Passage of test particles through oscillating spherically-symmetric dark matter configurations

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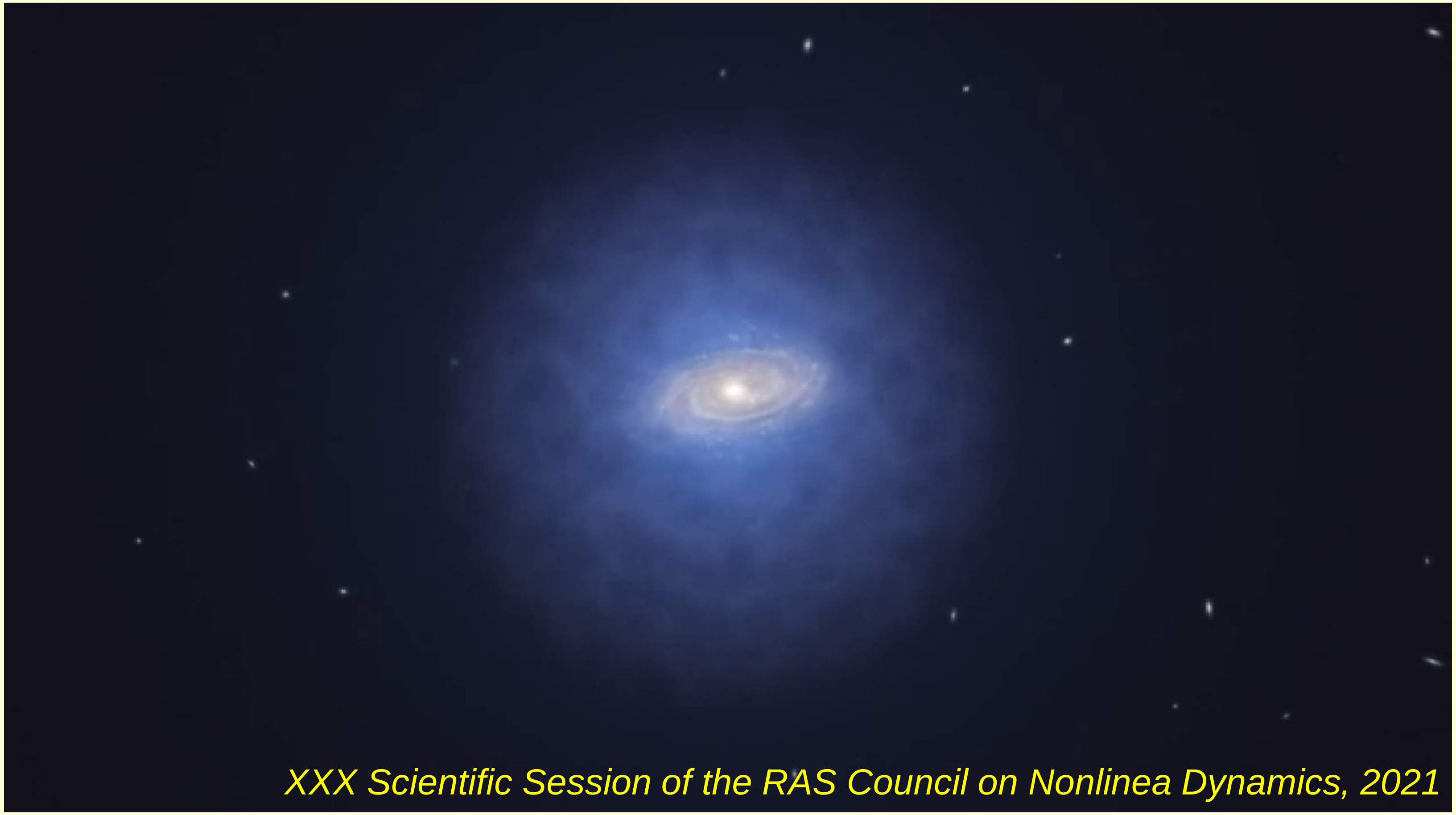


1 Oscillating dark matter

1.1 ΛCDM

Problems at galactic and subgalactic scales: the cusp profile of central densities in galactic halos, the overpopulation of substructures predicted by N-body simulations.

- J. R. Primack (2009)

1.2 SFDM

Fundamental nonlinear scalar field describing coherent state of ultra light particles, e.g., axions, with mass $m \sim 10^{-21} \div 10^{-23}$ eV and $\omega \sim m$ ($T \sim 0.1 \div 10$ years).

- M. S. Turner (1983)
- E. Seidel and W.-M. Suen (1991, 1994)
- P. J. E. Peebles (1999, 2000)
- D. J. E. Marsh (2016)

Primordial fluctuations

$\Downarrow$

Inflation

$\Downarrow$

Homogeneous oscillating background

$\Downarrow$

Parametric instability

$\Downarrow$

Oscillons (pulsons)

$\Downarrow$

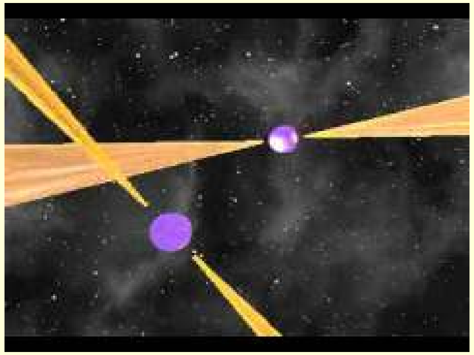
Selfgravity

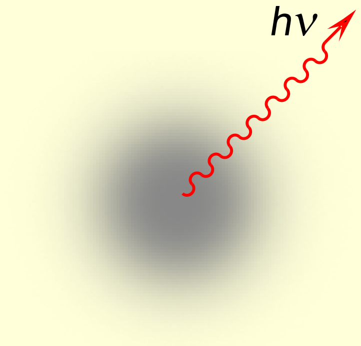
$\Downarrow$

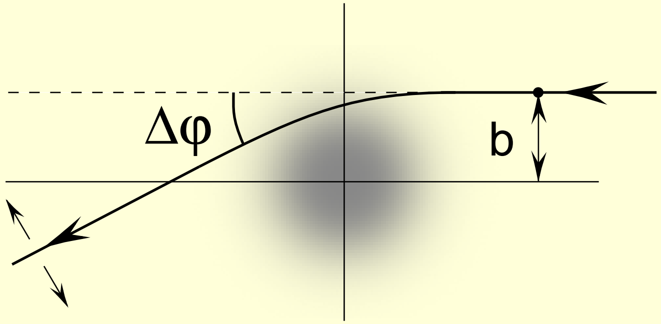
Oscillatons (gravipulsons)

1.3 Possible effects of the dark matter oscillations

- Periodic variations in pulsar timing array (A. Khmelnitsky and V. Rubakov (2014))
- Detecting axion dark matter wind with laser interferometers (A. Aoki and J. Soda (2017))
- Secular variation of the orbital period in binary pulsar systems (D. Blas, D. L. Nacir, and S. Sibiryakov (2017)
- Resonance effects in circular motion of stars at galactic center (M. Bosković et al (2018))
- Periodic variations of spectroscopic emission lines from the stars at the halo center caused by the gravitational frequency shift (M. Bosković et al (2018), V. Koutvitsky and E. Maslov (2019))
- Periodic variations of intensity of images when lensing the distant sources (V. Koutvitsky and E. Maslov (2020))







2 Motion of test particles in time-dependent spherically symmetric gravitational fields

According to the basic concepts of General Relativity, massive particles in a curved space-time move along the geodesics $x^\mu = x^\mu(s)$ satisfying the equation

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0,$$

where ds is a proper time. This equation determines all geodesic characteristics, including the deflection angle when passing near a gravitating mass.

In **static** case, for example, for Schwarzschild spacetime the deflection angle is given by

$$\Delta\varphi = \frac{2GM}{bv^2}\left(1+v^2\right) + O((r_g/b)^2),$$

where G is the gravitational constant, M is the total gravitating mass, b is the impact parameter, v is the initial particle velocity, $r_g = 2GM$, and $r_g/b \ll v^2$.

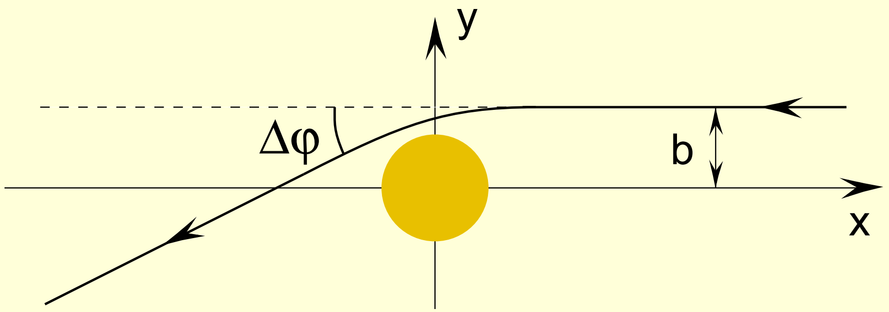


Fig. 1

Consider a spherically symmetric **nonstatic** metric of the form

$$ds^2 = B(t,r)\,dt^2 - A(t,r)\,dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2).$$

For the trajectories lying in the plane $\vartheta = \pi/2$, the geodesic equation reduces to

$$\frac{d}{ds}\ln\left(B\frac{dt}{ds}\right) = \frac{\dot{B}}{2B}\frac{dt}{ds} - \frac{\dot{A}}{2B}\left(\frac{dr}{ds}\right)^2\left(\frac{dt}{ds}\right)^{-1},$$

$$\frac{d^2r}{ds^2} + \frac{B'}{2A}\left(\frac{dt}{ds}\right)^2 + \frac{\dot{A}}{A}\frac{dt}{ds}\frac{dr}{ds} + \frac{A'}{2A}\left(\frac{dr}{ds}\right)^2 - \frac{r}{A}\left(\frac{d\varphi}{ds}\right)^2 = 0,$$

$$\frac{d^2\varphi}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\varphi}{ds} = 0,$$

where $(\cdot) = \partial/\partial t$, $(') = \partial/\partial r$. For a particle coming from infinity with initial velocity v and impact parameter b

$$\frac{d\varphi}{ds} = \frac{bv}{r^2\sqrt{1-v^2}}, \qquad A\left(\frac{dr}{ds}\right)^2 - B\left(\frac{dt}{ds}\right)^2 + \frac{b^2v^2}{r^2\,(1-v^2)} + 1 = 0.$$

In the weak field approximation

$$A = 1 - 2\psi + O(\varkappa^2), \quad B = 1 + 2\chi + O(\varkappa^2),$$

where $\psi(t, r)$ and $\chi(t, r)$ are time-periodic functions of order $\varkappa \ll 1$, and $\varkappa \sim G$.

Trajectory without gravitating mass (straight line):

$$x = x_0 + v(t_0 - t), \quad y = b,$$

$$r(t) = \sqrt{x^2(t) + b^2}, \qquad \frac{dt}{ds} = \frac{1}{\sqrt{1 - v^2}}.$$

With gravitating mass (deflected trajectory):

$$r(t) = (1 + \eta(t))\sqrt{x^2(t) + b^2},$$

$$B\frac{dt}{ds} = \frac{1 + \zeta(t)}{\sqrt{1 - v^2}},$$

$$(\eta \sim \zeta \sim \varkappa \ll 1).$$

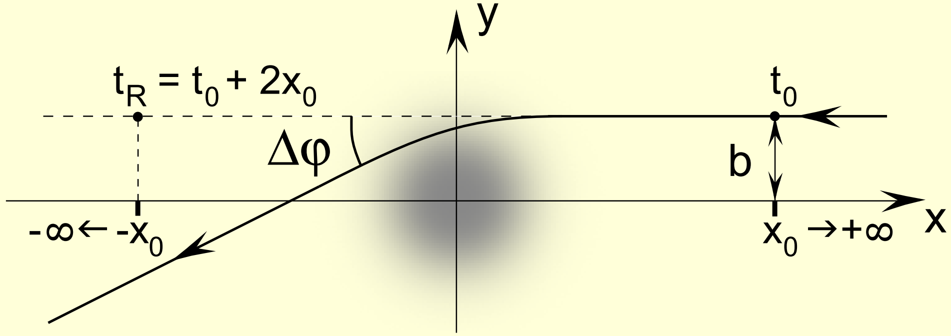


Fig. 1a

From the geodesic equations we obtain

$$\frac{d\zeta}{dt} = \dot{\chi}(t,r) + v^2\dot{\psi}(t,r)\left(1 - \frac{b^2}{r^2}\right),$$

$$vx(x^2+b^2)\frac{d\eta}{dt} - v^2(x^2-b^2)\eta + v^2x^2\psi(t,r) + \left[\left(2v^2-1\right)x^2-b^2\right]\chi(t,r) + \left[\left(1-v^2\right)x^2+b^2\right]\zeta(t) \;=\; 0,$$

$$\frac{d\varphi}{dt} = \frac{vb}{x^2+b^2}\left[1 + (2\chi - \zeta - 2\eta)\right].$$

Integration of these equations gives

$$\zeta = \frac{1}{v}\int_x^\infty \left[\dot{\chi}(t,r) + v^2\dot{\psi}(t,r)\left(1 - \frac{b^2}{r^2}\right)\right] dx,$$

$$\eta = \frac{x}{v^2\left(x^2+b^2\right)}\left\{\int \left[v^2x^2\psi(t,r) + \left(\left(2v^2-1\right)x^2-b^2\right)\chi(t,r) + \left(\left(1-v^2\right)x^2+b^2\right)\zeta(t)\right]\frac{dx}{x^2} + const\right\},$$

$$\varphi = \pi/2 - \mathrm{arctg}(x/b) + b\int_x^\infty \frac{2\chi - \zeta - 2\eta}{x^2+b^2}\,dx,$$

The obtained formula for the deflection angle,

$$\Delta\varphi = b\int_{-\infty}^{\infty} \frac{2\chi - \zeta - 2\eta}{x^2 + b^2}\, dx,$$

is valid not only for time-periodic metrics, but also for static ones. In the latter case $\zeta = 0$.

Consider, for example, the Schwarzschild metric. Assuming $r_g/b = \varkappa \ll 1$, where $r_g = 2GM$ is the gravitational radius, we have

$$\psi = \chi = -\varkappa\frac{b}{2r}.$$

$$\eta = -\varkappa\frac{bx}{2v^2(x^2+b^2)}\left(\frac{\sqrt{x^2+b^2}}{2x} + \left(3v^2 - 1\right)\operatorname{arsh}\frac{x}{b} + const\right).$$

Integration gives the well-known result

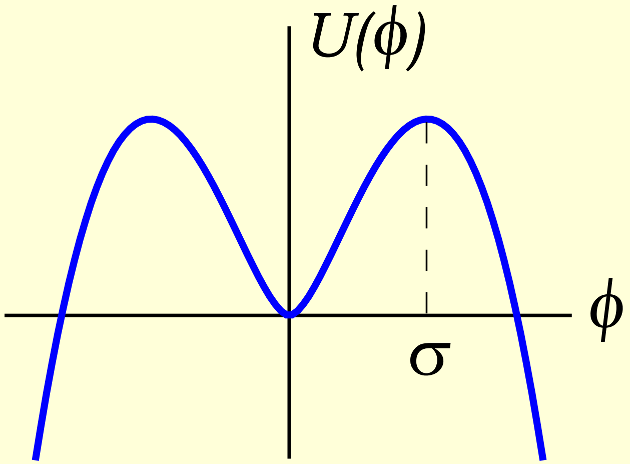
$$\Delta\varphi = \frac{2GM}{bv^2}\left(1 + v^2\right) + O((r_g/b)^2).$$

In the case of a time-periodic metric, the deflection angle will generally depend on the initial time t_0 or, which is the same, on the observation time $t_R = t_0 + 2x_0$.

3 Deflection of particles when passing through a time-periodic spherically symmetric scalar field

As a deflecting mass, we consider a pulsating dark matter halo made from the self-gravitating real scalar field with the potential

$$U(\phi) = \frac{1}{2} m^2 \phi^2 \left(1 - \ln \frac{\phi^2}{\sigma^2}\right)$$



● quantum field theory [G. Rosen (1969), Bialynicki-Birula & Mycielski (1975)]

● inflationary cosmology [Linde (1982, 1992), Albrecht & Steinhardt (1982), Barrow & Parsons (1995)]

● supersymmetric extensions of the Standard Model (flat direction potentials in the gravity mediated supersymmetric breaking scenario) [Enqvist & McDonald (1998)]

Einstein-Klein-Gordon system

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \left[\phi_{,\mu}\phi_{,\nu} - \left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} - U(\phi) \right) g_{\mu\nu} \right],$$

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x_\mu} \left(\sqrt{-g} \frac{\partial \phi}{\partial x^\mu} \right) + \frac{dU(\phi)}{d\phi} = 0.$$

The case of spherical symmetry: $ds^2 = Bdt^2 - Adr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)$:

$$\begin{aligned} \frac{A_r}{A} + \frac{A-1}{r} &= 4\pi G r A \left[\frac{1}{B}\phi_t^2 + \frac{1}{A}\phi_r^2 + m^2\phi^2 \left(1 - \ln\frac{\phi^2}{\sigma^2} \right) \right], \\ \frac{B_r}{B} - \frac{A-1}{r} &= 4\pi G r A \left[\frac{1}{B}\phi_t^2 + \frac{1}{A}\phi_r^2 - m^2\phi^2 \left(1 - \ln\frac{\phi^2}{\sigma^2} \right) \right], \end{aligned}$$

$$\frac{1}{B}\phi_{tt} - \frac{1}{A}\left(\phi_{rr} + \frac{2}{r}\phi_r \right) + \frac{1}{2B}\left(\frac{A_t}{A} - \frac{B_t}{B} \right)\phi_t + \frac{1}{2A}\left(\frac{A_r}{A} - \frac{B_r}{B} \right)\phi_r = m^2\phi \ln\frac{\phi^2}{\sigma^2}.$$

Boundary conditions

$$\phi(t,\infty) = 0,\ A(t,\infty) = 1,\ B(t,\infty) = 1,\ \phi_r(t,0) = 0,\ A(t,0) = 1.$$

This system has a pulsating solution of the form

$$\phi(t,r) = \sigma[a(\theta) + \varkappa Q(\theta,\rho) + O(\varkappa^2)]e^{(3-\rho^2)/2},$$

$$A(t,r) = \left(1 - \frac{\rho_g}{\rho}\right)^{-1}, \quad B(t,r) = \left(1 - \frac{\rho_g}{\rho}\right)e^{-s},$$

where

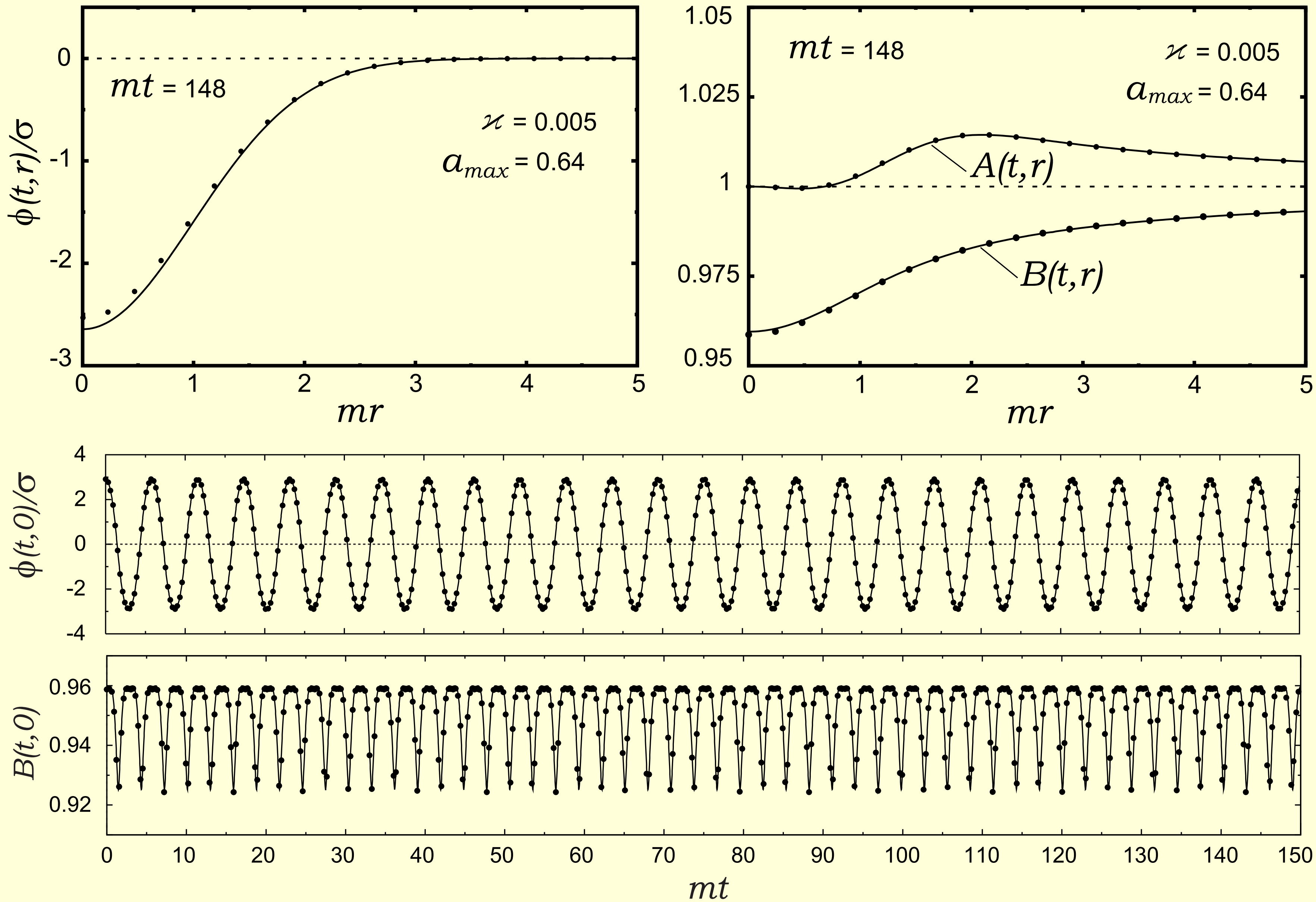
$$\rho_g(\tau,\rho) = -\varkappa\rho\left[V_{\max}\left(1 - \frac{\sqrt{\pi}\,\mathrm{erf}\rho}{2\rho}e^{\rho^2}\right) + a^2\rho^2\right]e^{3-\rho^2} + O(\varkappa^2),$$

$$s(\tau,\rho) = \varkappa(2V_{\max} + a^2\ln a^2 + a^2\rho^2)e^{3-\rho^2} + O(\varkappa^2),$$

$\tau = mt$, $\rho = mr$, $\varkappa = 4\pi G\sigma^2 \ll 1$ (G is the gravitational constant). The function $a(\theta(\tau))$ oscillates in the range $-a_{\max} \leqslant a(\theta) \leqslant a_{\max}$ in the local minimum of the potential $V(a)$:

$$a_{\theta\theta} = -dV/da, \qquad V(a) = (a^2/2)\left(1 - \ln a^2\right) \leqslant V_{\max} = V(a_{\max}),$$

where $\theta_\tau = 1 + \varkappa\Omega + O(\varkappa^2)$, and the constant $\varkappa\Omega$ is the pulson frequency correction due to gravitational effects. The function $Q(\theta,\rho)$ is a series in Hermite polynomials whose coefficients are periodic (in θ) solutions of nonhomogeneous Hill equations.



mt = 148
ϰ = 0.005
a_max = 0.64
ϕ(t,r)/σ
mr
A(t,r)
B(t,r)
ϕ(t,0)/σ
B(t,0)
mt

Since the metric found is everywhere regular and has no horizon, we can rewrite the functions $A(t,r)$ and $B(t,r)$ with the required accuracy in the form

$$A = 1 - 2\psi + O(\varkappa^2), \quad B = 1 + 2\chi + O(\varkappa^2),$$

where

$$\psi(t,r) = \frac{\varkappa}{2}\left[V_{\max}\left(1 - \frac{\sqrt{\pi}\mathrm{erf}\rho}{2\rho}e^{\rho^2}\right) + a^2\rho^2\right]e^{3-\rho^2},$$

$$\chi(t,r) = -\frac{\varkappa}{2}\left[V_{\max}\left(1 + \frac{\sqrt{\pi}\mathrm{erf}\rho}{2\rho}e^{\rho^2}\right) + a^2\ln a^2\right]e^{3-\rho^2},$$

and $\rho = mr$, $\tau = mt$. Calculating $\dot{\psi}(t,r)$, $\dot{\chi}(t,r)$ and setting

$$\begin{aligned} \tau &= \tau_R - (\xi_0 + \xi)/v, \quad \tau_R = mt_R, \quad \xi_0 = mx_0 \to \infty \\ \rho^2 &= \xi^2 + \beta^2, \quad \xi = mx, \quad \beta = mb \qquad d/d\tau = -d/d\xi, \end{aligned}$$

we find

$$\zeta = \frac{\varkappa}{2}e^{3-\beta^2}\int_{\xi}^{\infty}\left[\frac{d}{d\xi}\left(a^2\ln a^2\right) - v^2\xi^2\frac{d}{d\xi}a^2\right]e^{-\xi^2}d\xi.$$

On the other hand,

$$v^2\frac{d^2a^2}{d\xi^2} = \frac{d^2a^2}{d\tau^2} = \frac{d^2a^2}{d\theta^2}\theta_\tau^2 = 4V_{\max} - 2a^2 + 4a^2\ln a^2 + O(\varkappa).$$

Using these relations and integrating by parts, we finally obtain

$$\zeta = -\frac{\varkappa}{4}e^{3-\beta^2}\left[v^2e^{-\xi^2}\left(\frac{1}{2}\frac{d^2}{d\xi^2} + \xi\frac{d}{d\xi}\right)a^2 - \left(1-v^2\right)\int_\xi^\infty \frac{da^2}{d\xi}e^{-\xi^2}d\xi\right] + O(\varkappa^2).$$

Now we substitute ψ, χ and ζ into the formula for η and integrate over ξ. This gives

$$\eta = \frac{\varkappa}{4v^2}e^{3-\beta^2}\left\{\sqrt{\pi}V_{\max}\left[\left(1+v^2\right)\frac{\xi\operatorname{erf}\xi}{\rho^2} - e^{\beta^2}\left(\left(3v^2-1\right)\frac{\xi}{\rho^2}\int_0^\xi \frac{\operatorname{erf}\rho}{\rho}d\xi + \frac{\operatorname{erf}\rho}{\rho}\right)\right]\right.$$

$$- v^2e^{-\xi^2}\left(a^2 + v^2\frac{\xi}{2\rho^2}\frac{da^2}{d\xi}\right) - \frac{1-v^2}{\rho^2}\left[\left(v^2-2\beta^2\right)\xi\int_\xi^\infty a^2e^{-\xi^2}d\xi\right.$$

$$\left.\left. - 2\left[\left(1-v^2\right)\xi^2-\beta^2\right]\int_\xi^\infty a^2\xi e^{-\xi^2}d\xi + 2\left(1-v^2\right)\xi\int_\xi^\infty a^2\xi^2e^{-\xi^2}d\xi\right] + const\,\frac{\xi}{\rho^2}\right\} + O(\varkappa^2).$$

Now we rewrite the general expression for the deflection angle as

$$\Delta\varphi = \beta \int_{-\infty}^{\infty} \frac{2\chi - \zeta - 2\eta}{\xi^2 + \beta^2}\, d\xi$$

and substitute there

$$2\chi - \zeta - 2\eta = \varkappa e^{3-\beta^2} \Bigg\{ \frac{\sqrt{\pi} V_{\max}}{2v^2} \Bigg[\frac{\xi}{\rho^2} \Bigg(\left(3v^2 - 1\right) e^{\beta^2} \int_0^{\xi} \frac{\mathrm{erf}\rho}{\rho}\, d\xi - \left(1 + v^2\right) \mathrm{erf}\xi \Bigg)$$

$$+ \left(1 - v^2\right) e^{\beta^2} \frac{\mathrm{erf}\rho}{\rho} \Bigg] - \frac{v^2}{4} e^{-\xi^2} \Bigg[\frac{1}{2} \frac{d^2 a^2}{d\xi^2} - \left(1 + \frac{1}{\rho^2}\right) \xi \frac{da^2}{d\xi} \Bigg] + \frac{1}{4} \left(1 - v^2\right) a^2 e^{-\xi^2}$$

$$+ \frac{1 - v^2}{2v^2} \Bigg[\left(v^2 - 2\beta^2\right) \frac{\xi}{\rho^2} \int_{\xi}^{\infty} a^2 e^{-\xi^2} d\xi - \left(2 - v^2\right) \left(1 - \frac{2\beta^2}{\rho^2}\right) \int_{\xi}^{\infty} a^2 \xi e^{-\xi^2} d\xi$$

$$+ 2\left(1 - v^2\right) \frac{\xi}{\rho^2} \int_{\xi}^{\infty} a^2 \xi^2 e^{-\xi^2} d\xi \Bigg] + const\, \frac{\xi}{\rho^2} \Bigg\} + O(\varkappa^2).$$

As a result, after integrating, we arrive at a simple formula

$$\Delta\varphi = \frac{2GM}{bv^2}\left(1+v^2\right)\left(1-e^{-m^2b^2}\right)+\varkappa\frac{mb}{2v^2}\left(1-v^2\right)e^{3-m^2b^2}\int_{-\infty}^{\infty}a^2e^{-\xi^2}d\xi+O(\varkappa^2)$$

where G is gravitational constant, M is the halo mass,

$$M = \left(e\sqrt{\pi}\right)^3\sigma^2m^{-1}V_{\max}\left(1+O(\varkappa)\right),$$

b is the impact parameter, v is the initial particle velocity, $v^2 \gg 2GM/b$, and $\varkappa = 4\pi G\sigma^2$ is small parameter. The function $a(\theta)$ is found from the equation $a_{\theta\theta} = -dV/da$ followed by the substitution $\theta = (1+\varkappa\Omega)(\tau_R - \xi/v)$.

For $a^2_{\max} \ll 1$ we find $a(\theta) \approx a_{\max}\cos\omega_0\theta$ with $\omega_0 \approx \sqrt{1-\ln a^2_{\max}}$, and

$$\frac{\Delta\varphi}{\varkappa} \approx \frac{e^3\sqrt{\pi}}{4v^2}a^2_{\max}\left[\frac{\omega_0^2}{\beta}\left(1+v^2\right)\left(1-e^{-\beta^2}\right)+\left(1-v^2\right)\beta e^{-\beta^2}\left(1+e^{-(\omega_0/v)^2}\cos 2\omega_0\theta_R\right)\right],$$

where $\theta_R = (1+\varkappa\Omega)\,\tau_R$, $\beta = mb$.

In general, averaging $\Delta\varphi$ over the period, we find

$$\overline{\Delta\varphi} = \varkappa\frac{e^3\sqrt{\pi}}{2\beta v^2}\left[V_{\max}\left(1+v^2\right)\left(1-e^{-\beta^2}\right) + \left(1-v^2\right)\beta^2e^{-\beta^2}\overline{a^2}\right]+O(\varkappa^2).$$

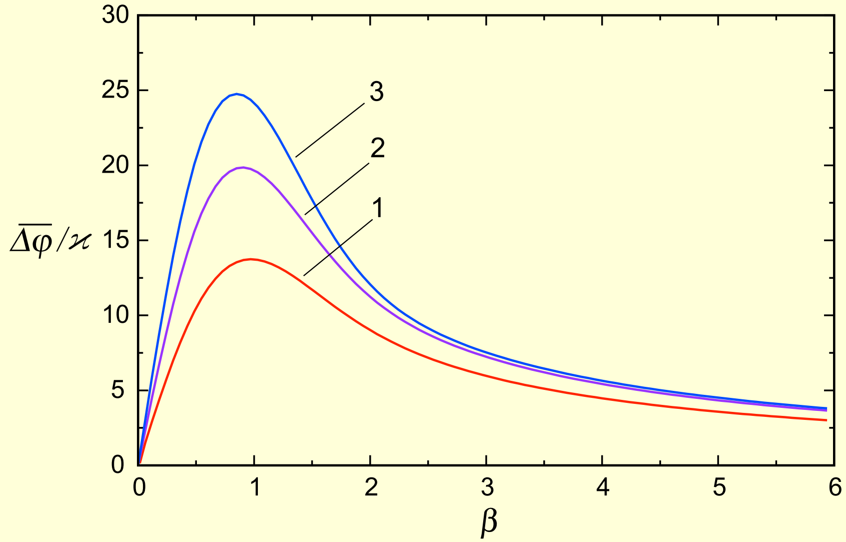


Fig. 2. Dependence of $\overline{\Delta\varphi}$ on the impact parameter for $a^2_{\rm max} = 0.42$ (1), $a^2_{\rm max} = 0.705$ (2), and $a^2_{\rm max} = 0.86$ (3); $\quad v = 0.8.$

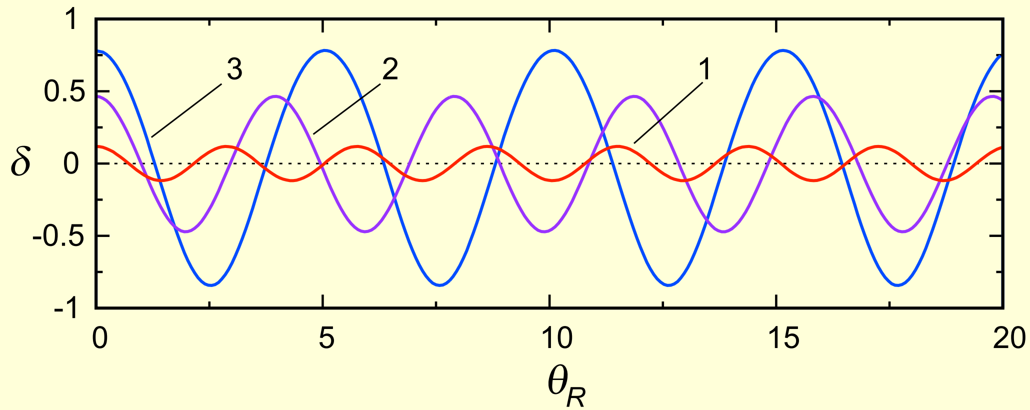


Fig. 3. Deviation of the deflection angle from its averaged value, $\delta = (\Delta\varphi - \overline{\Delta\varphi})/\varkappa$, for $a^2_{\rm max} = 0.42$ (1), $a^2_{\rm max} = 0.705$ (2), and $a^2_{\rm max} = 0.86$ (3); $\quad \beta = 1,\ v = 0.8.$

4 Cosmological implications

The energy density of the lump oscillates in time and decays as

$$T_0^0 \sim m^2 \sigma^2 a^2(\theta) \rho^2 e^{3-\rho^2},$$

where $\theta \simeq mt, \quad \rho = mr \gg 1.$

Therefore, the characteristic size of the lump is $\sim m^{-1}$, and the oscillation period $T_g \approx (2m)^{-1} T$, where $T \sim 10$ is the oscillation period of $a(\theta)$.

For example, for $m \sim 10^{-22}$eV we have the lump of the size ~ 0.06 pc, oscillating with the period ~ 1 year.

To obtain the Sun-sized lump we need to assume $m \approx 2.7 \times 10^{-16}$eV, that gives $T_g \approx 12$ seconds.


Thank you
for your attention!