$$a(t) = A_{00}\exp(\ \gamma_0 t) \quad b(t) = A_{m0}\exp(\ \gamma_m t)$$

Phase curve: a circle $(X - a')^2 + (Y - a'')^2 = |b|^2$ (3) $X = \operatorname{Re} E(x), Y \quad \operatorname{Im} E(x)$ $\{a' = \operatorname{Re} a, a'' \quad \operatorname{Im} a\}$

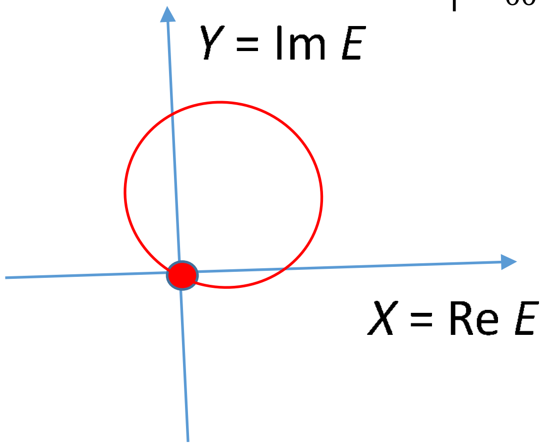
If $|A_{m0}| > |A_{00}|$

$$t = T \quad \frac{L^2}{8\pi^2 d m^2} \ln\left|\frac{A_{m0}}{A_{00}}\right|^2 \qquad (4)$$

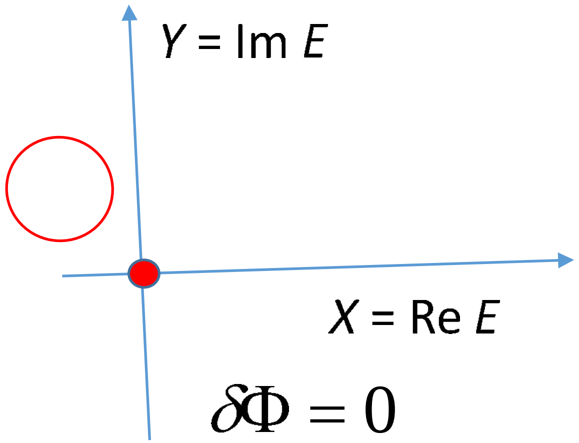
$t < T$



$\delta\Phi = 2\pi m$



$t > T$



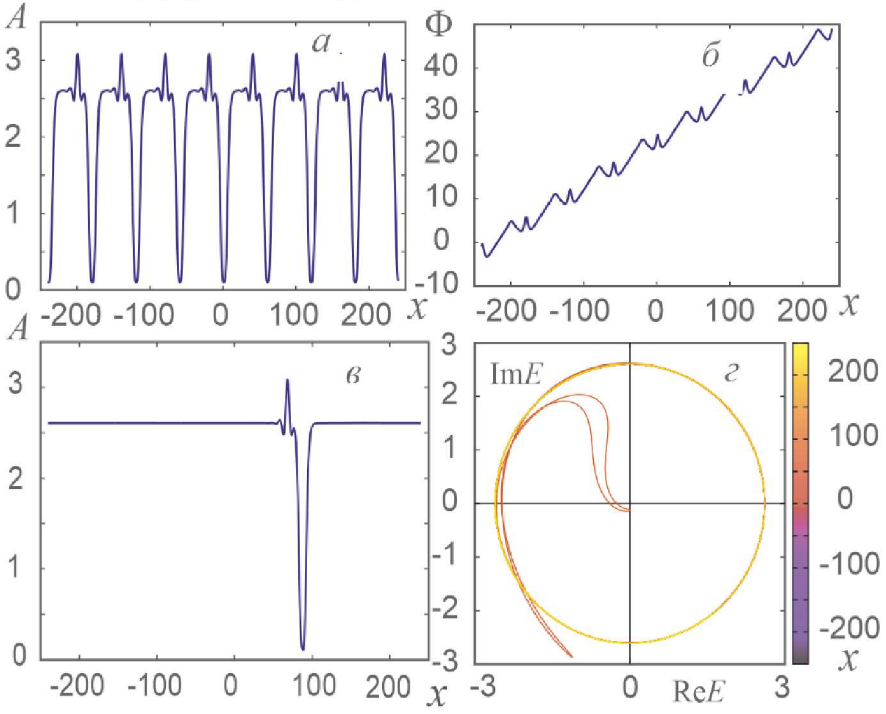
$\delta\Phi = 0$

Only two variants of topological charge

1D-coaxial laser, $L > L_{cr}$

1 – real amplitude $A=|E|$
2 – phase $\Phi = \arg E$
3 – energy flow $S = I(d\Phi / dx)$
at fixed time moment t

$m = 8$



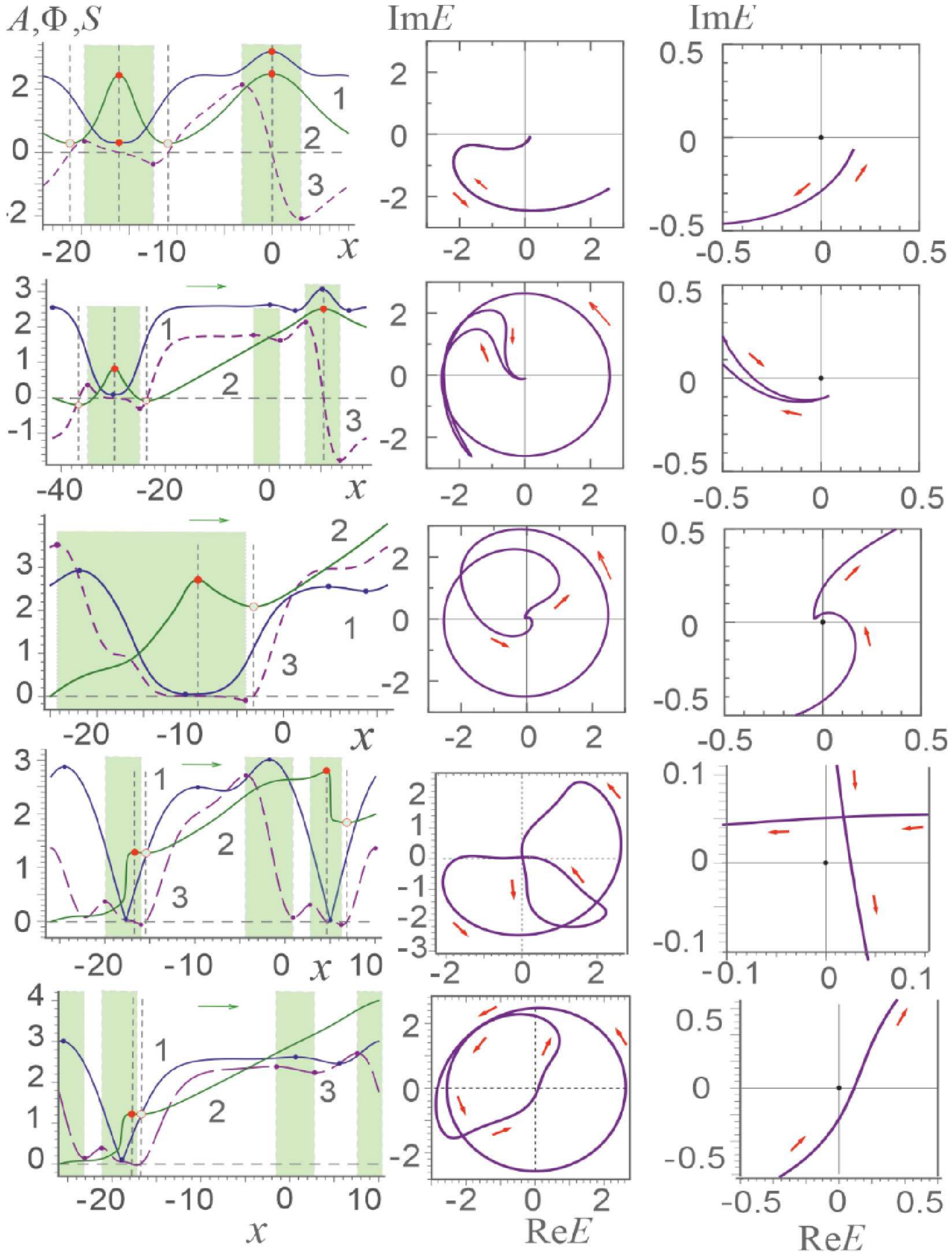
$m = 0$

$m = 1$

$m = 2$

$m = 1*$

$m = 2*$



Shaded are zones of energy sinks, dS/dx < 0

Elementary topological reactions

A-C: Dynamics of fragments of the phase curve.

A: phase curve crosses the origin, $\delta m = \pm 1$

B: self-crossing of phase curve; crossing number *Cr* changes by 2; not critical event.

C: cusp event with formation of additional loop; crossing number *Cr* changes by 1.

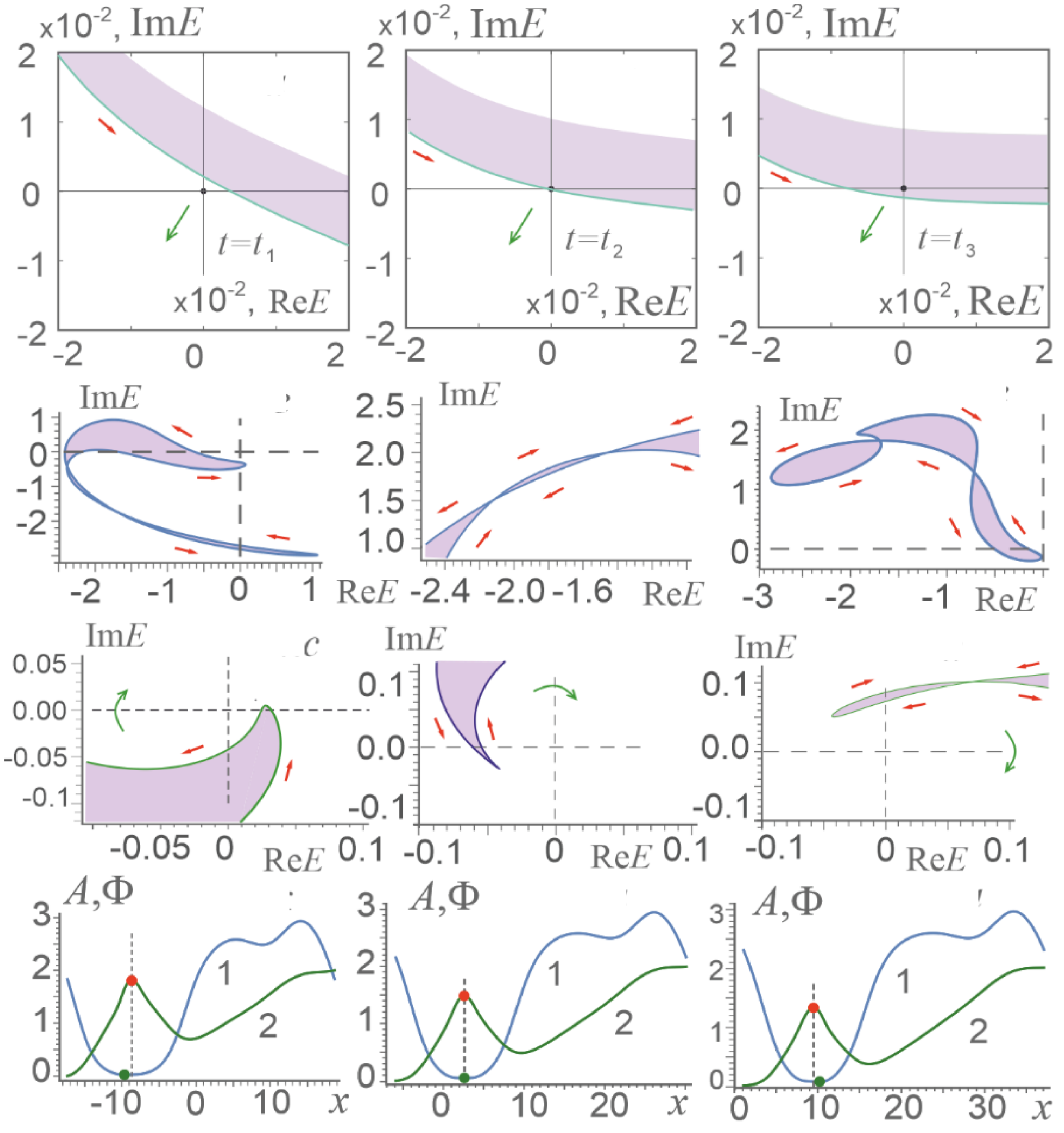
D: profiles of amplitude *A* and phase Φ; cusps arise when extrema of these two dependencies coincide (at discrete time moments at discrete coordinate values).

A

B

C

D



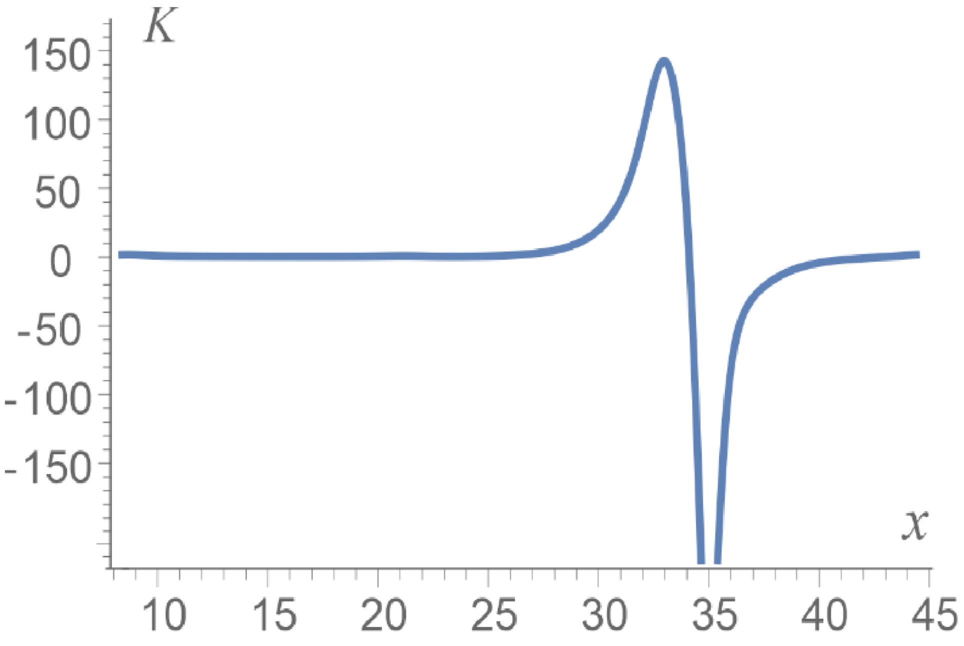
Curvature of the phase curve near cusp at $x = x_0$

$$X = \mathrm{Re}\,E(x),\; Y = \mathrm{Im}\,E(x)$$

$$X(x) = X_0 + X_0'(x - x_0) + \frac{1}{2}X_0''(x - x_0)^2 + \frac{1}{6}X_0'''(x - x_0)^3 + \ldots,$$
$$Y(x) = Y_0 + Y_0'(x - x_0) + \frac{1}{2}Y_0''(x - x_0)^2 + \frac{1}{6}Y_0'''(x - x_0)^3 + \ldots \qquad (1)$$

$$X_0' = 0, \quad Y_0' = 0 \qquad (2)$$

$$K = \frac{X_0''Y_0''' - Y_0''X_0'''}{2(X_0''^2 + Y_0''^2)^{3/2}}\frac{1}{x - x_0} \qquad (3)$$

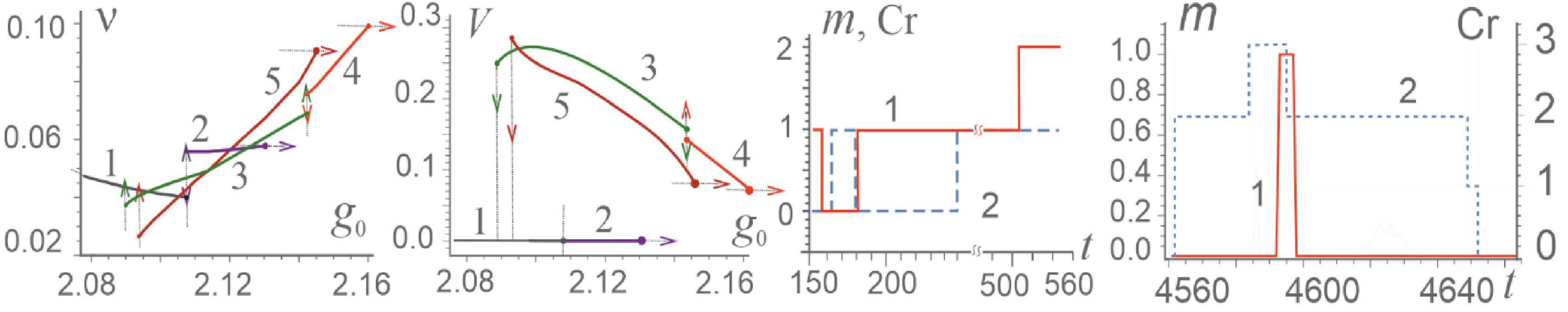


Variation of velocity V, topological charge m, crossing number Cr, instantaneous frequency shift ν and power P

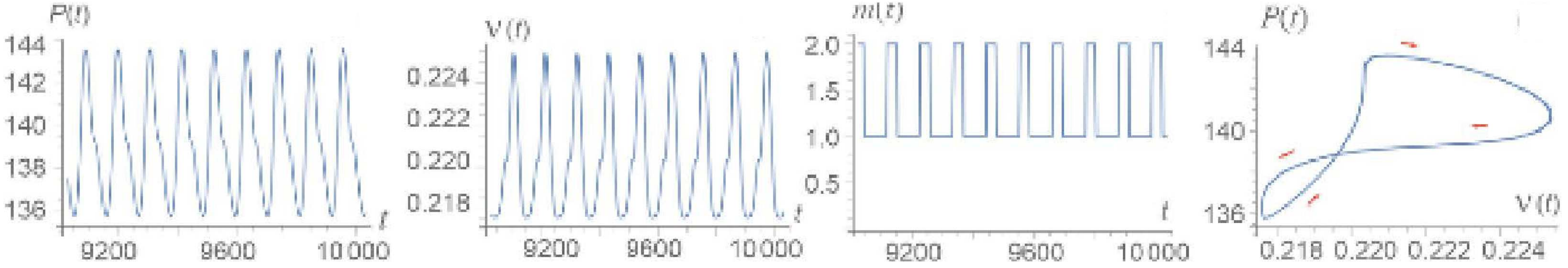
1 – one-humped symmetric structure, m = 0;
2 – three-humped symmetric structure, m = 0;
3 – asymmetric structure, m = 1;
4 - asymmetric structure, m = 2, two phase jumps by ~π;
5 - asymmetric structure, m = 2, one phase jumps by ~π

$$\nu(t) = \int_0^L (\partial\Phi / \partial t) I \, dx / P(t) \qquad (1)$$
$$P(t) = \int_0^L I \, dx$$

Transient



Variation of gain g_0



Periodic process for fixed parameters

CONCLUSION

- For a coaxial laser (not simply connected aperture), it is possible to introduce the topological charge of structures as radiation phase incursion $\delta\Phi / 2\pi$.
- Topological protection is not absolute here: Topological charge conserves only at not large deviations from the steady-state modes and for not large variations of the scheme parameters.
- If the phase curve is nondegenerate, without self-intersections and smooth everywhere, topological change can vary during the structure evolution only by 1. However, this variation can be arbitrary large, if cusps of the phase curve arise.

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