ДИНАМИКА РАСПРОСТРАНЕНИЯ КОРОНАВИРУСА В РАМКАХ ПРОСТЫХ ЛОГИСТИЧЕСКИХ МОДЕЛЕЙ

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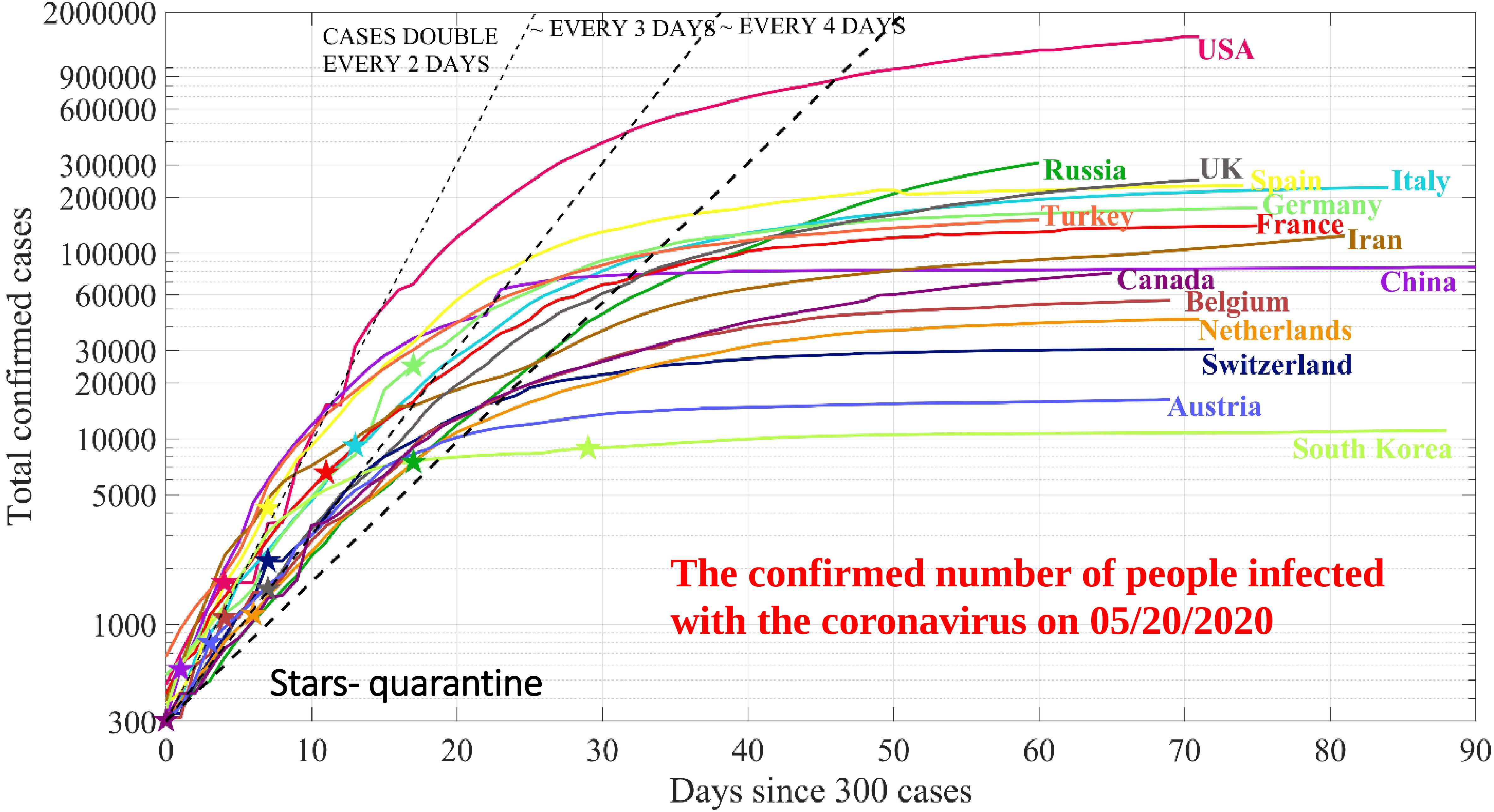
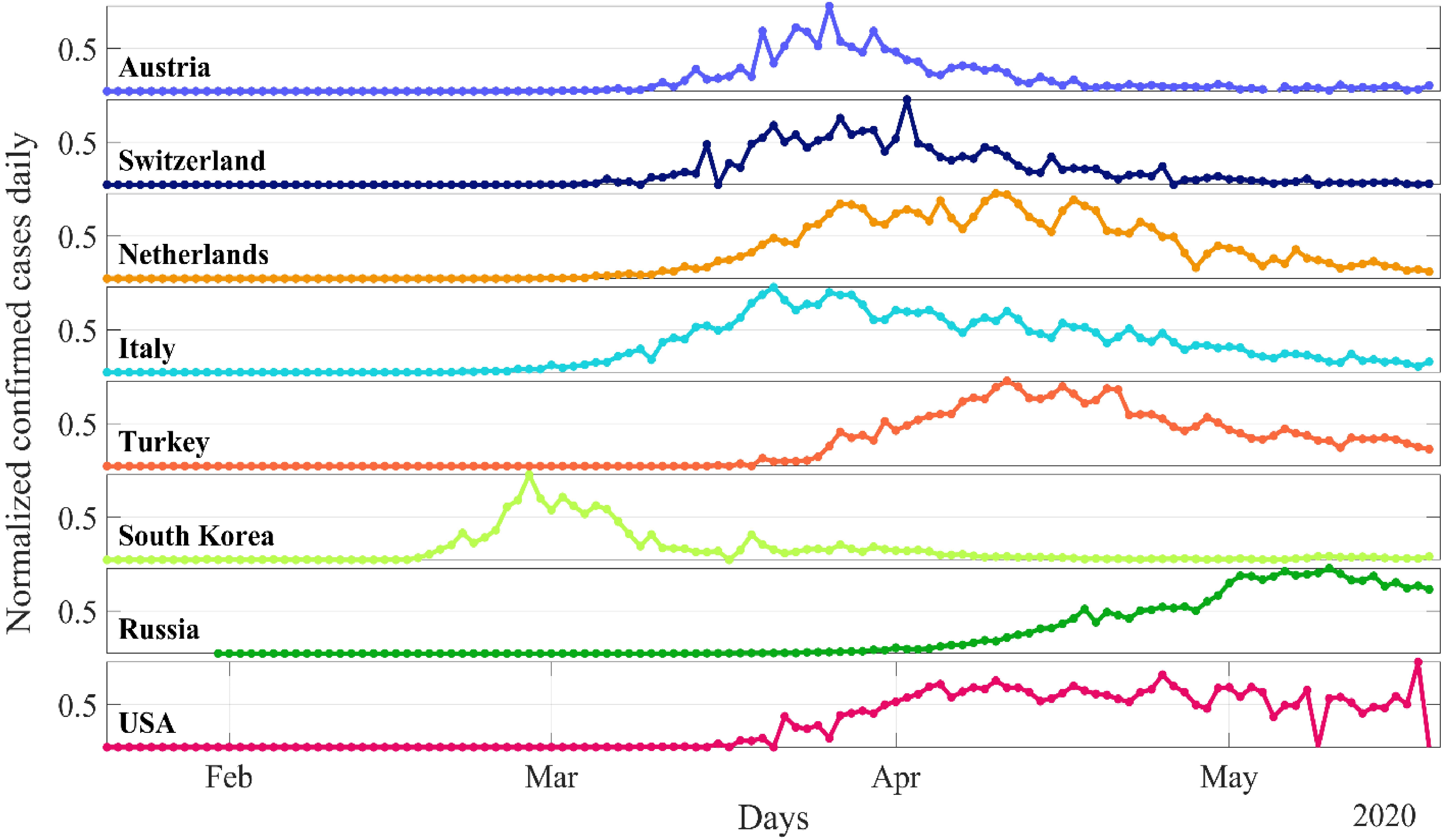
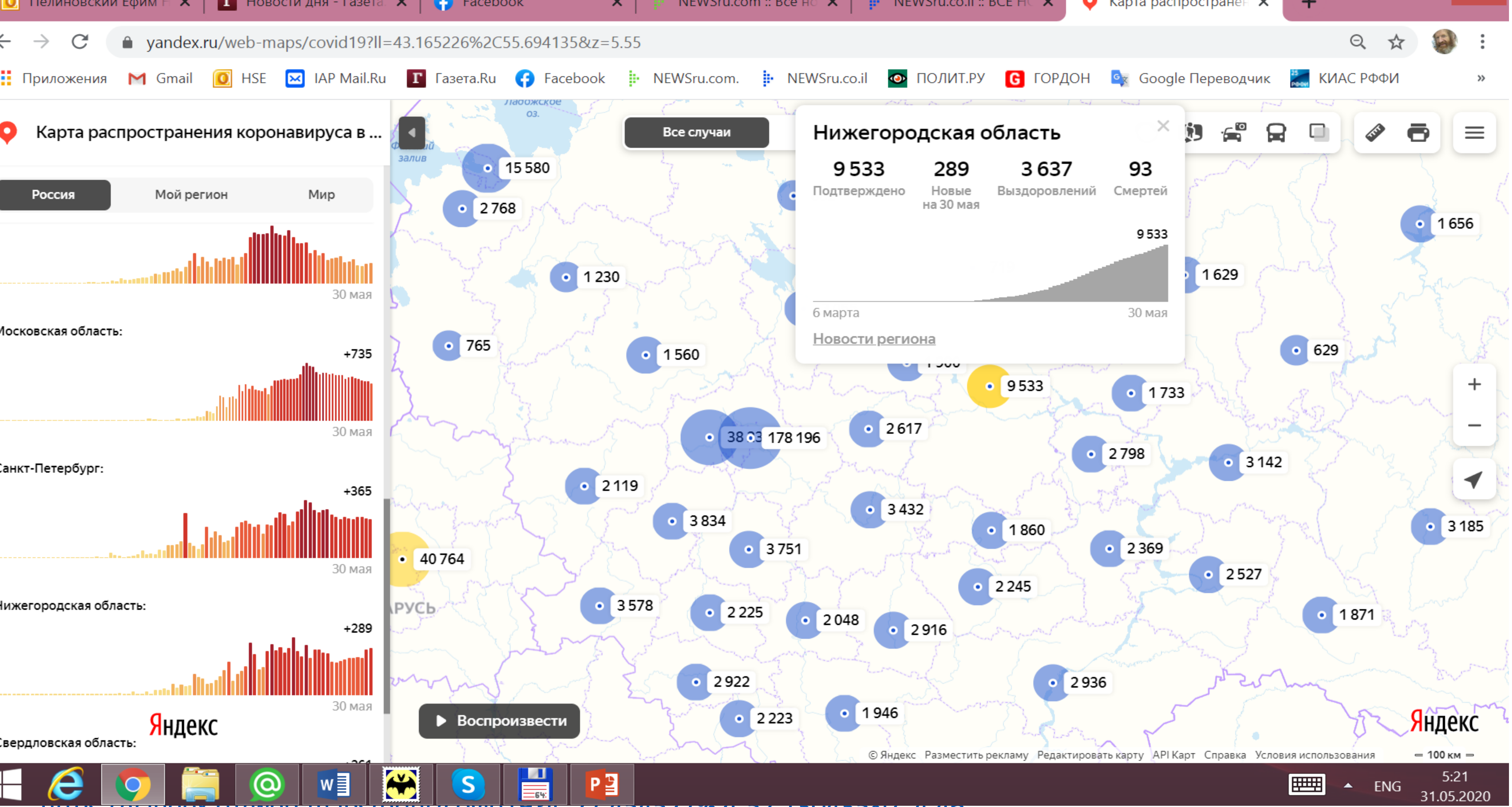
Сессия научного совета РАН 14 *декабря 2020 года*



WHO data https://www.who.int/emergencies/diseases/novel-coronavirus-2019/situation-reports



The number of infected people per day, normalized to the maximum value for each country



https://yandex.ru/web-maps/print/covid19?ll=27.454327%2C52.130435&z=4.46

31 May 2020

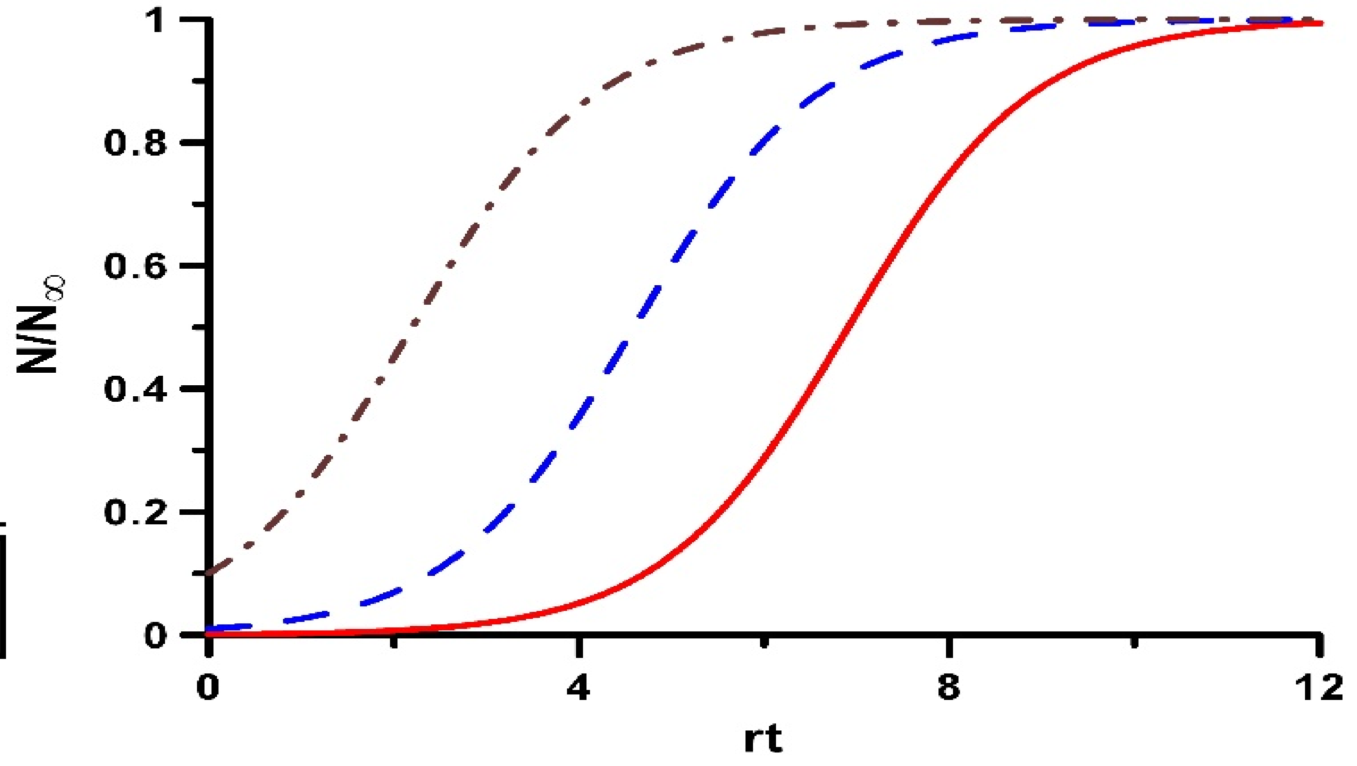
Logistic Equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{N_\infty}\right)$$

Two unknown constants!

$$N(t) = \frac{N_0 N_\infty \exp(rt)}{N_\infty + N_0\left[\exp(rt) - 1\right]}$$

Verhulst P.F. **Notice sur la loi que la population poursuit dans son accroissement.** **Correspondance Mathematique et Physique. 1838. V. 10. P. 113 – 121.**



Correctly

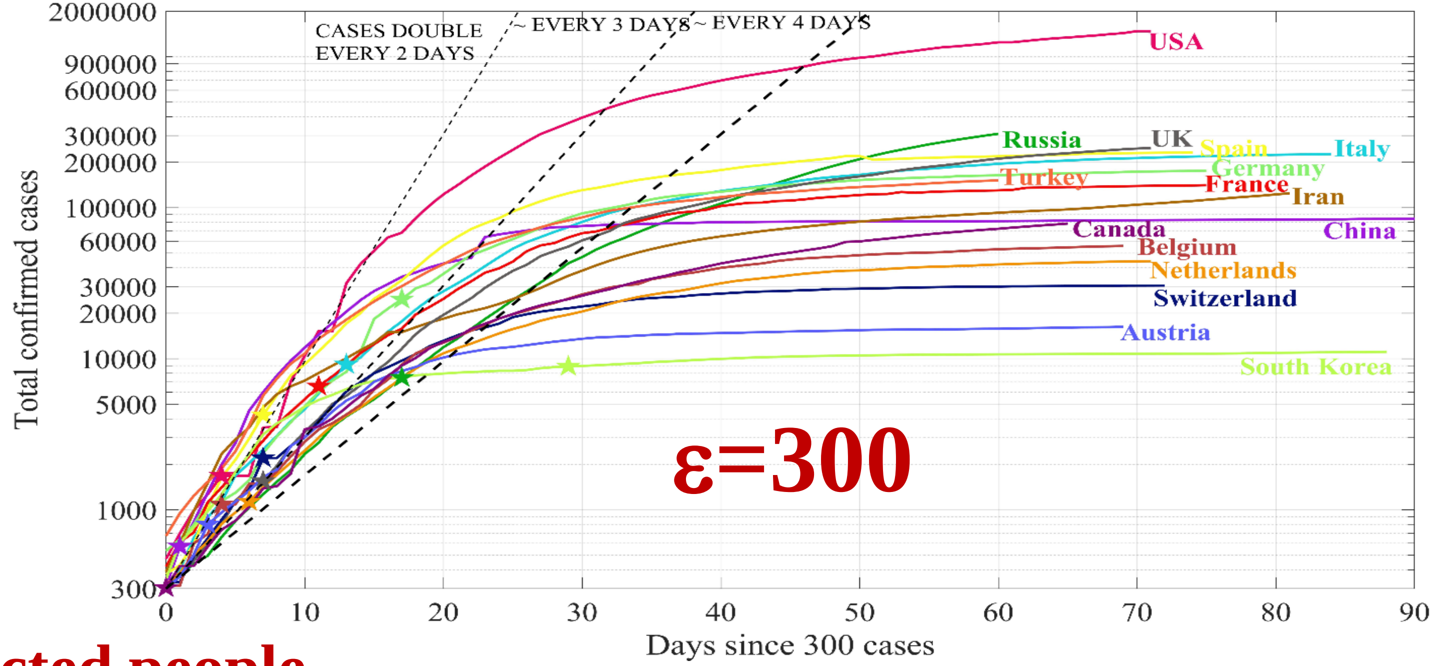
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{N_\infty}\right) - \varepsilon$$

ε - threshold of infection

Initial Stage

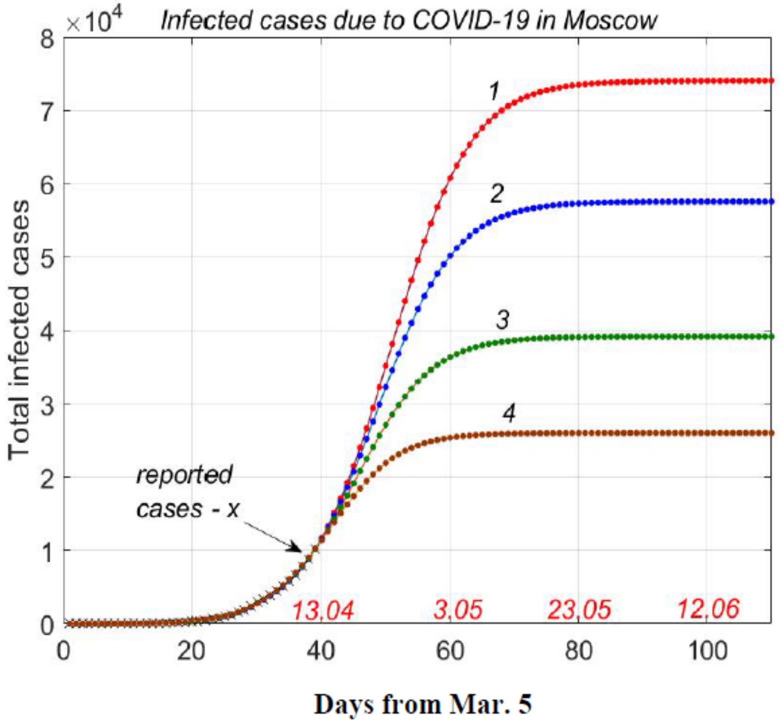
$$N(t) = N_0 \exp(rt)$$

Coefficient **r** is trivially found



N_∞ **Maximum possible number of infected people**

can be estimated only at the stage of the noticeable difference between the data and the exponential curve, when the number of sick people is already not small



E.M. Koltsova, E.S. Kurkina, A.M. Vasetsky.

Mathematical Modeling of the Spread of COVID-19 in Moscow and Russian Regions. *arXiv:2004.10118*

Initial Stage

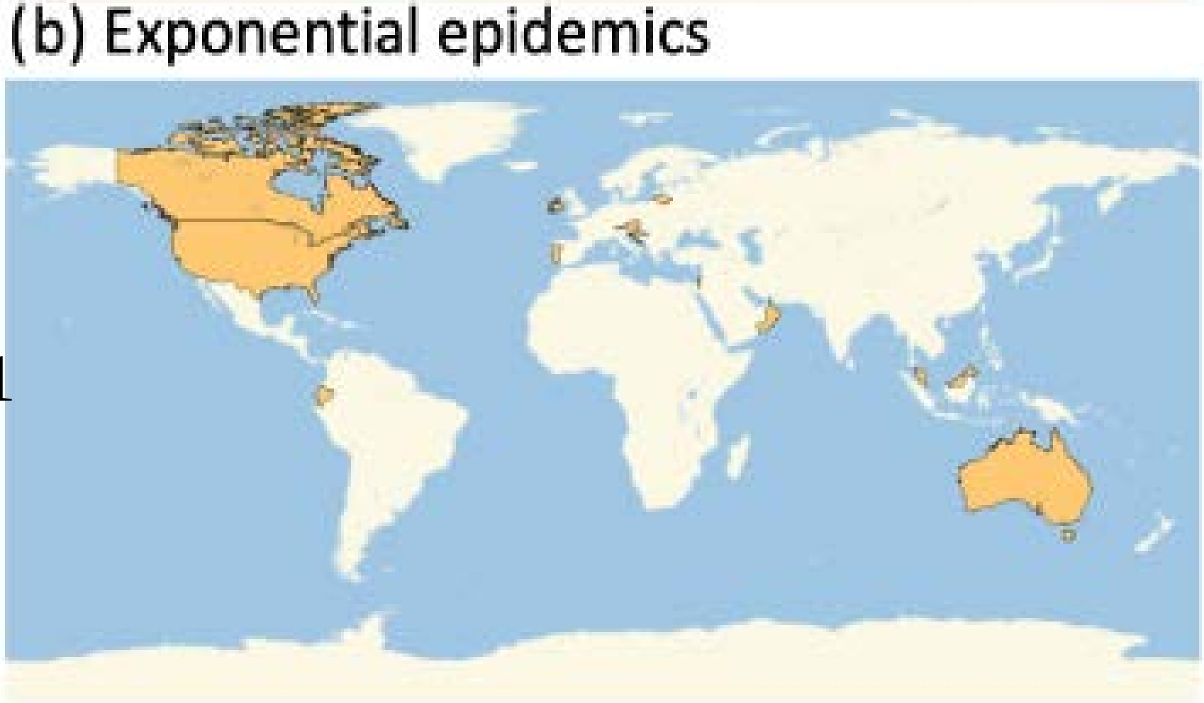
Natalia Komarova and Dominik Wodarz

Patterns of the COVID19 epidemic spread around the world: exponential vs power Laws. *medRxiv preprint doi:*

https://doi.org/10.1101/2020.03.30.20047274.

$$\frac{dN}{dt} = rN^{\alpha}$$

$$N = \begin{cases} [(1-\alpha)rt]^{\frac{1}{1-\alpha}}, & 0 < \alpha < 1, \\ N_0 \exp(rt), & \alpha = 1, \\ \dfrac{1}{[(\alpha-1)r(t_0 - t)]^{\frac{1}{\alpha-1}}}, & \alpha > 1 \end{cases}$$



But firstly

Logistic Equation

Daily infected cases

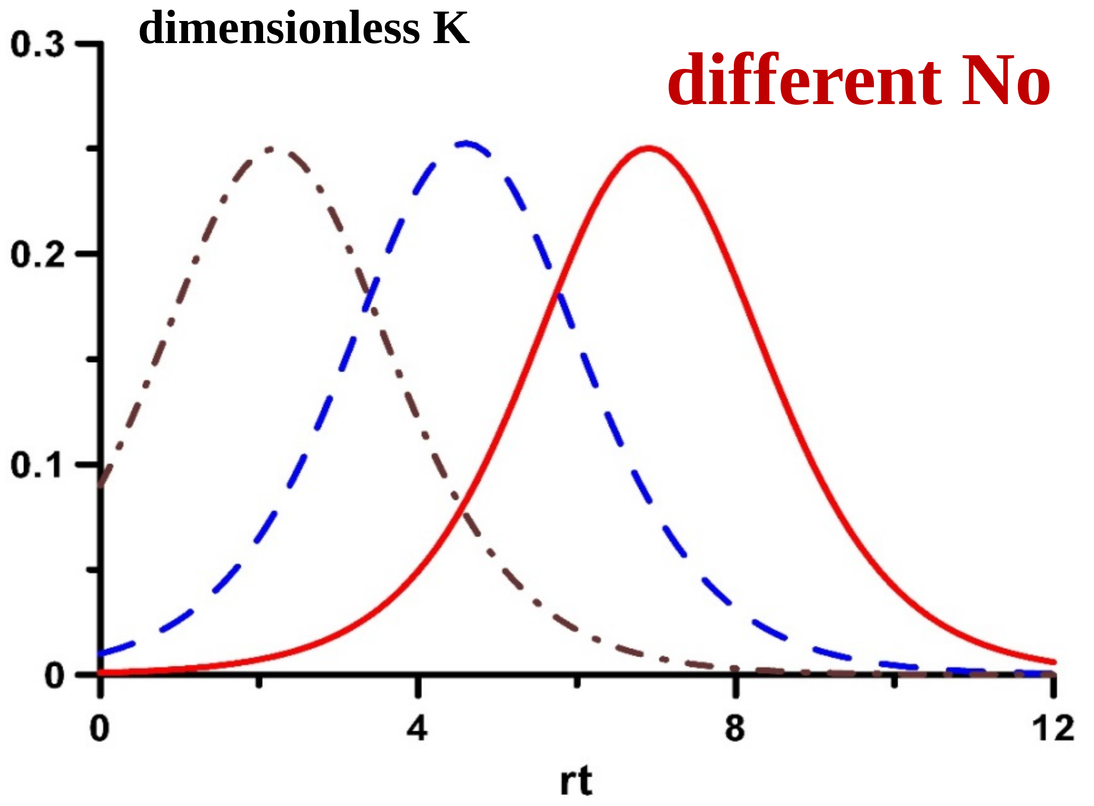
$$K = \frac{dN}{dt} = \frac{N_0 N_\infty (N_\infty - N_0) r \exp(rt)}{\left(N_\infty + N_0\left[\exp(rt) - 1\right]\right)^2}$$

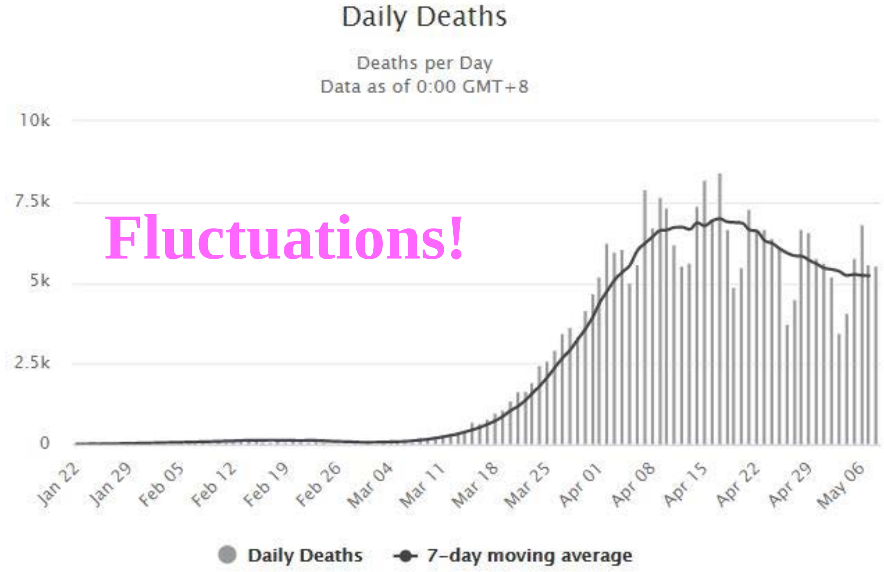
$$\max(K) = \frac{rN_\infty}{4}$$

Number of hospitals

$$T = \frac{1}{r} \ln \frac{N_\infty - N_0}{N_0}$$

Epidemic peak





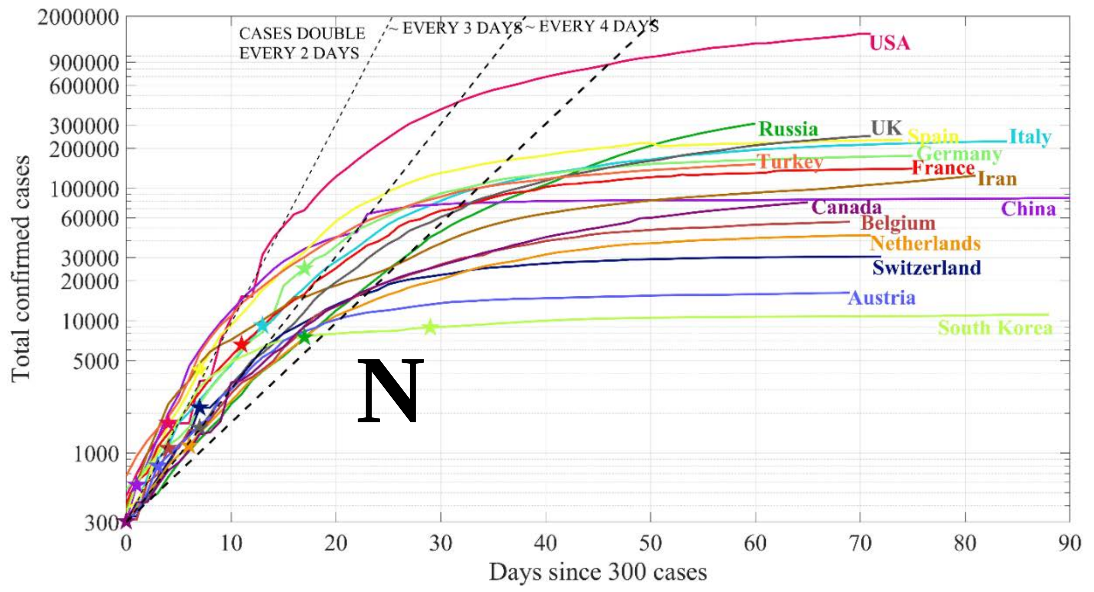
"Practical" Logistic Curve

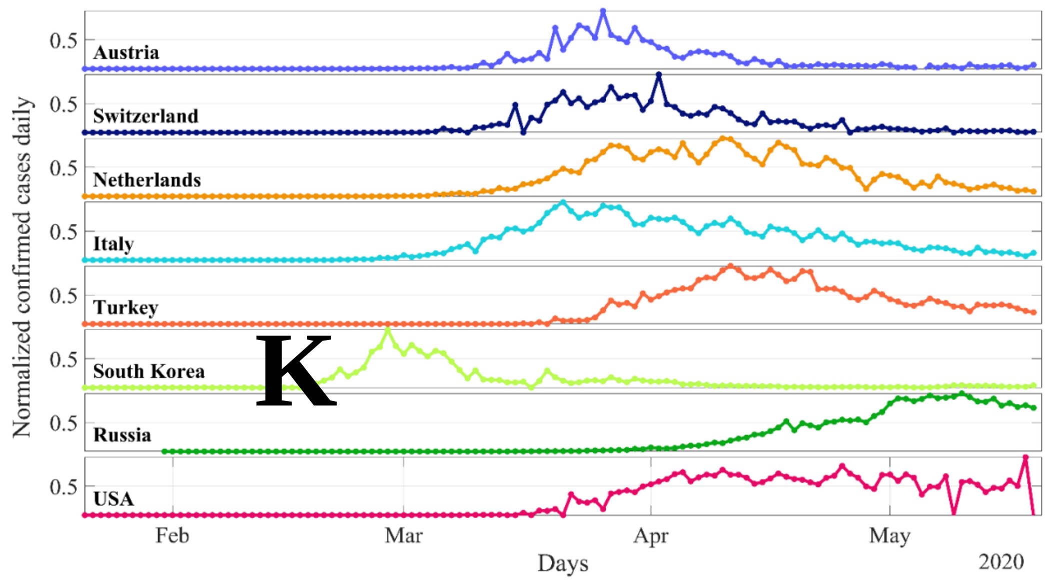
$$K_n = N_{n+1} - N_n = rN_n\left(1 - \frac{N_n}{N_\infty}\right)$$

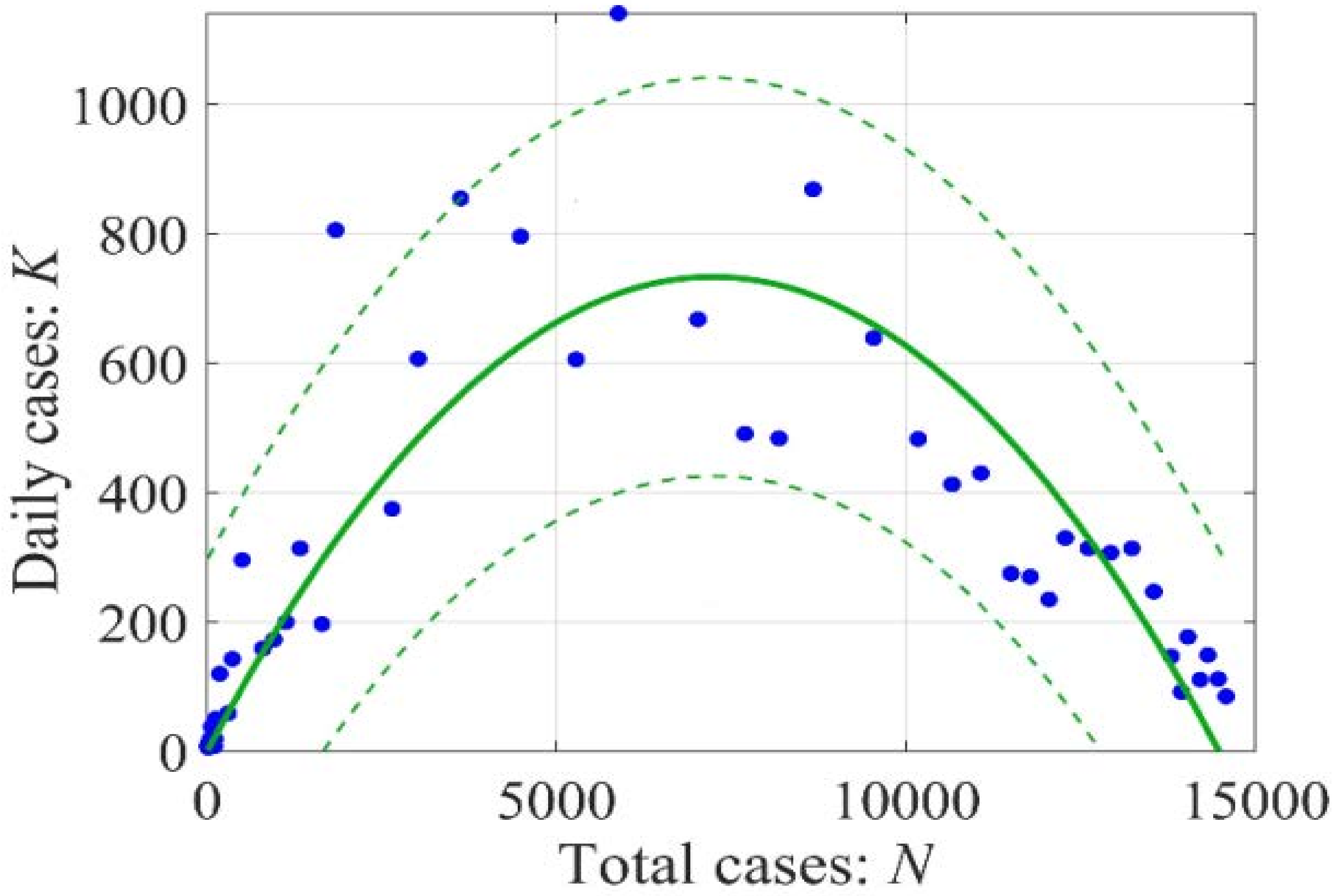
$$K = rN\left(1 - \frac{N}{N_\infty}\right)$$

Algebraic problem – using parabola approximation to find coefficients

Number of points > 50







Austria

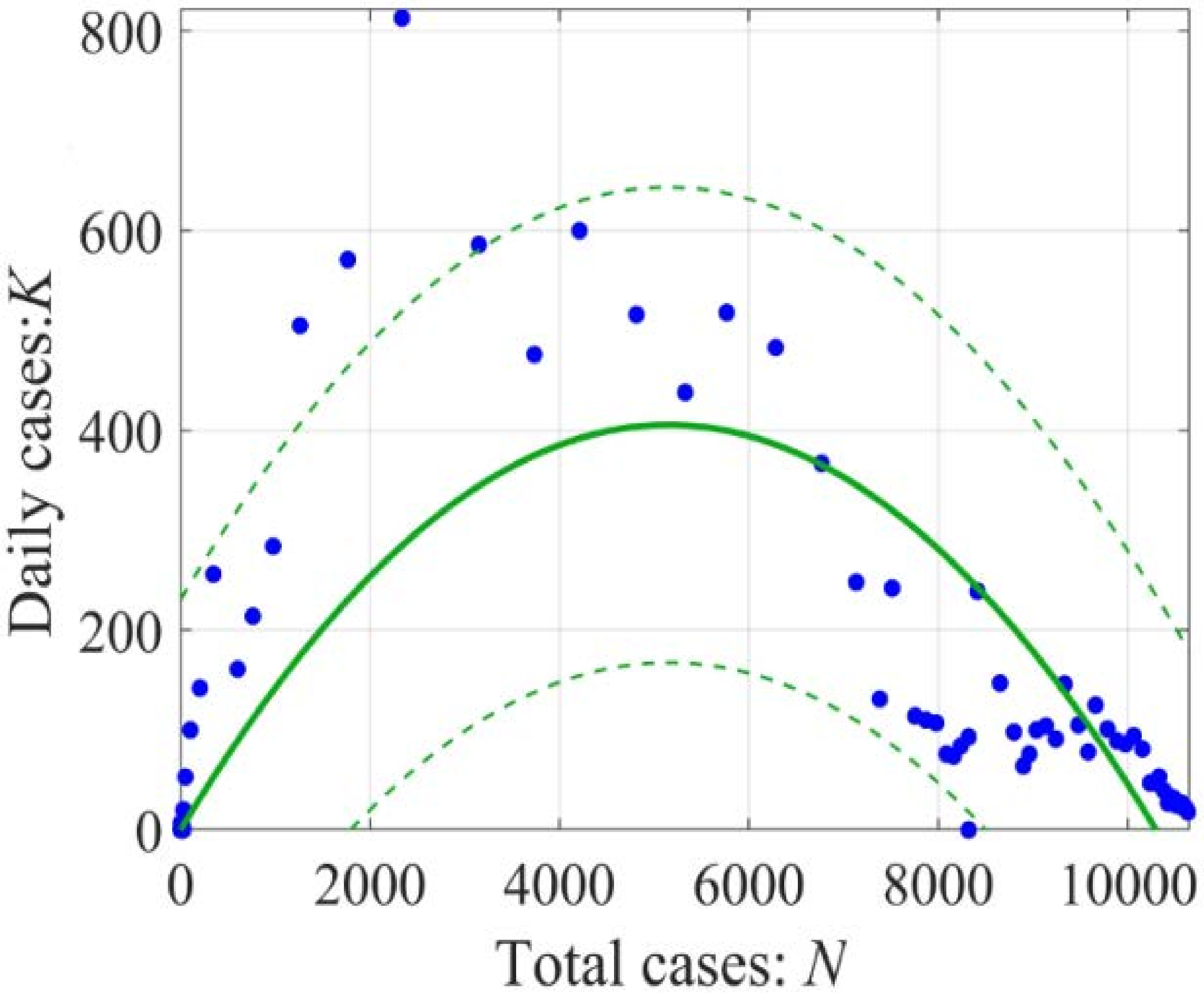
$N_\infty = 14700$

***r* = 0.195**

R^2= 0.81

Good

solid green line is the regression, dashed lines give 95% prediction bounds



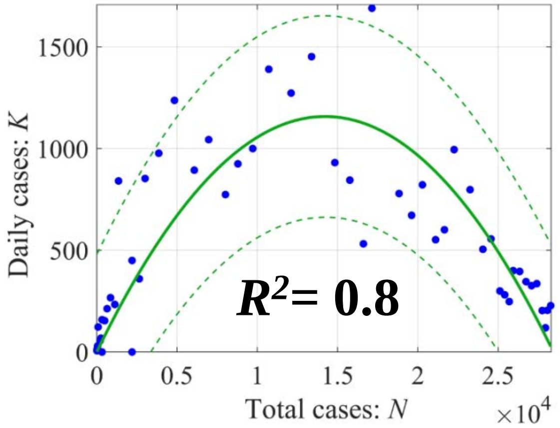
South Korea

$$N_\infty = 10300$$

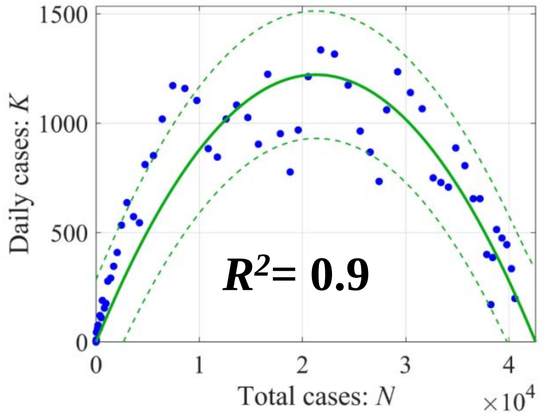
$r = 0.16$

$R^2 = 0.55$

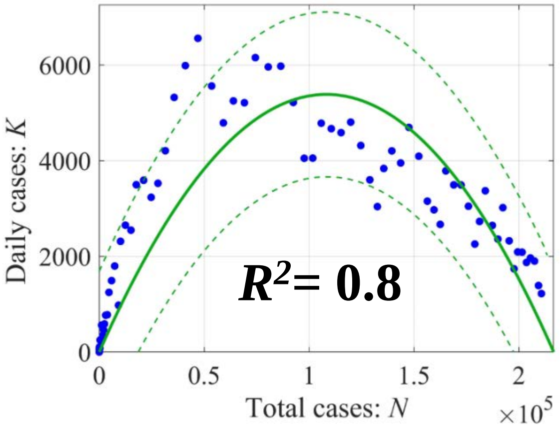
Bad



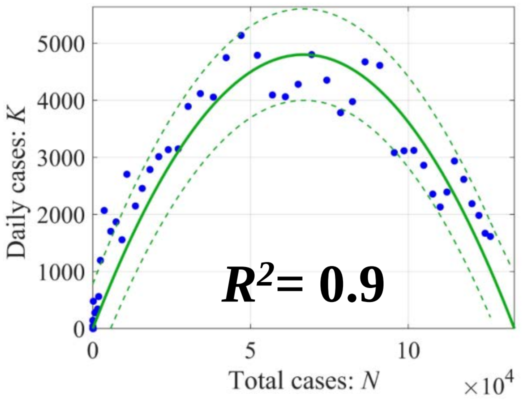
Switzerland



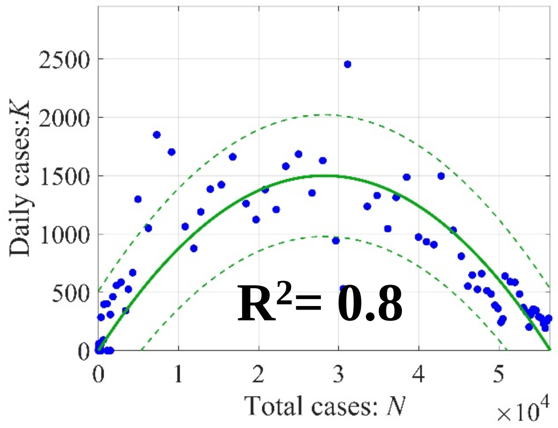
Netherlands



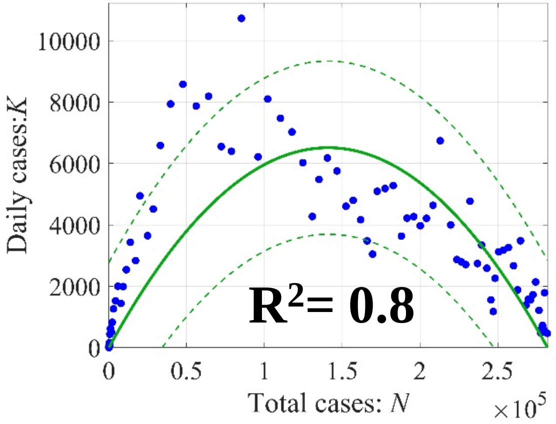
Italy



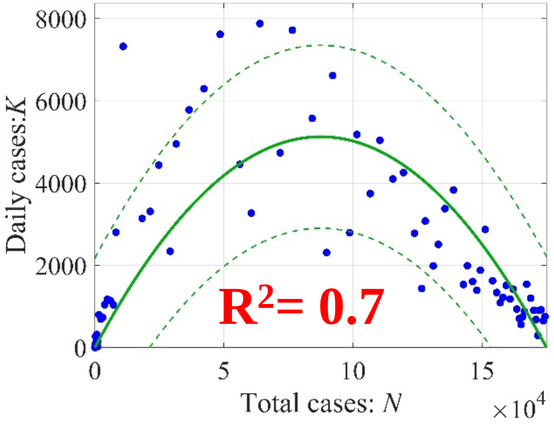
Turkey



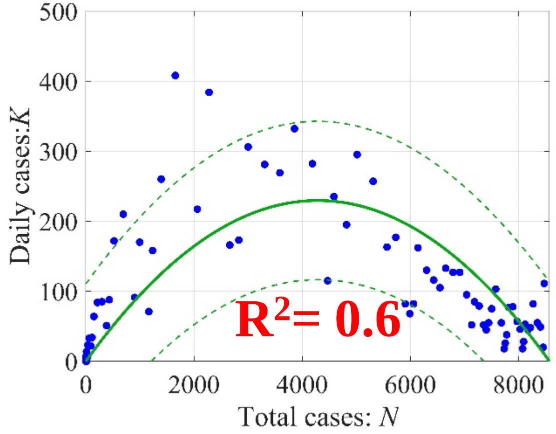
Belgium



Spain



Germany



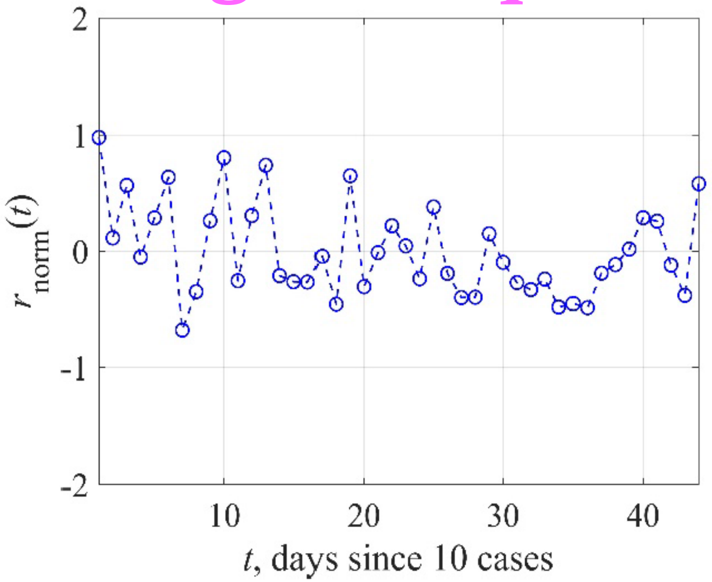
Czech

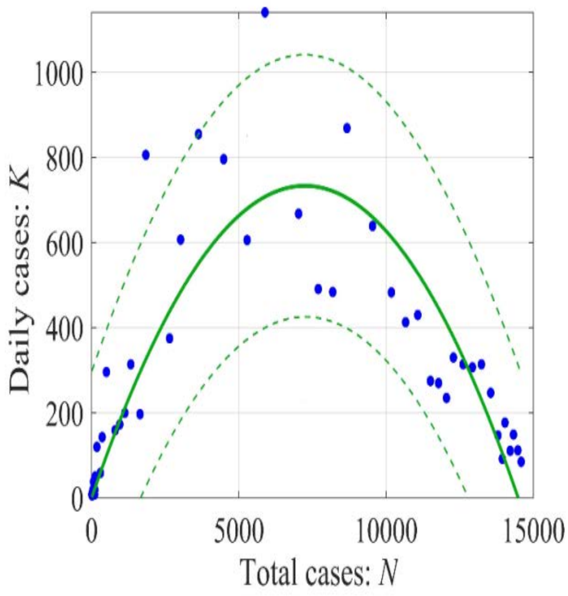
Australia, Norway **R^2= 0.7**

Variable-coefficient “Stochastic” Logistic Equation

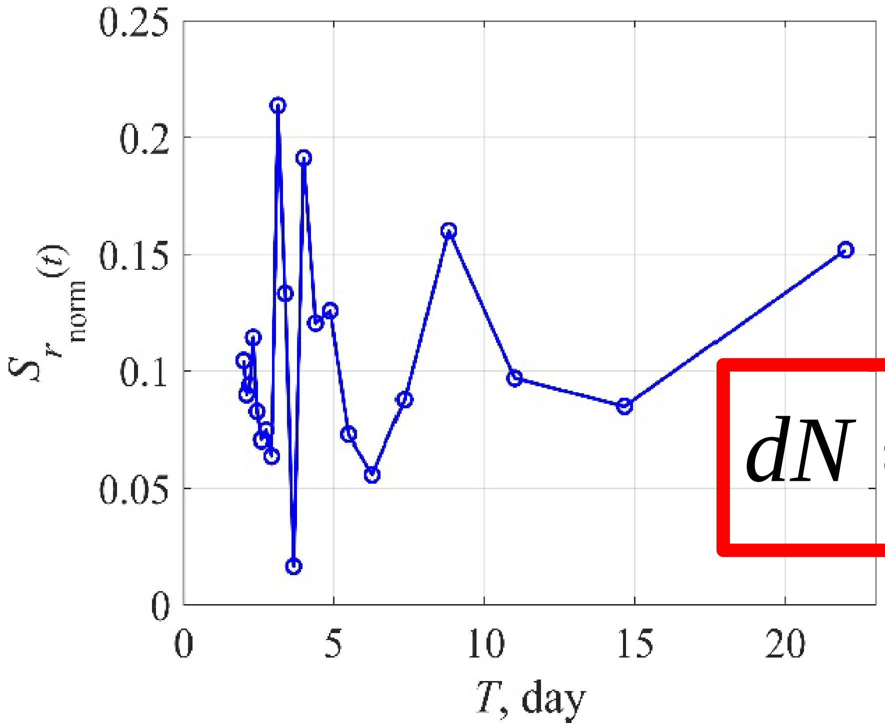
$$\frac{dN}{dt} = r(t)N - p(t)N^2$$

$$r_{norm} = \left(r - \langle r \rangle\right) / \langle r \rangle$$

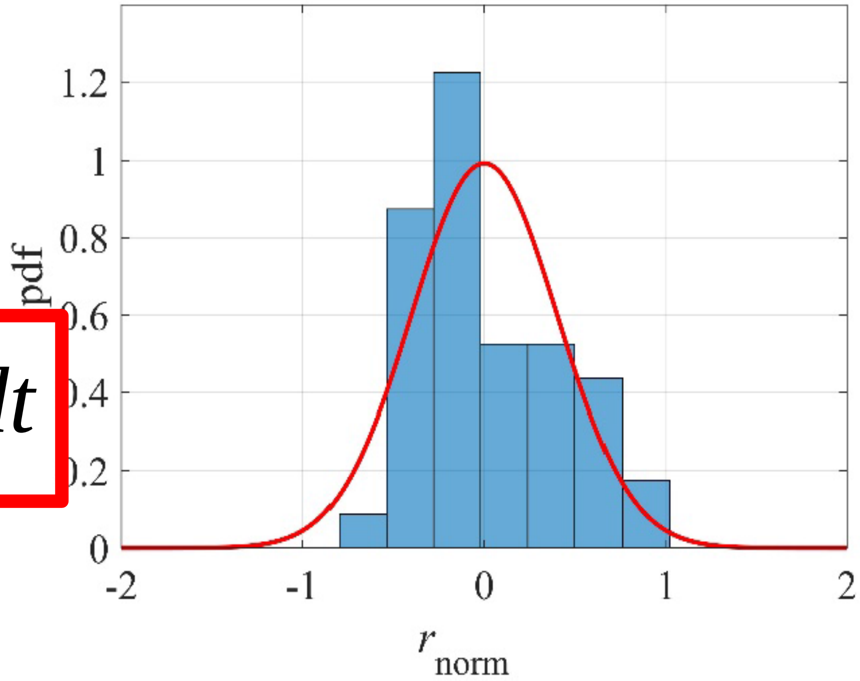




Austria



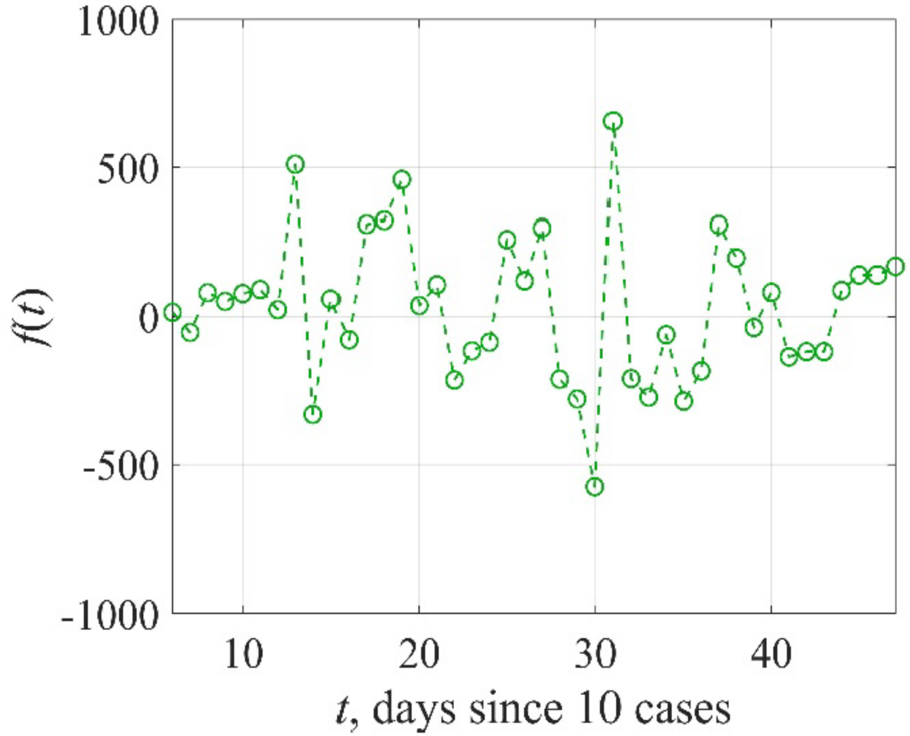
$$dN = \left[r(t)N - p(t)N^2\right]dt$$

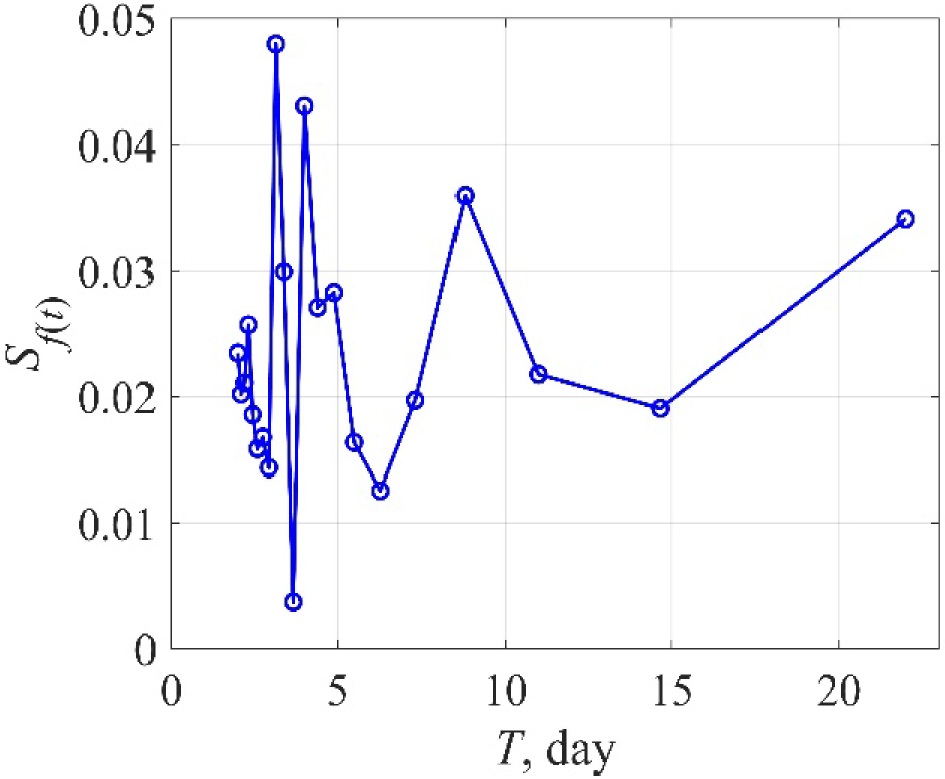


Stochastic

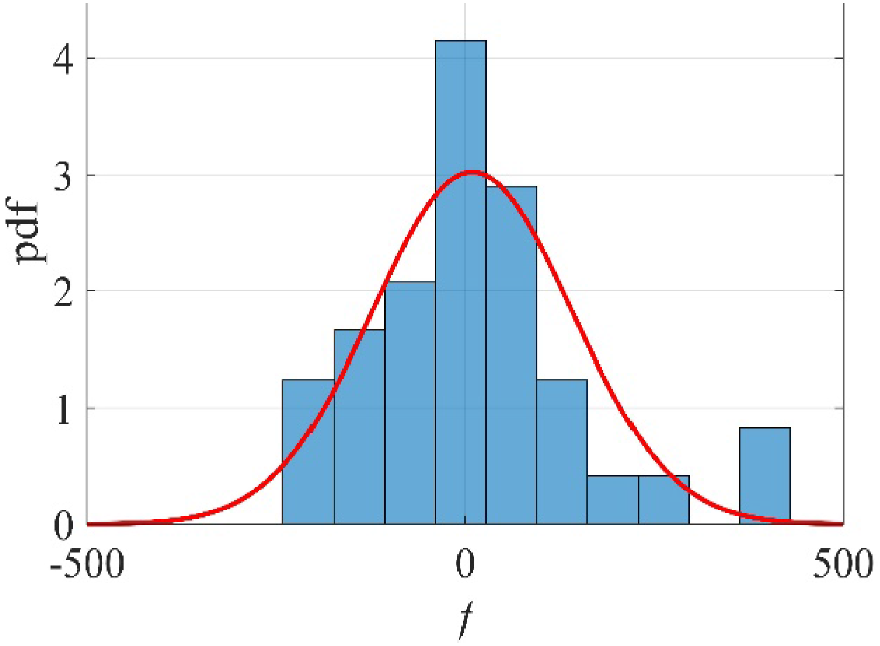
Forced Stochastic Logistic Equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{N_\infty}\right) + f(t)$$





Austria



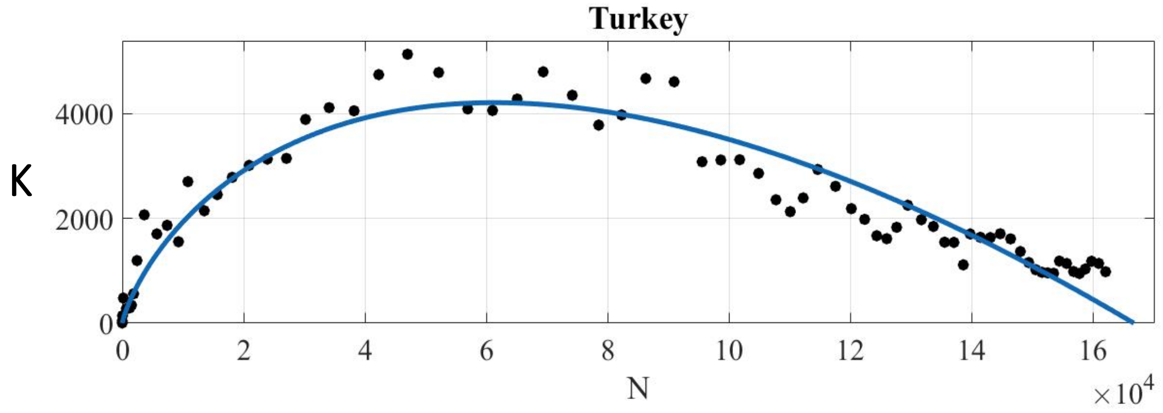
Benjamin Gompertz, 1779-1865

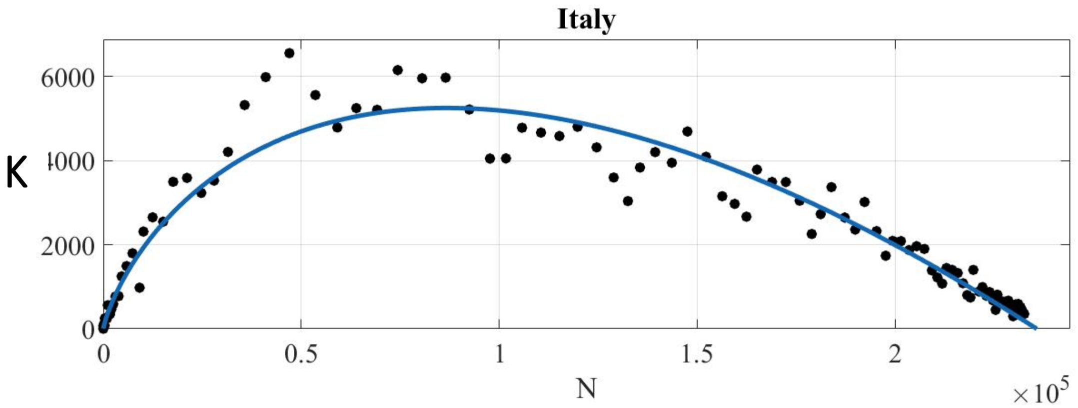
$$\frac{dN}{dt} = rN\left(1 - \frac{\ln N}{\ln N_\infty}\right)$$

Two constants

It is a linear ODE respect to ln(N)

$$\ln(N) = \ln(N_\infty)\left[1 - \exp\left(-\frac{r(t-t_0)}{\ln(N_\infty)}\right)\right]$$





Better agreement for some countries

Generalized Logistic Equation

$$\frac{dN}{dt} = rN^{\alpha}\left(1 - \frac{N}{N_{\infty}}\right)^{\beta}$$

Equilibrium points are not “rough”

Generally, no exact solutions

For approximation of real data, four constants - better

$$\frac{dN}{dt} = rN^{\alpha}\left(1 - \frac{N}{N_{\infty}}\right)^{\beta} - \varepsilon$$

ε - threshold of infection

$$K_n = N_{n+1} - N_n = rN_n^{\alpha}\left(1 - \frac{N_n}{N_{\infty}}\right)^{\beta} - \varepsilon$$

Properties of discrete version are not well known.

Generalized Logistic Equation

Initial stage $$\frac{dN}{dt} = rN^{\alpha}$$

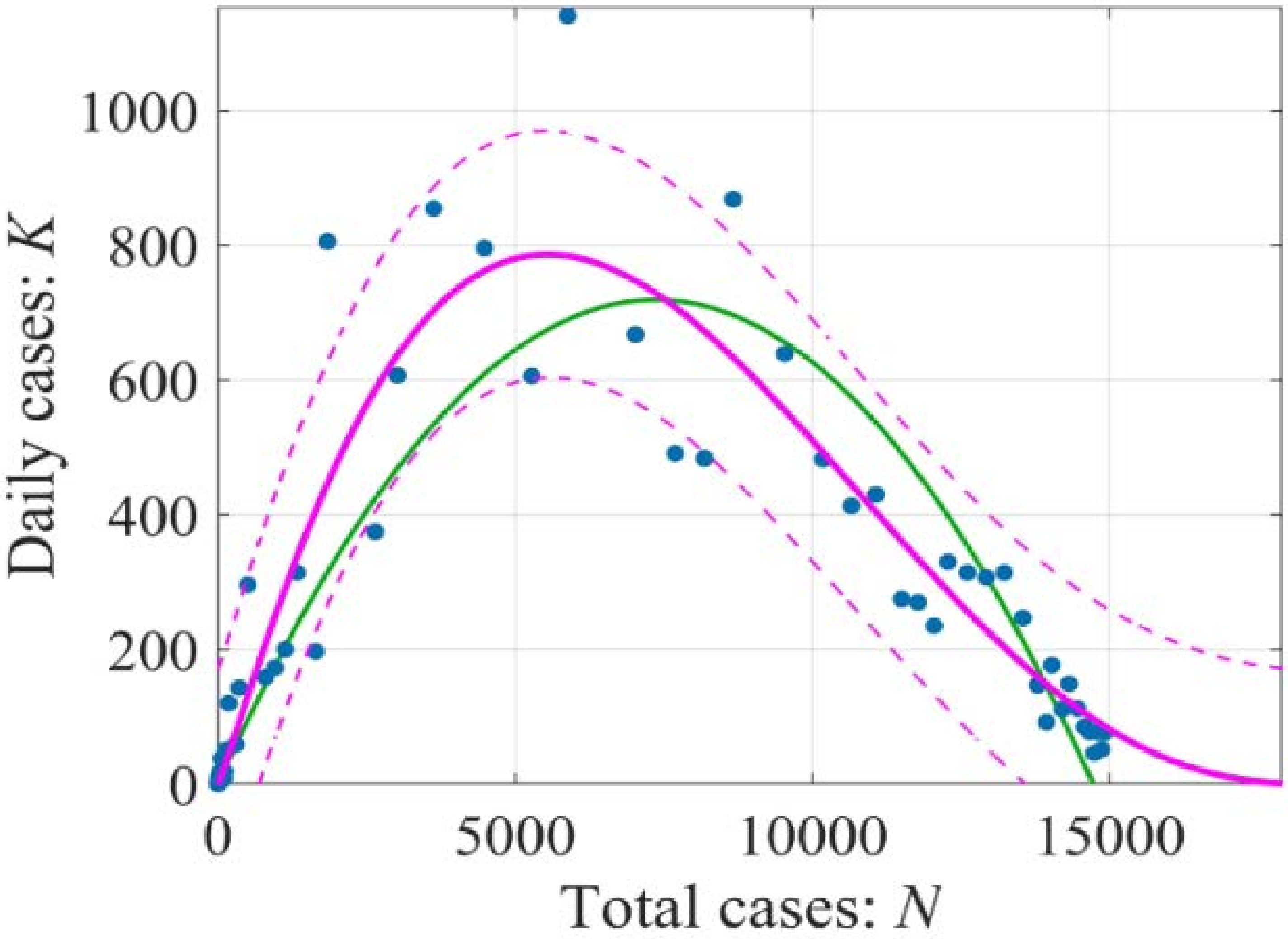
Power growth, two constants

$$N = \begin{cases} \left[(1-\alpha)rt\right]^{\frac{1}{1-\alpha}}, & 0 < \alpha < 1, \\ N_0 \exp(rt), & \alpha = 1, \\ \dfrac{1}{\left[(\alpha-1)r(t_0 - t)\right]^{\frac{1}{\alpha-1}}}, & \alpha > 1 \end{cases}$$

$$K = rN^{\alpha}\left(1 - \frac{N}{N_{\infty}}\right)^{\beta}$$

$$K_n = N_{n+1} - N_n = rN_n^{\alpha}\left(1 - \frac{N_n}{N_{\infty}}\right)^{\beta}$$

For approximation of real data, four constants - better



Pink line – generalized model, the dashed line - 95% prediction bounds. The **green line** - simple logistic model

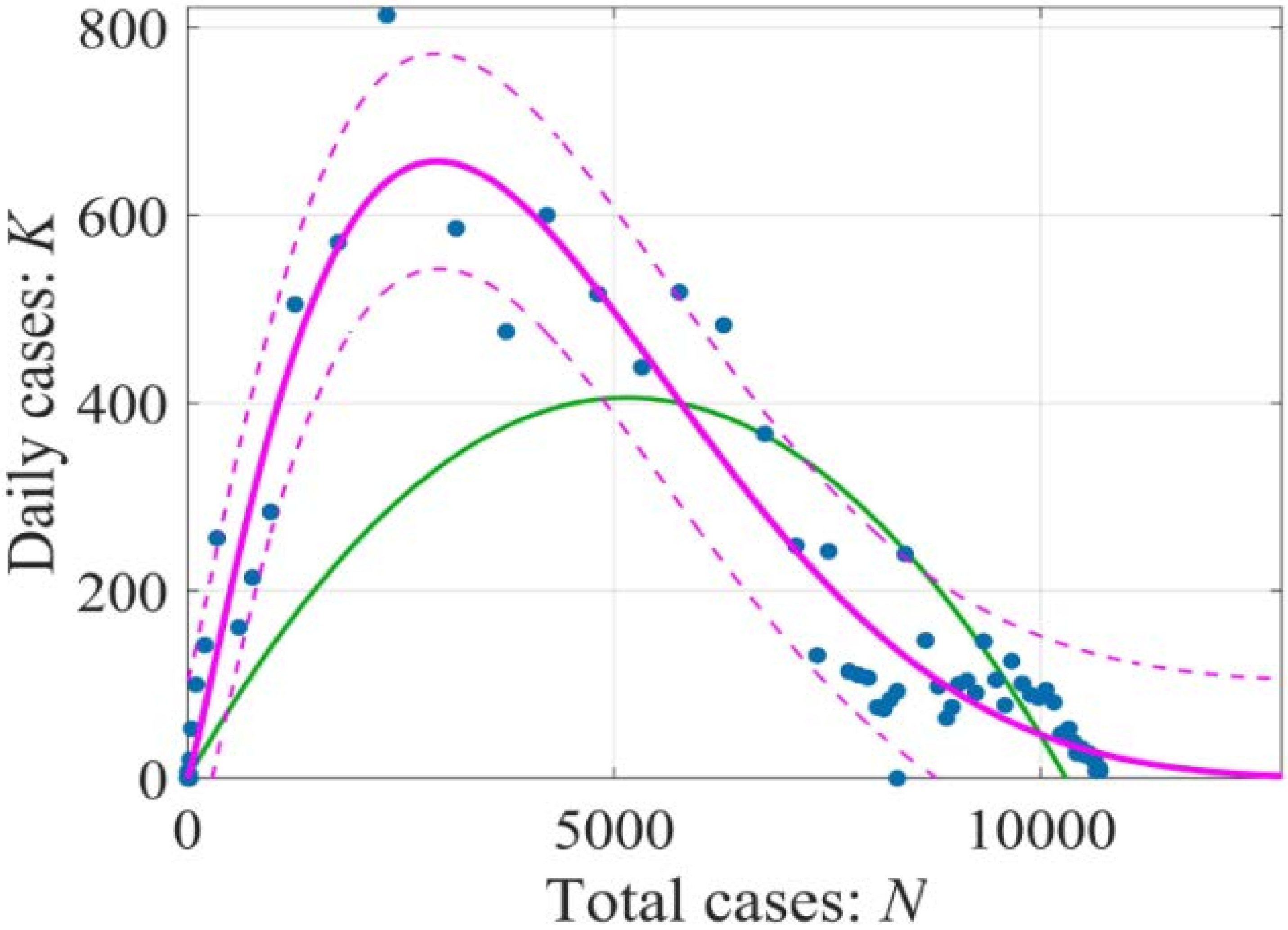
Austria

$$\alpha = 1.1$$

$$\beta = 2.6$$

$$R^2 = 0.88$$

was $R^2 = 0.81$



Pink line – generalized model, the dashed line - 95% prediction bounds. The **green line** - simple logistic model

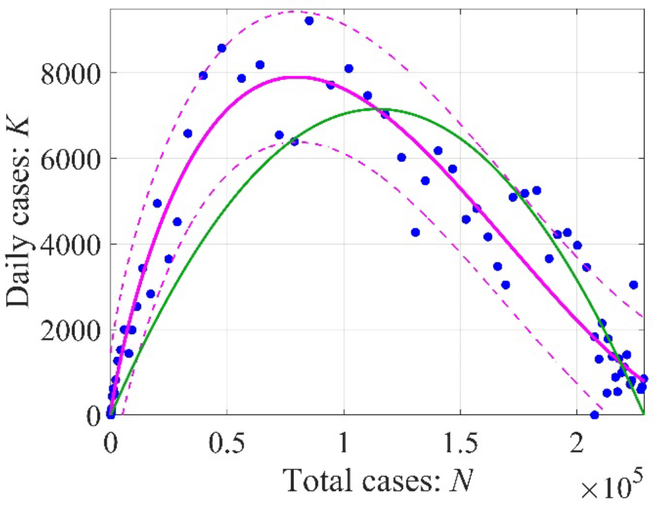
South Korea

$$\alpha = 1.2$$

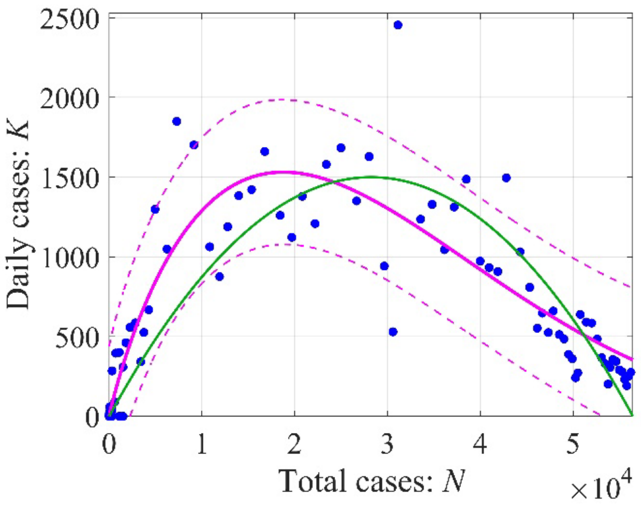
$$\beta = 5.4$$

$R^2 = 0.91$

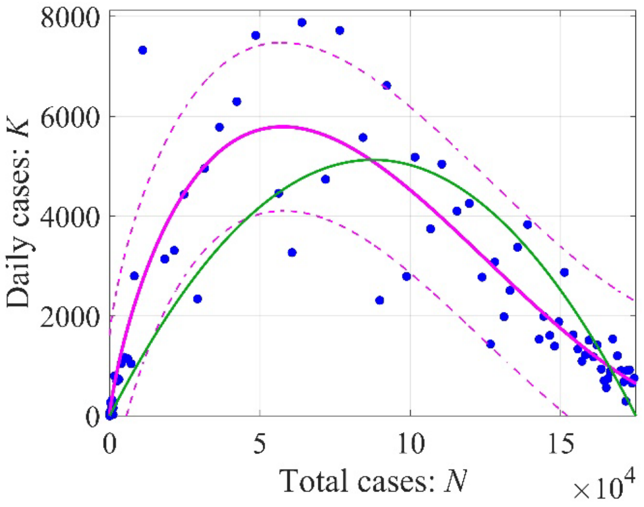
was $R^2 = 0.55$



Spain

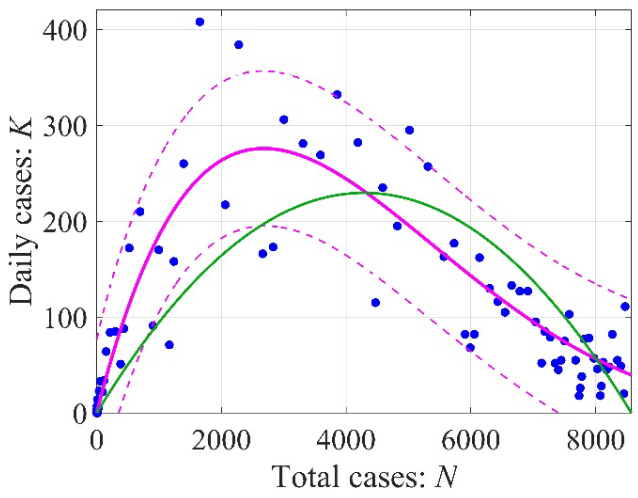


Belgium

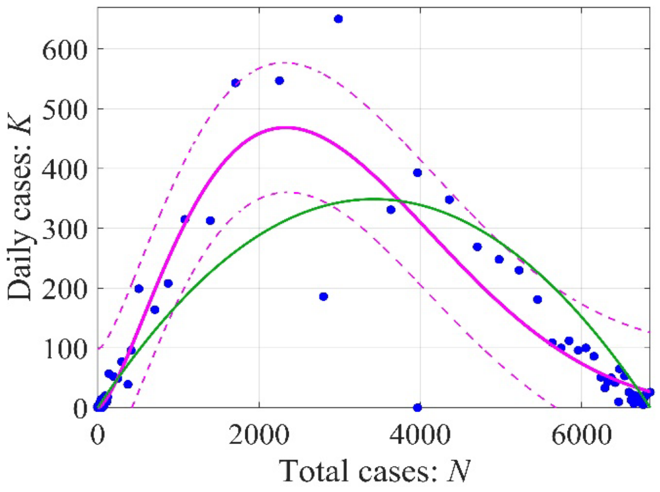


Germany

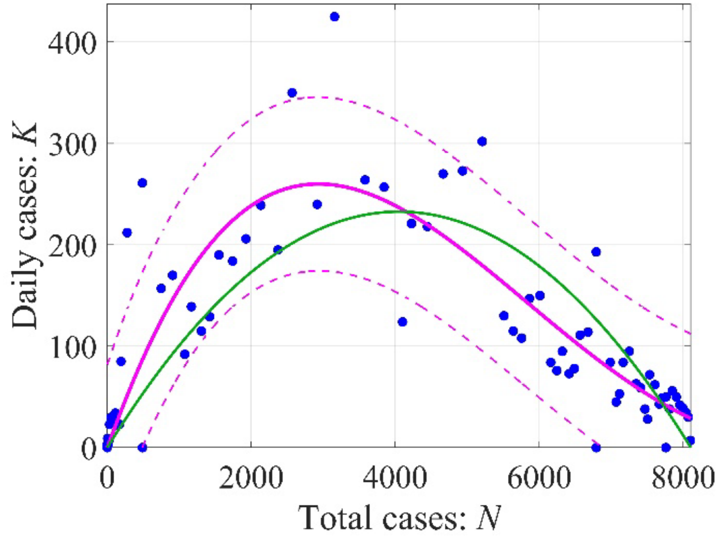
$$R^2 = 0.8 - 0.9$$



Czech

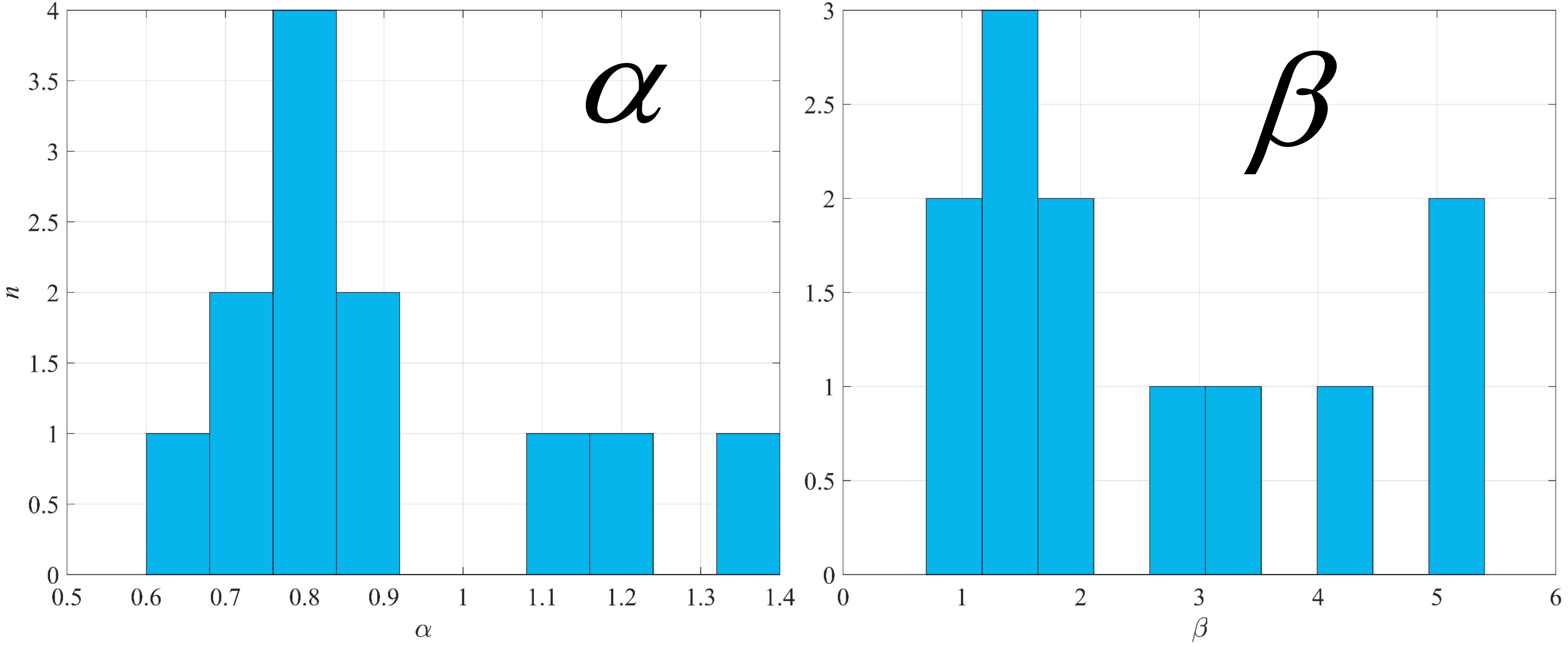


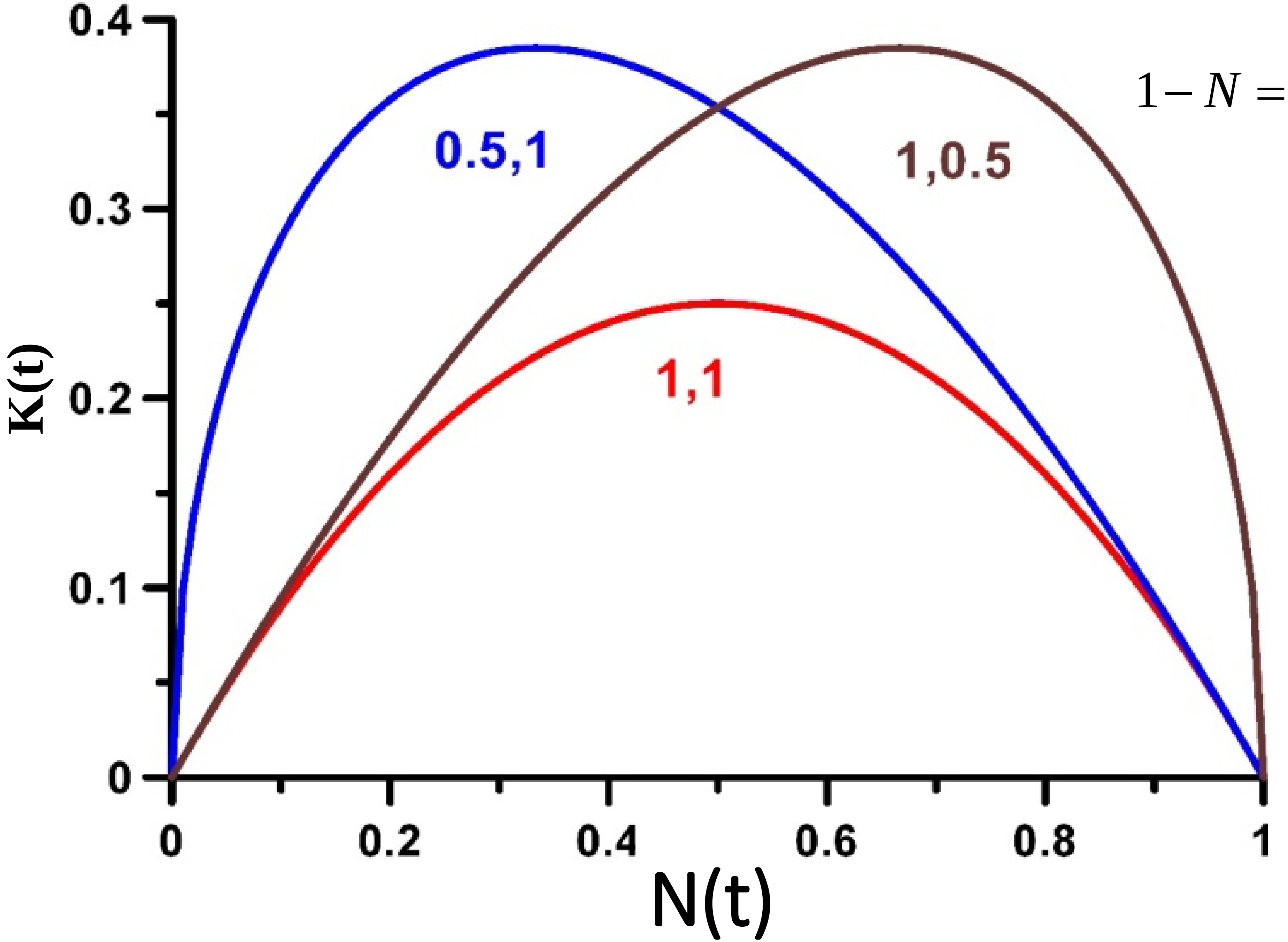
Australia



Norway

Similar analysis performed for 12 countries

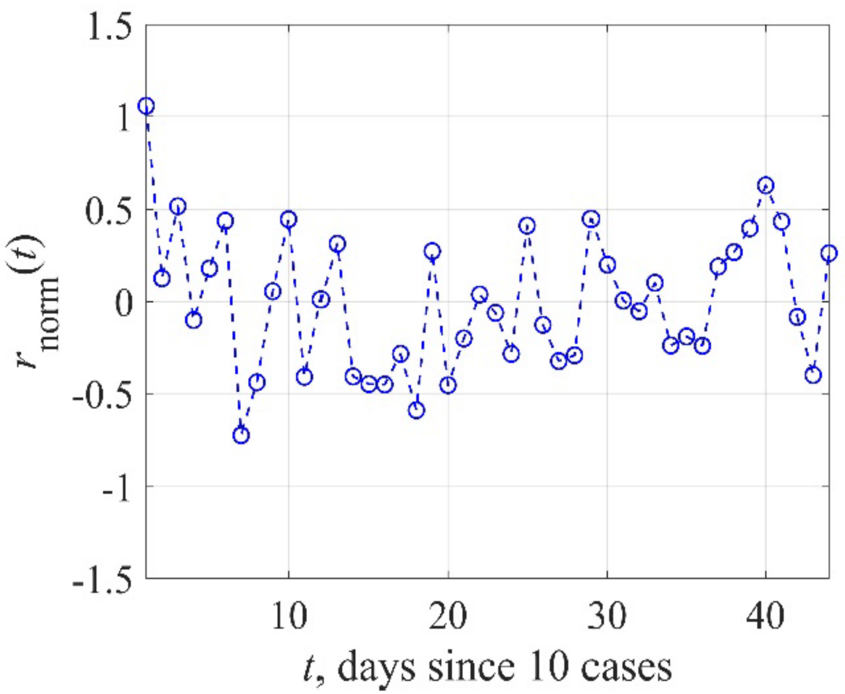


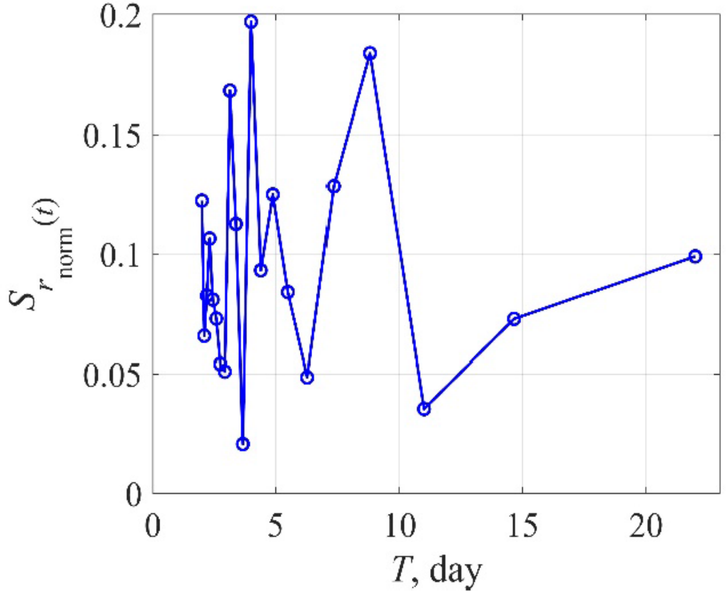


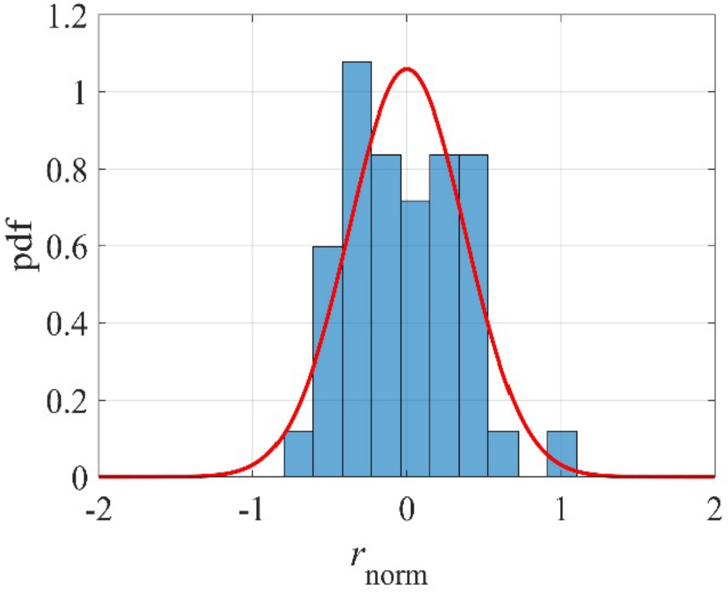
$$1 - N = \begin{cases} \exp(-t), & \beta = 1, \\ \left(\dfrac{\beta - 1}{t}\right)^{\frac{1}{\beta - 1}}, & \beta > 1. \end{cases}$$

Stochastic Generalized Logistic Equation

$$\frac{dN}{dt} = r(t) N^{\alpha} \left(1 - \frac{N}{N_{\infty}} \right)^{\beta}$$



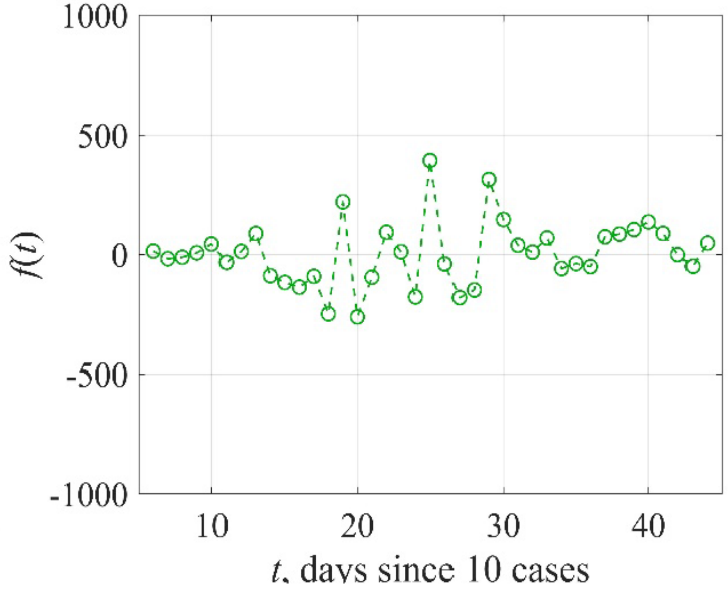


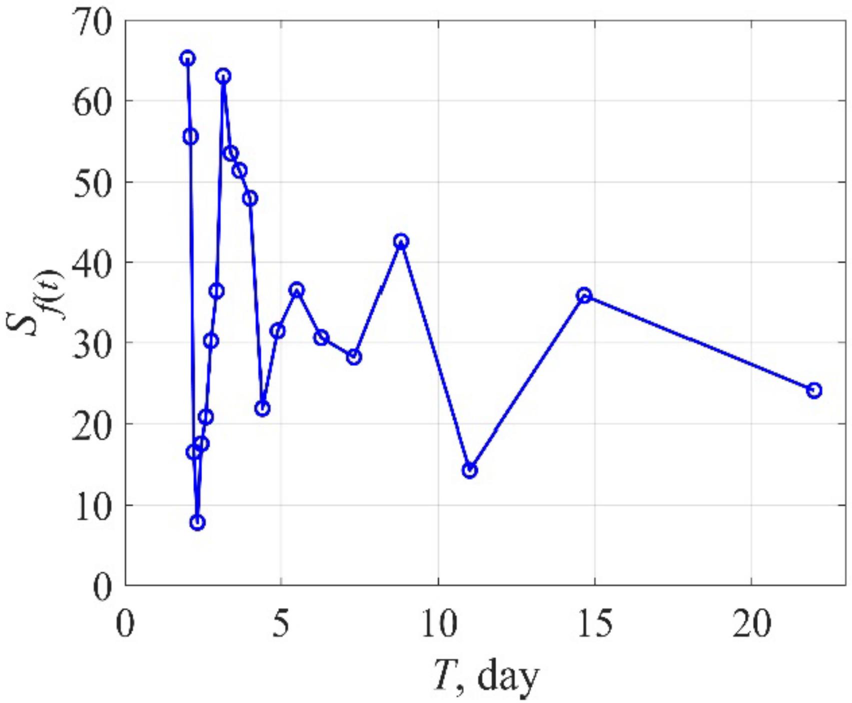


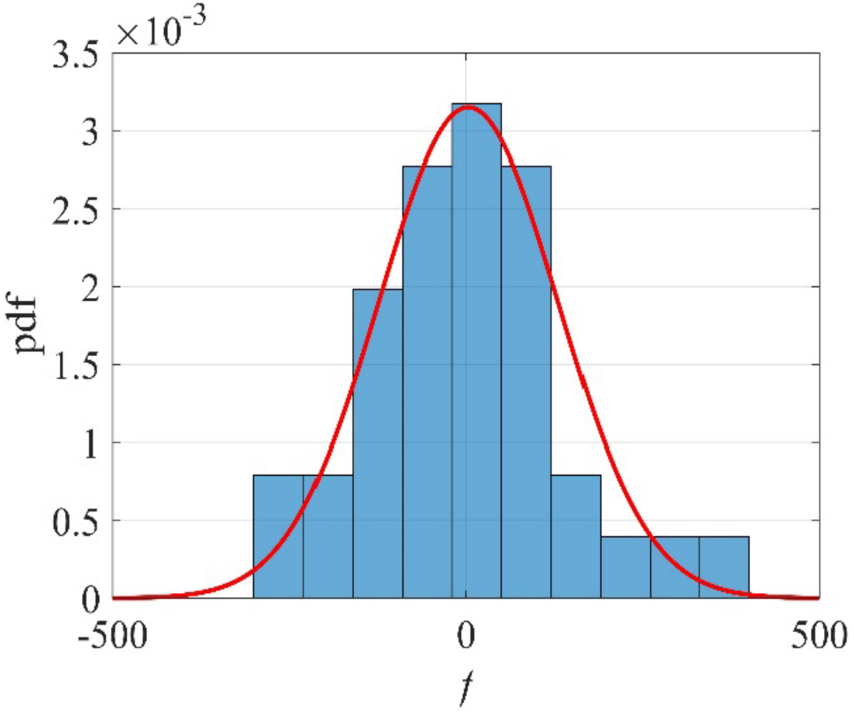
Austria

Forced Stochastic Generalized Logistic Equation

$$\frac{dN}{dt} = r(t) N^{\alpha} \left(1 - \frac{N}{N_{\infty}}\right)^{\beta} + f(t)$$



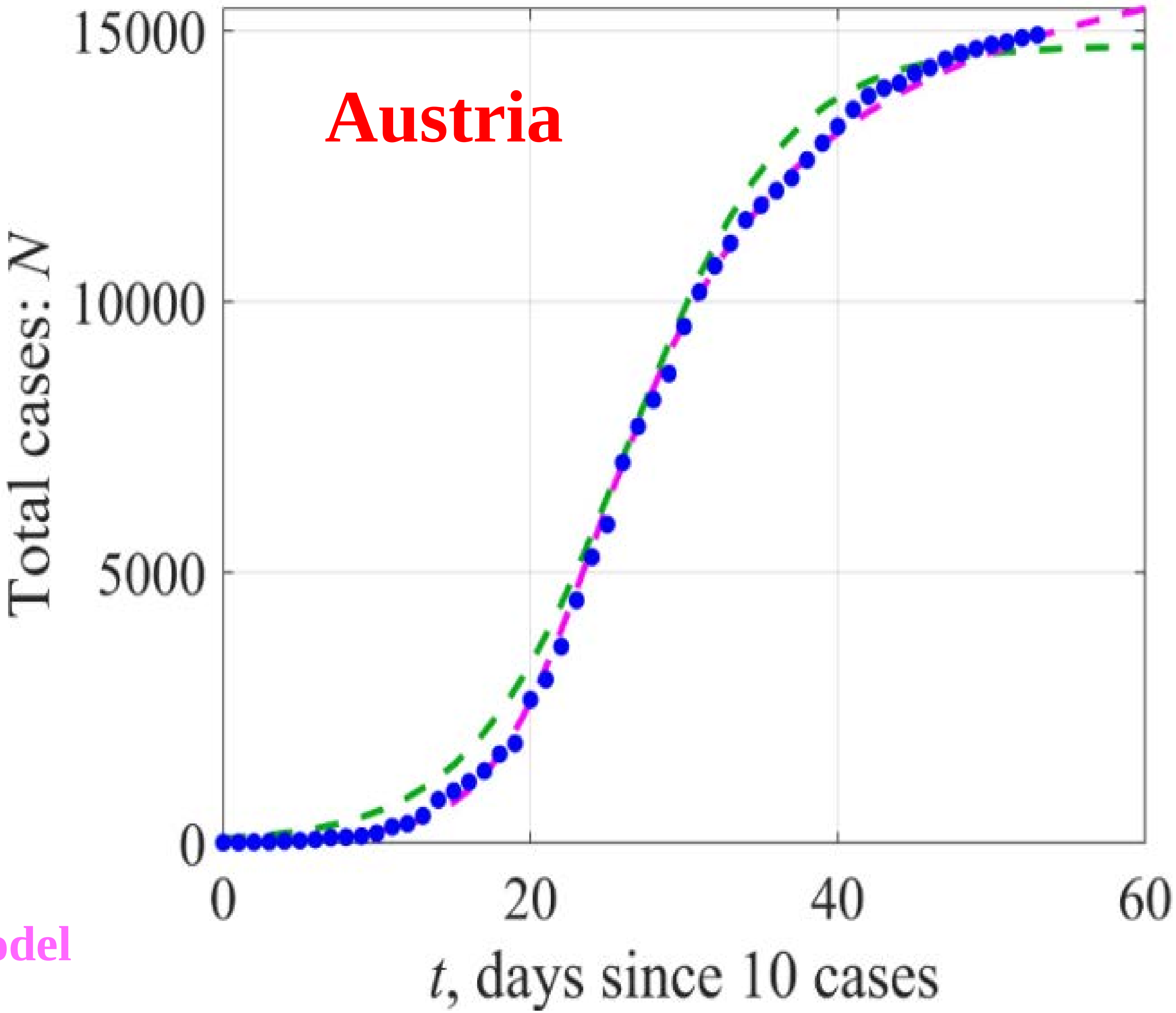




Austria

Epidemic Spread

Almost ideal coinciding in generalized logistic model

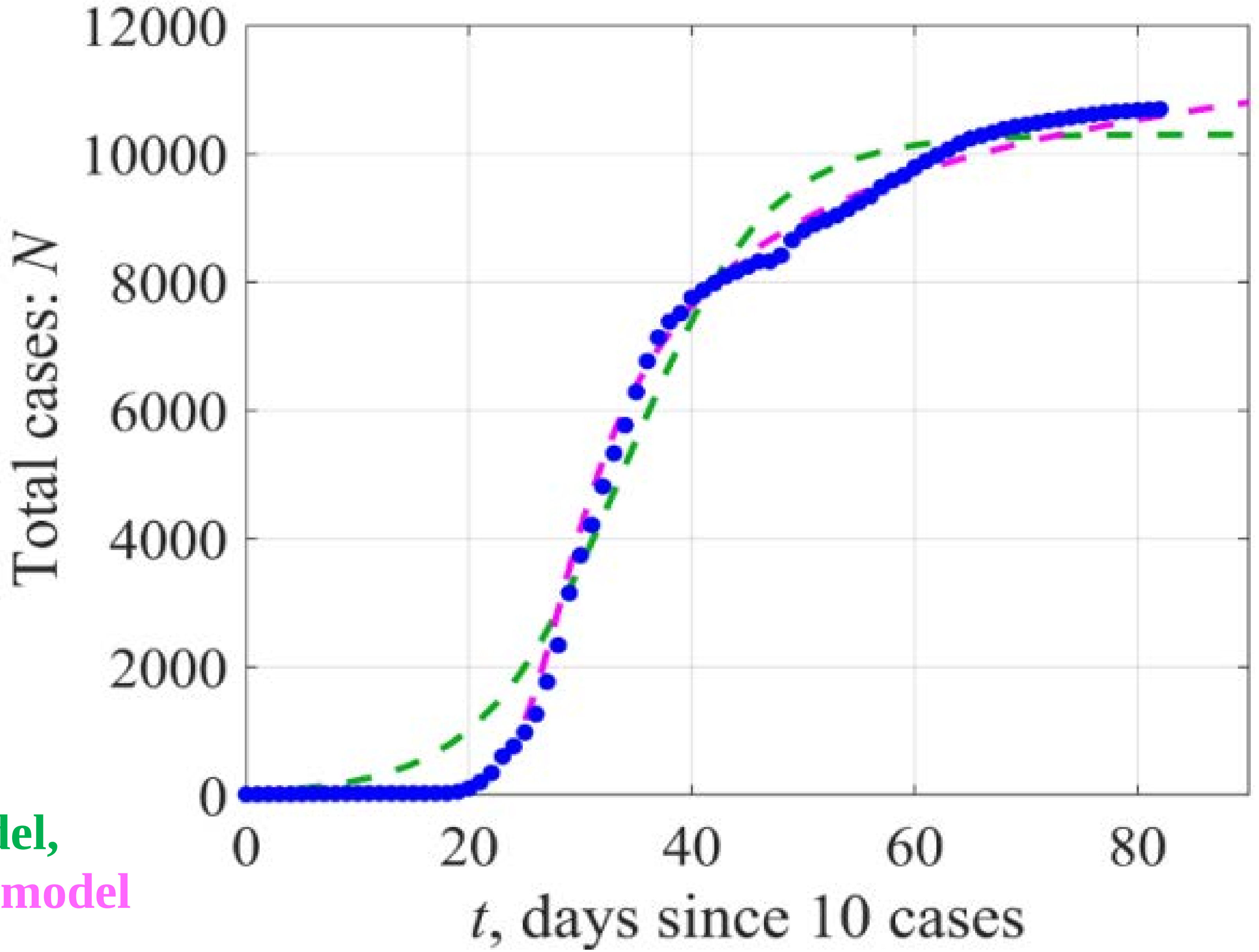


Blue - data,
green – simple logistic model,
pink – generalized logistic model

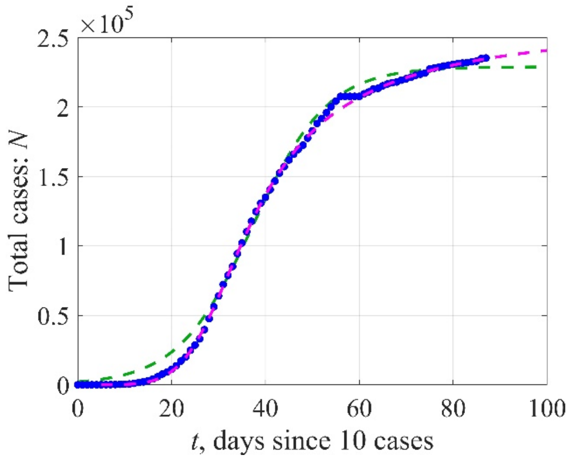
Epidemic Spread

Almost ideal coinciding in generalized logistic model

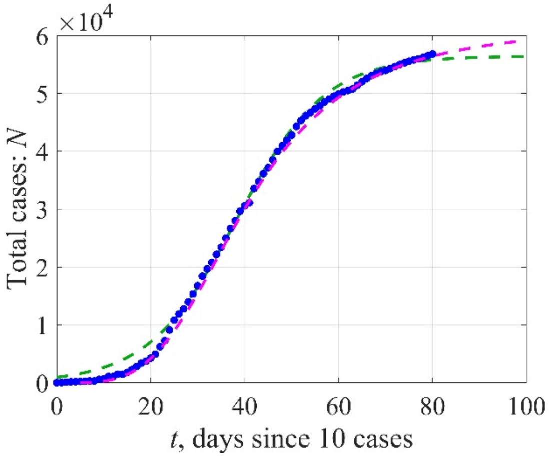
South Korea



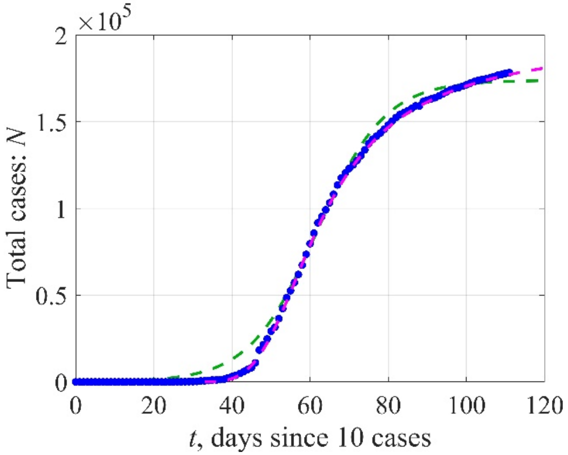
Blue - data,
green – simple logistic model,
pink – generalized logistic model



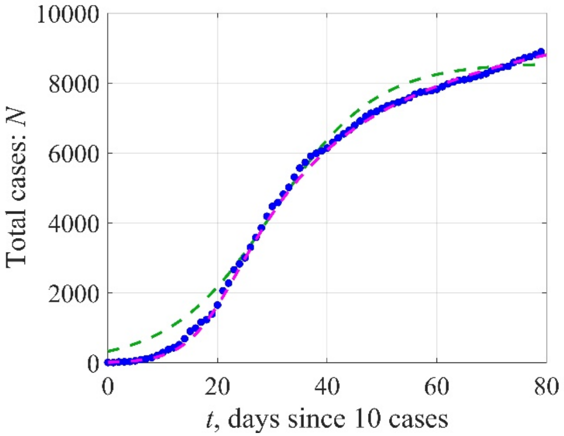
Spain



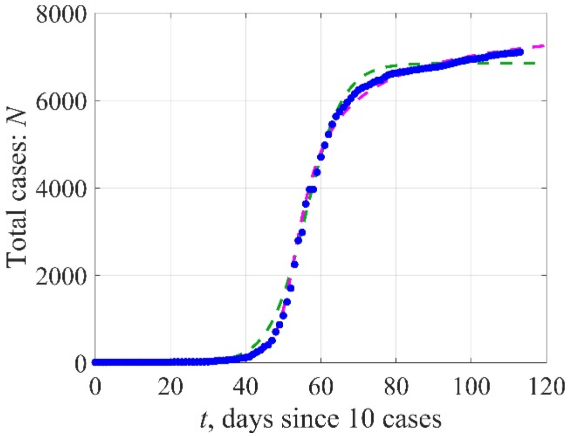
Belgium



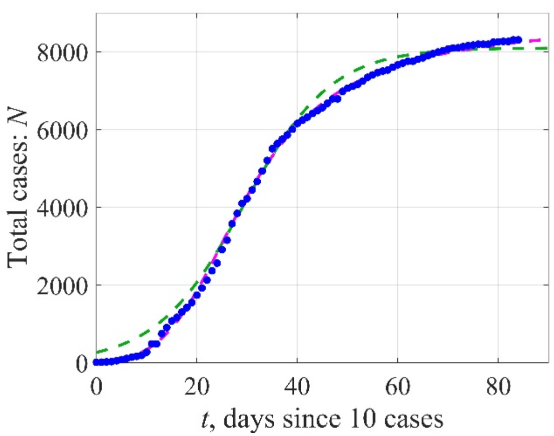
Germany



Czech



Australia

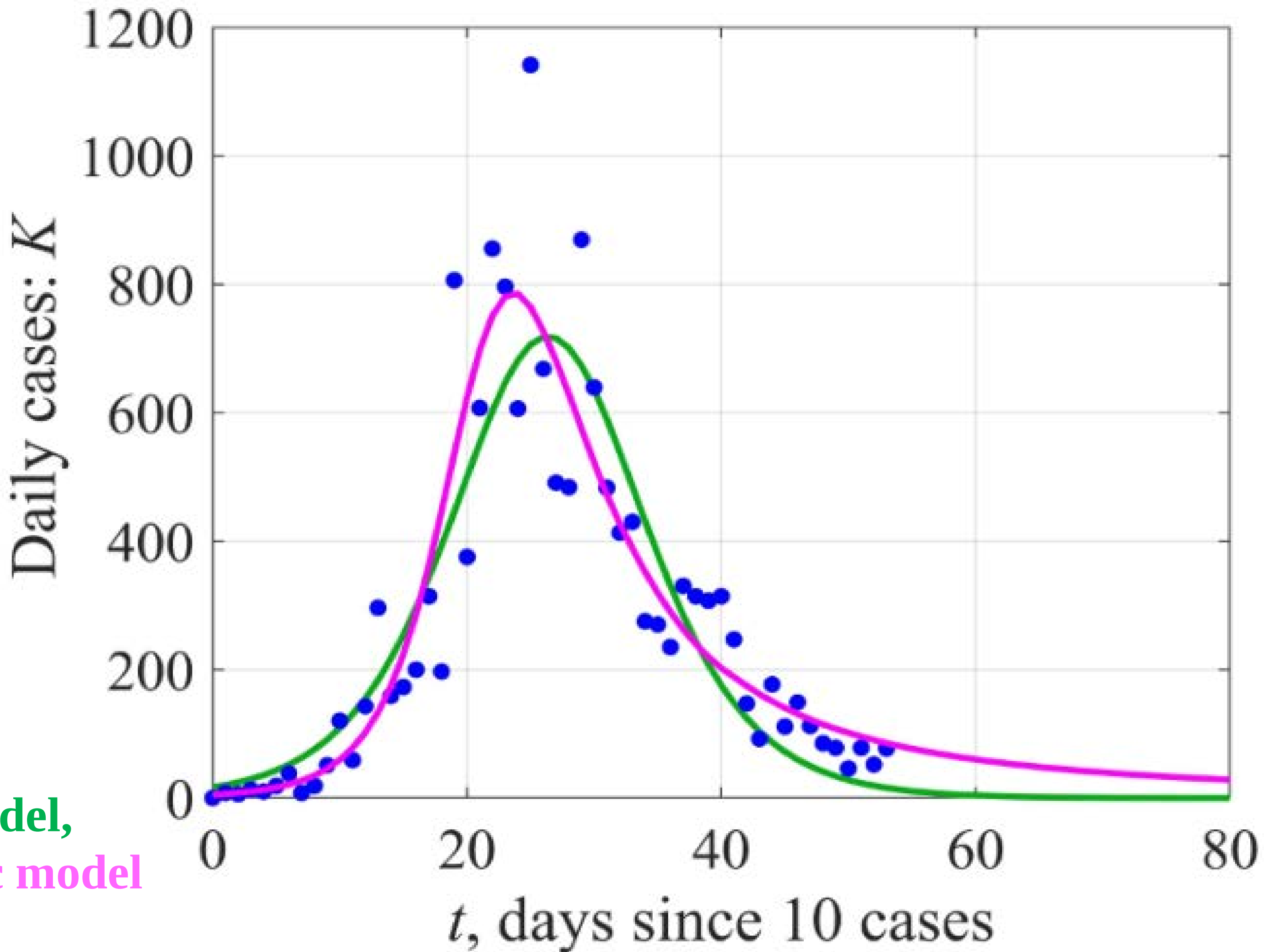


Norway

Daily Epidemic

Non-ideal coinciding in generalized logistic model

Austria



Blue - data,
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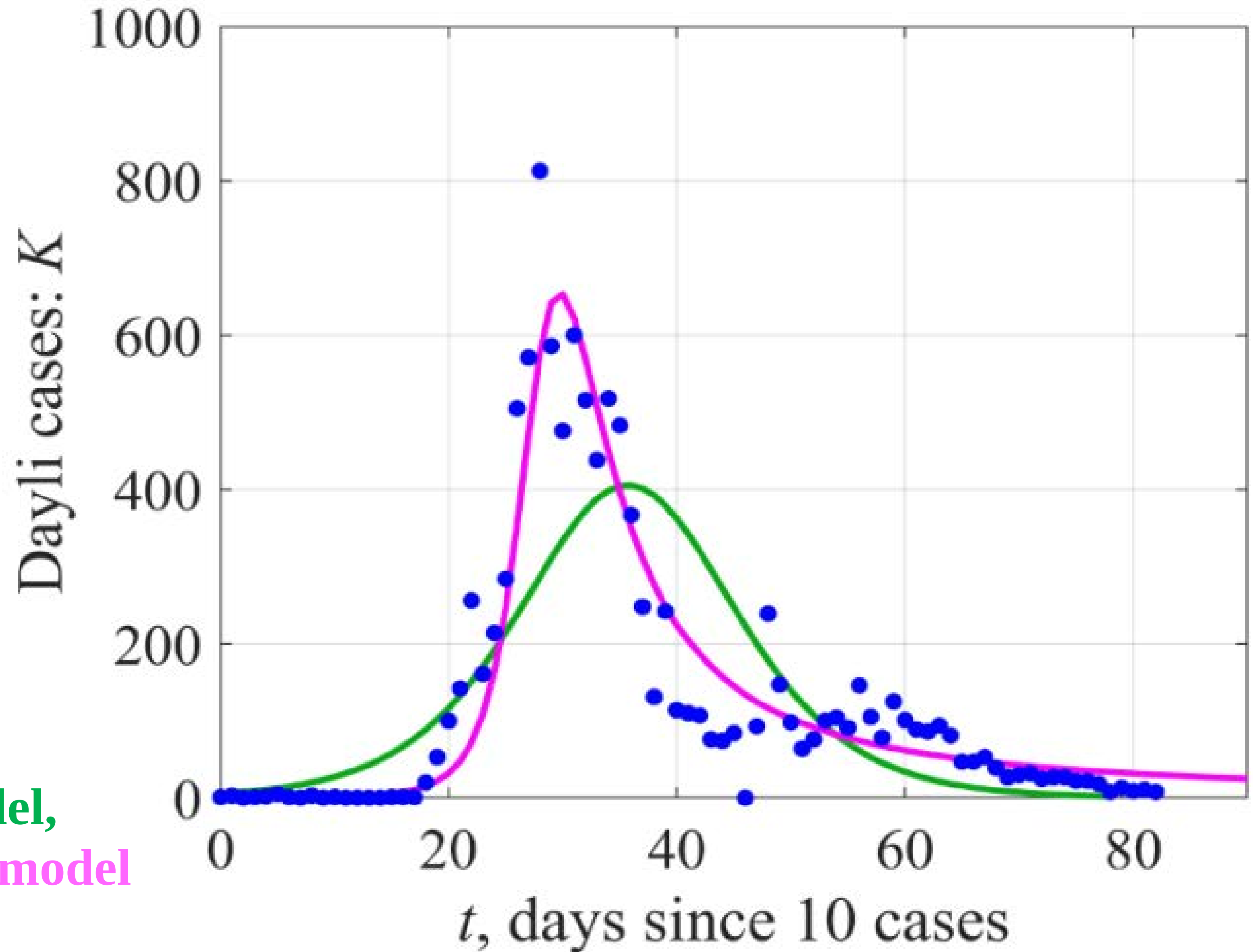
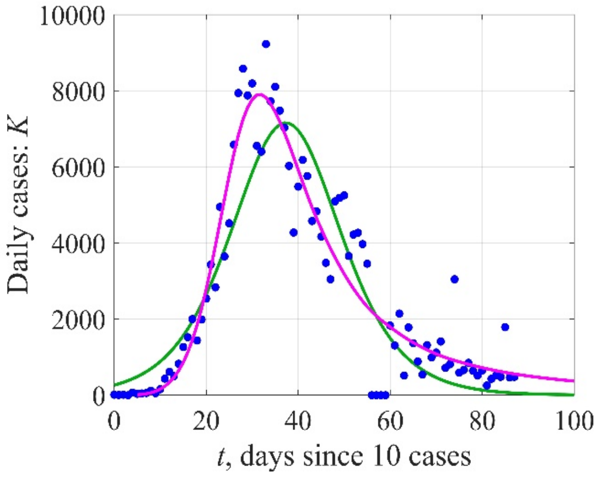
Daily Epidemic

South Korea

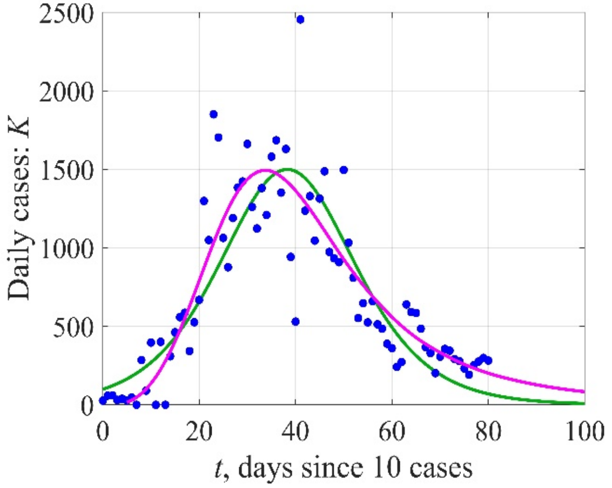
Non-ideal coinciding in generalized logistic model



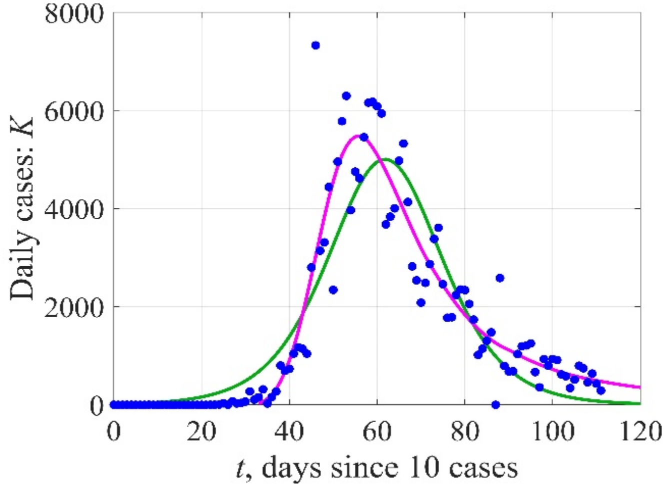
Blue - data,
green – simple logistic model,
pink – generalized logistic model



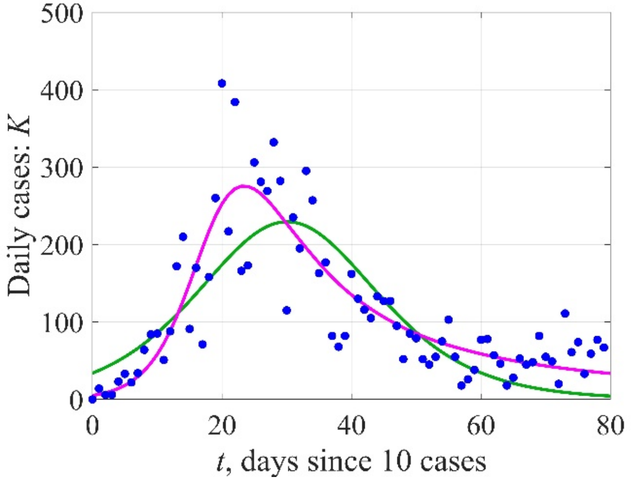
Spain



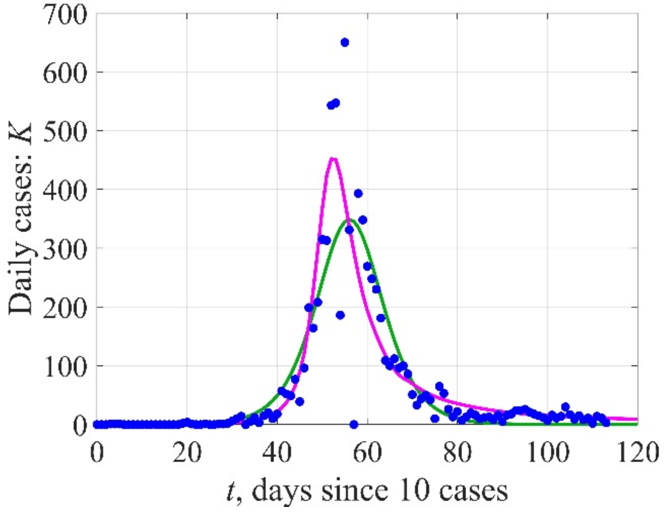
Belgium



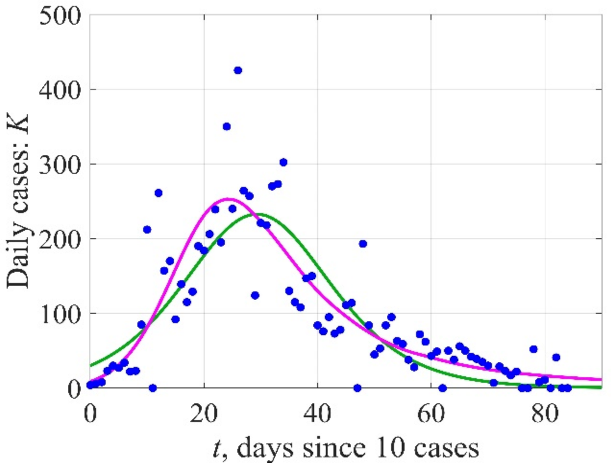
Germany



Czech



Australia



Norway

Pelinovsky, E., Kurkin, A., Kurkina, O., Kokoulina, M., and Epifanova, A. Logistic equation and COVID-19. *Chaos, Solitons and Fractals* (*Nonlinear Science, and Nonequilibrium and Complex Phenomena)*, 2020, vol. 140, 110241.

Kokoulina M.V., Epifanova A.S., Pelinovsky E.N., Kurkina O.E., Kurkin A.A. Analysis of coronavirus dynamics using the generalized logistic model. *Transactions of NNSTU n.a. R.E. Alekseev*, 2020, № 3, 28-41 (in Russian).

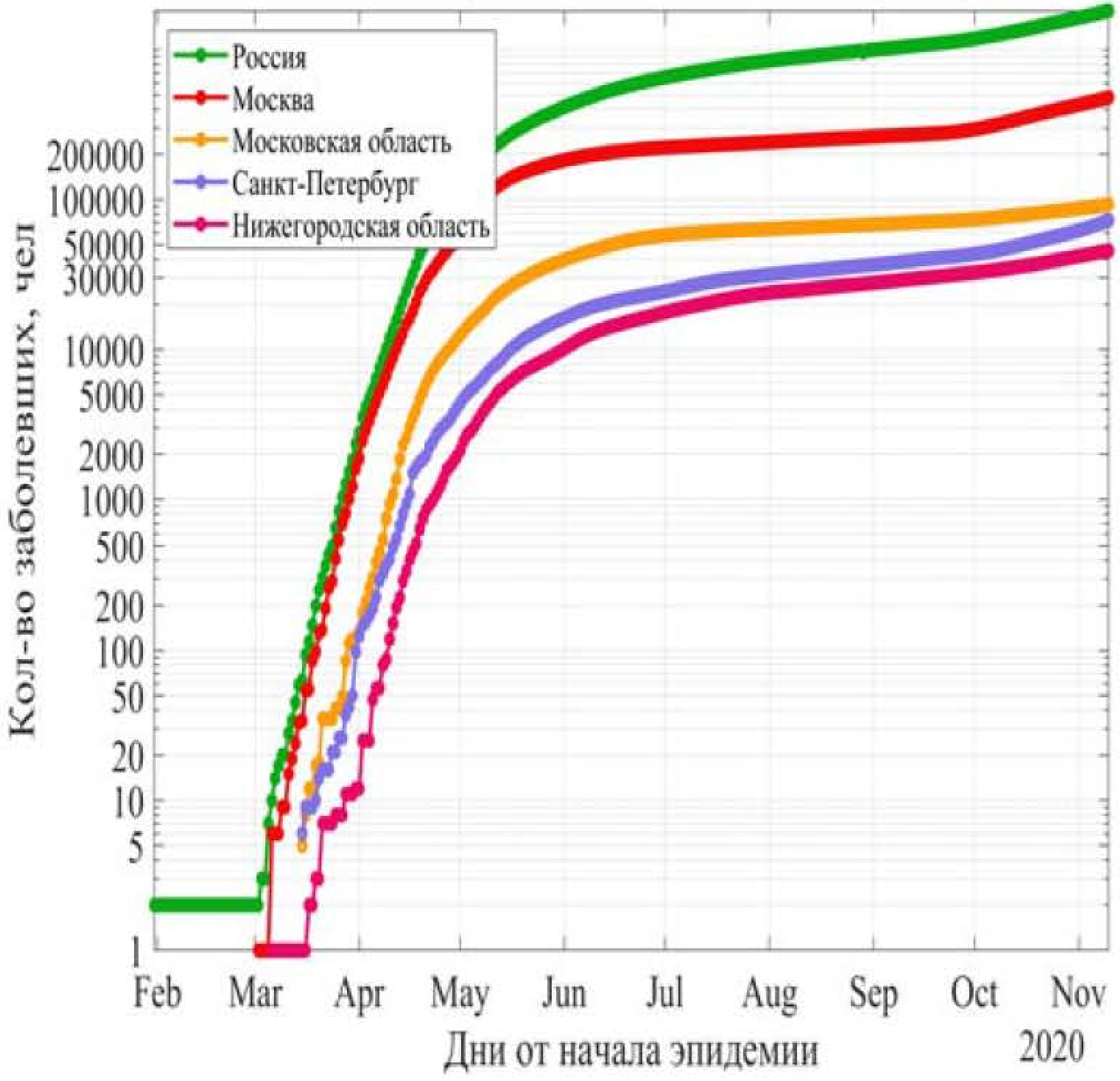
Consolini C., Materassi M. A stretched logistic equation for pandemic spreading. *Chaos, Solitons and Fractals*. 2020. Vol. 140. Art. No. 110113

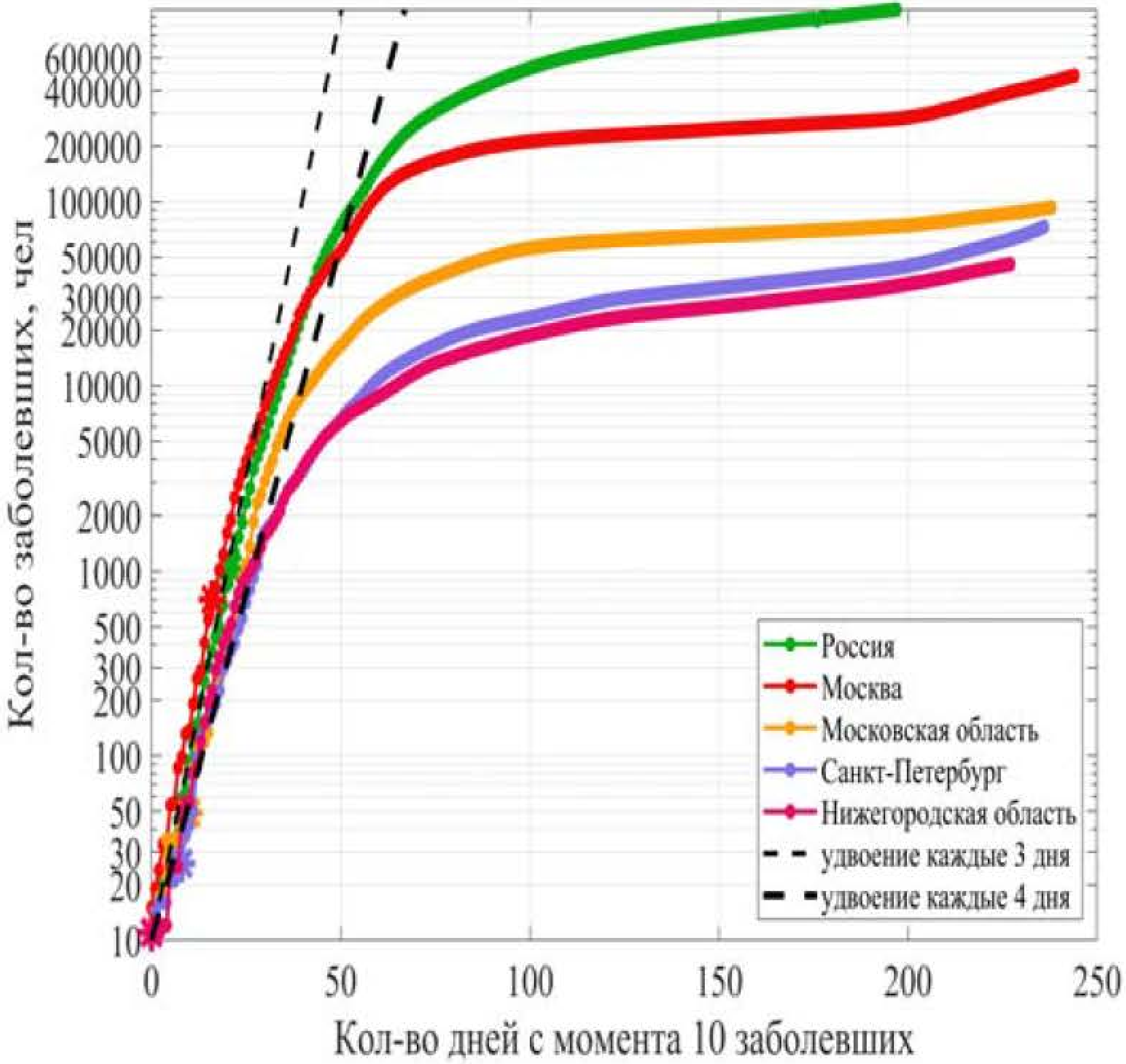
Wu K., Darcet D., Wang Q., Sornette D. Generalized logistic growth modeling of the COVID-19 outbreak: comparing the dynamics in the 29 provinces in China and in the rest of the world. *Nonlinear Dynamics*, 2020, vol. 101, 1561–1581

Carletti T., Fanelli D., Piazza F. COVID-19: The unreasonable effectiveness of simple models. *Chaos, Solitons and Fractals*. 2020. Vol. 140. Art. No. 100034.

Две волны COVID-19 в России

https://lmnad.nntu.ru/ru/projects/covid19/

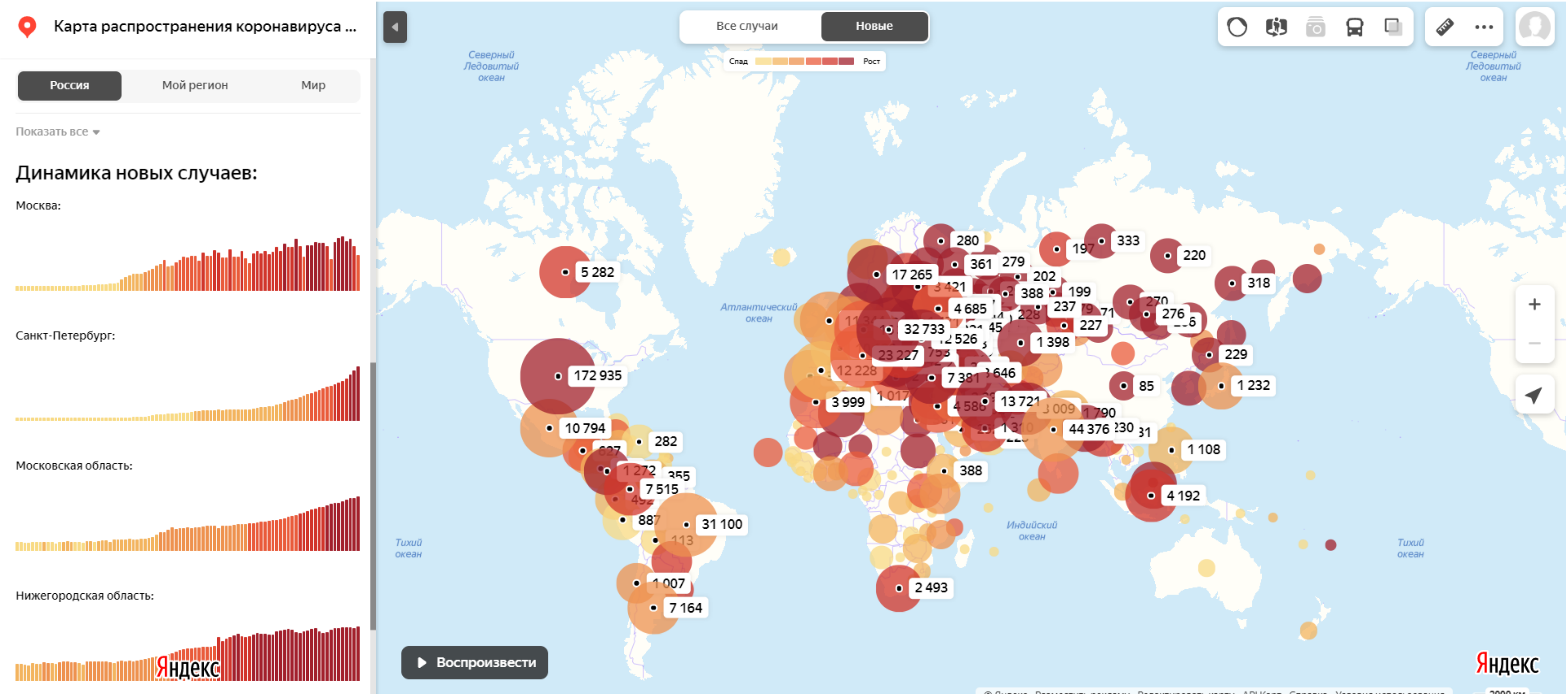




Лаборатория моделирования природных и техногенных катастроф, ННТУ им. Р. Алексеева

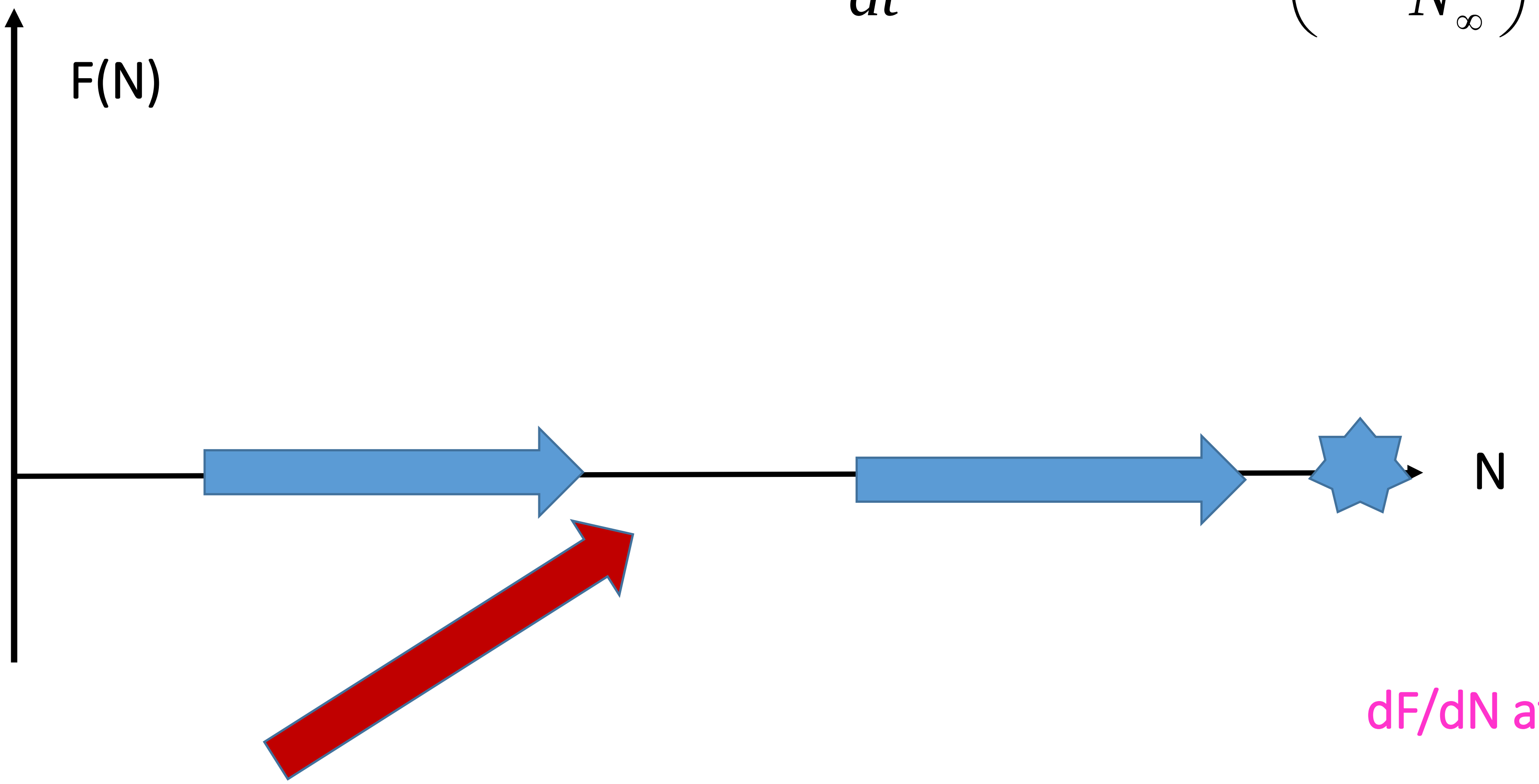
Second Wave of COVID-19

26 November 2020



Second Wave of COVID -19

$$\frac{dN}{dt} = F(N) \neq rN\left(1 - \frac{N}{N_\infty}\right)$$



dF/dN at this point do not exist!

There is “semi-stable” equilibrium point

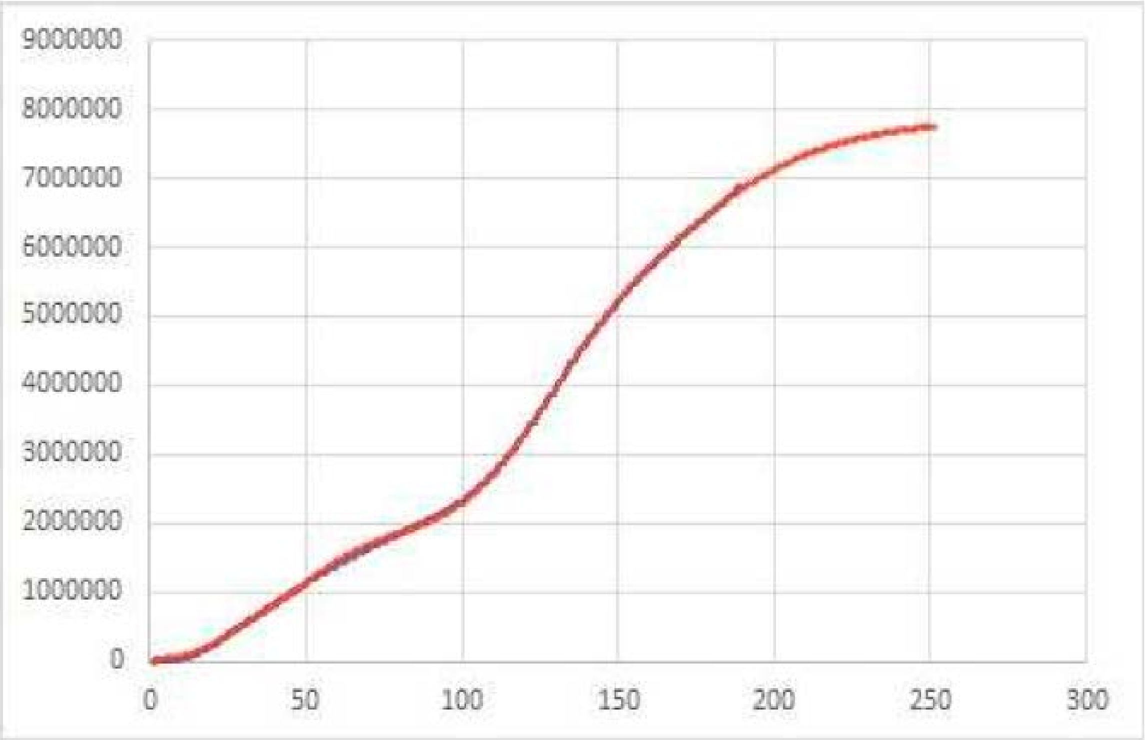
$$F(N) = \begin{cases} r_1 N(1 - N/N_1) & 0 < N < N_1 \\ r_2 (N - N_1)(1 - N/N_2) & N_1 < N < N_2 \end{cases}$$

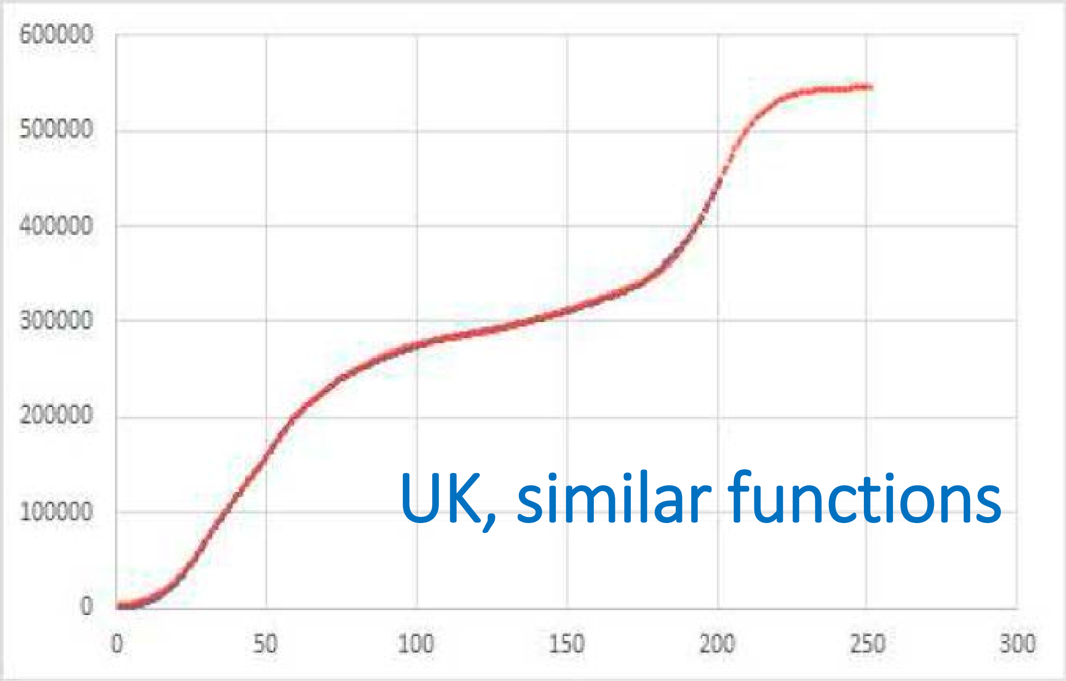
USA, March – September 2020

Grzegorz Rzadkowski, 2020

Logistic wavelets and logistic function: An application to model the spread of SARS-CoV-2 virus infections

http://arxiv.org/abs/2010.09085v1





$$f(x) = \frac{630,913}{1+\exp(-\frac{x-25}{6.6})} + \frac{1,085,184}{1+\exp(-\frac{x-52}{9})} + \frac{3,288,916}{1+\exp(-\frac{x-126}{14.6})} + \frac{2,846,457}{1+\exp(-\frac{x-174}{23.3})}$$

Conclusions:

- **Generalized Logistic model can describe past epidemic**

- **Variable-coefficient models are required to describe daily characteristics**

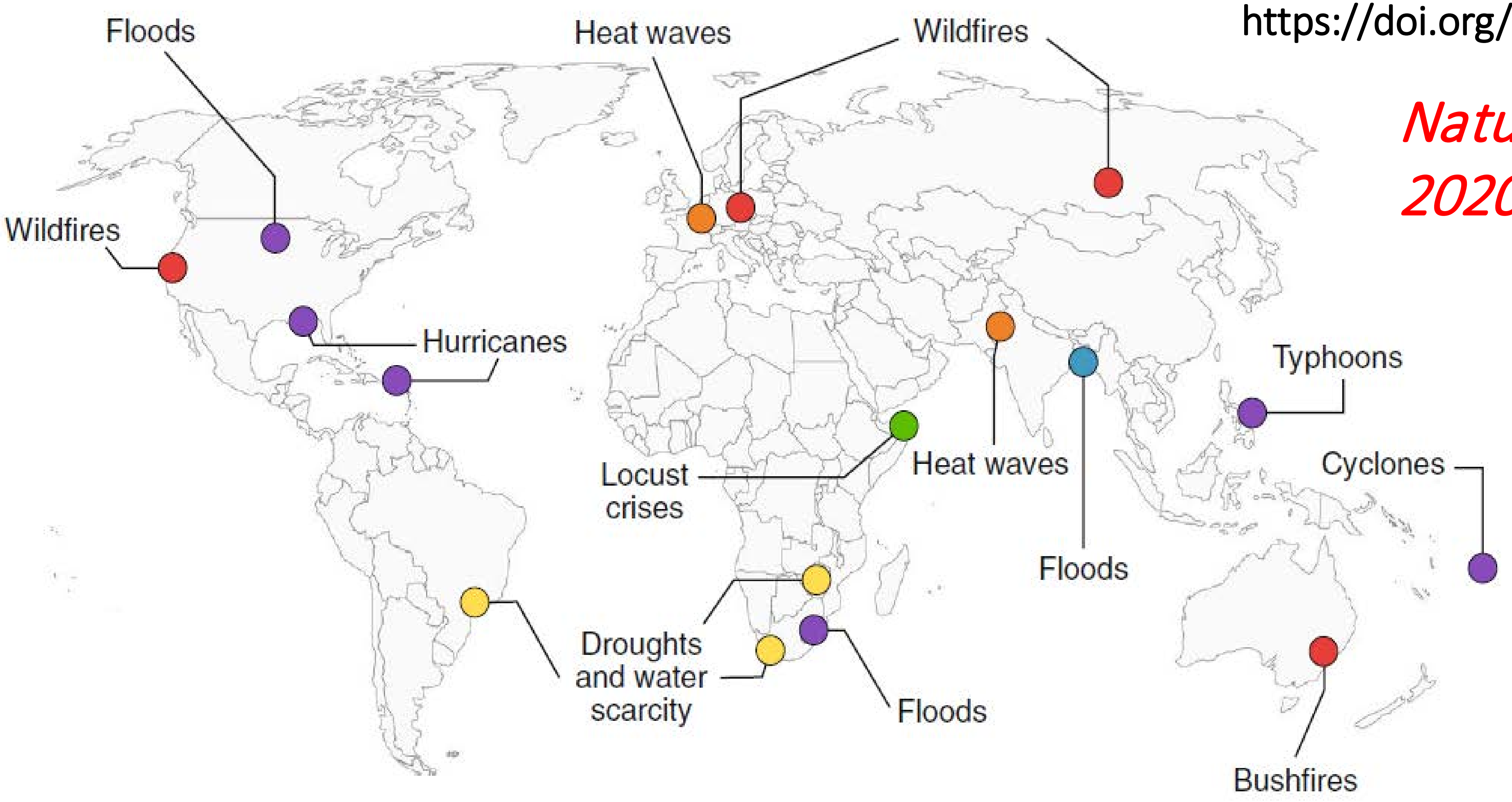
- **Stochastic models can be also applied**

- **But FUTURE is with transport models (PDE)**

– М. Кириллин (ИПФ)

https://doi.org/10.1038/s41558-020-0804-2

Nature Climate Change 2020



Likely upcoming climate hazards during the COVID-19 pandemic. Climate-attributable risks are likely to intersect with the COVID-19 crisis all around the world, with many already causing disruptions or likely to do so over the next 12 to 18 months